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A NOVEL CONCEPT FOR FINDING THE FUNDAMENTAL RELATIONS BETWEEN STREAM FUNCTION AND VELOCITY POTENTIAL IN REAL NUMBERS IN TWO-DIMENSIONAL FLUID MOTIONS

Prabir Chandra Bhattacharyya

Professor, Department of Mathematics and Director of Research and Development, Institute of Mechanics of Continua and Mathematical Sciences

P-9/1, LIC Township, Madhyamgram, Kolkata - 700129, India

E-mail : prabirbhattacharyya@yahoo.com

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Abstract

In this paper, the author has presented the fundamental relations between stream function or current function, Ψ *and velocity potential or velocity function,* φ *which are* $\frac{\partial \varphi}{\partial x} = \frac{\partial \Psi}{\partial y}$ *and* $\frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x}$ *where* $x, y, \varphi(x, y)$, $\Psi(x, y)$ *are all real in two-dimensional fluid motions using real variables only whereas these relations had been established by using complex variables by Cauchy – Riemann which are known as Cauchy – Riemann equations in fluid dynamics.*

Keywords: Cauchy – Riemann equations, Quadratic equations, Rectangular Bhattacharyya's Coordinates, Stream function, Theory of Dynamics of Numbers, Velocity potential.

I. Introduction

After the invention of fundamental three new concepts in mathematics namely

1. Theory of Dynamics of Numbers [Prabir Chandra Bhattacharyya. : 'AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS: A NEW CONCEPT'. *J. Mech. Cont. & Math. Sci.*, Vol.-16, No.-11, January (2022). pp 37-53.]
2. (a) Theory of Quadratic Equation – I [Prabir Chandra Bhattacharyya, : 'A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION'. *J. Mech. Cont. & Math. Sci.*, Vol.-17, No.-3, March (2022) pp 41-63.]
2. (b) Theory of Quadratic Equation – II [Prabir Chandra Bhattacharyya, : 'AN OPENING OF A NEW HORIZON IN THE THEORY OF QUADRATIC EQUATION: PURE AND PSEUDO QUADRATIC EQUATION – A NEW CONCEPT', *J. Mech. Cont. & Math. Sci.*, Vol.-17, No.-11, November (2022) pp 1-25]

3. Rectangular Bhattacharyya's Coordinates [Prabir Chandra Bhattacharyya, 'AN INTRODUCTION TO RECTANGULAR BHATTACHARYYA'S CO ORDINATES: A NEW CONCEPT'. *J. Mech. Cont. & Math. Sci.*, Vol.-16, No.-11, November (2021). pp 76-86.]

The author becomes successful to establish the fundamental relations between stream function, Ψ and potential velocity, φ in two-dimensional fluid motions,

i. e. $\frac{\partial \varphi}{\partial x} = \frac{\partial \Psi}{\partial y}$ and $\frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x}$

where, $x, y, \varphi\ (x, y),\ \Psi(x, y)$ are all real in two-dimensional fluid motions using real variables only.

Before going to establish the fundamental relations between stream function and velocity potential it is necessary to discuss fundamental three new concepts in brief.

According to the theory of Dynamics of Numbers 0 (zero) is defined as the starting point of any number. There are two types of countable numbers (1) Count up Numbers (2) Countdown Numbers.

The numbers which move away from the starting point 0 (zero) are called count up numbers. Symbolically denoted as $\overset{\blacktriangle}{1}$, $\overset{\blacktriangle}{2}$, $\overset{\blacktriangle}{3}$, $\overset{\blacktriangle}{4}$, etc, or without any symbol over the head. The numerical value of count up number is defined as a count up number with a prefixed addition operator plus + such as. $\overset{\blacktriangle}{3}$ = + 3

The numbers which move towards the starting point 0 (zero) are defined as countdown numbers. Symbolically denoted as $\overset{\blacktriangledown}{1}$, $\overset{\blacktriangledown}{2}$, $\overset{\blacktriangledown}{3}$, $\overset{\blacktriangledown}{4}$, etc. The numerical value of the countdown number is defined as a countdown number with a prefixed subtraction operator minus, –, such as. $\overset{\blacktriangledown}{3}$ = – 3 These two new symbols '$\blacktriangle$' and '$\blacktriangledown$' have been introduced by the author to make difference with the vectorial notation '$\rightarrow$' and '$\leftarrow$'. Also, the author has developed three laws of the theory of dynamics of numbers. [XXVI]

In the Theory of Quadratic Equation – I, the author solved the quadratic equation $x^2 + 1 = 0$ by introducing the Theory of Dynamics of Numbers. The author found that the inherent nature of unknown quantity *x* is countdown *x*, so the solution of the quadratic equation will be $\overset{\blacktriangledown}{x} = \overset{\blacktriangledown}{1}$ i.e. x = – 1. Here the author finds that the solution of the quadratic equation has one root only though according to the convensional method a quadratic equation must pass two roots. The author introspects the matter of why it happened so.

In the Theory of Quadratic Equation -- II the author proved that the inherent structure of the quadratic equation $\mathrm{x}^2 + 1 = 0$ is two-dimensional. So, the equation $\mathrm{x}^2 + 1 = 0$ is a pure quadratic equation. According to the new concept of the Theory of Quadratic Equation – II, pure quadratic equation has one and only one root. Hence the root x = – 1 of the quadratic equation $x^2 + 1 = 0$ is true and the reason behind it is genuine and scientific. Therefore, the concept of a complex number is not necessary for the solution of the quadratic equation $x^2 + 1 = 0$.

Moreover, to prove the genuinity of the inherent structure of the quadratic equation $x^2 + 1 = 0$, the author developed the 'Rectangular Bhattacharyya's Coordinates where all the abscissas and ordinates in a rectangular plane are positive and all coordinates in the plane are positive on the basis of the Theory of Dynamics of Numbers whereas in Cartesian Coordinates one abscissa is positive and the other abscissa is negative and also one ordinate is positive and the other ordinate is negative in the rectangular plane and the coordinates in the rectangular plane may be both positive or both negative or one positive and the other negative in the same braces by putting a comma between them. The author becomes successful to find the distance between any two points in a rectangular plane with this new concept of Bhattacharyya's Coordinates as it has been done by the Cartesian Coordinates.

In the case of an imaginary number 0 + i0 is undefined and there is no order relation in imaginary numbers.

Infine, the author states that it is a novel approach to establish the fundamental relations between stream function and velocity potential in fluid dynamics considering the three basic new concepts in real numbers only.

II. Literature Review

Incompressible (divergence–free) flow in two dimensions and also in three dimensions with axisymmetry is defined as a stream function. The velocity components of the flow can be expressed as the derivatives of the scalar stream function. The stream function is used to plot streamlines, which may be considered for the representation of the trajectories of particles in a steady flow. The two-dimensional Langrage stream function was introduced by Joseph Louis Langrage in 1781 [XVI]. The stokes stream function which is an axisymmetrical three-dimensional flow is named after George Gabriel Stokes [XXXVII]. In the particular case of fluid dynamics, the difference between the stream function values at any two points gives the volumetric flow rate or volumetric flux through a line connecting the two points. The stream function can be used to derive a complex potential when taken together with velocity potential.

The Cauchy-Riemann equations consist of a system of two partial differential equations. These equations together with certain continuity and differentiability criteria form a necessary and sufficient condition for a complex function to be holomorphic. This system of equations was first found in the work of Jean le Rond D'Alembert's [V]. After that Leonhard Euler connected this system to analytic functions [VII]. Cauchy then used these equations to construct his theory of functions [III]. Riemann dissertation on the theory of functions was founded in 1851 [XXXI].

On a pair of real-valued functions of two real variables u (x, y), v (x, y). The Cauchy-Riemann equations are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \tag{1}$$

and

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \tag{2}$$

Typically u and v are taken to be the real and imaginary parts respectively of complex-valued functions of a single complex number :

$$z = x + iy,\ \ f(x + iy) = u\,(x, y) + iv\,(x, y) \text{ and } w = \varphi + i\Psi$$

The property of a complex function of being differentiable at every point of an open set and connected subset of **C** is called holomorphy. Holomorphic functions are analytic and vice versa.

Fluid flow is a complex problem still today. We cannot satisfactorily predict turbulent flow such as the circulation of water in the kitchen from a pressure drop in the pipe. The answer is known to us only by experience, not by reasoning. But how our Indus valley ancestors of India who passed water through cities more than 4000 years ago clearly knew things.

The most appealing thing about turbulence is that a flow we consider random or chaotic may possess concealed order. This type of flow was convincingly demonstrated by Brown and Roshko in 1970. But the man who saw most clearly the fundamentally mathematical nature of the problem was Von Neumann. He thought that if the turbulence problem were solved, its impact on pure mathematics would be even greater than on fluid dynamics.

The role of a complex number is very important in fluid dynamics. Complex numbers were introduced by the Italian mathematician Gerolamo Cardano (1501 – 1576). Euler (1707 – 1783) visualized complex numbers as points with rectangular coordinates. Gauss (1777 – 1855) introduced the term ‘Complex Number’. Cauchy, a contemporary of Gauss extended the concept of complex numbers to complex functions.

Naiver, Stokes, Euler, Neumann, Cauchy, Riemann, and others could not find the solution to the quadratic equation, $x^2 + 1 = 0$ in real numbers, so they used complex numbers to solve any problem in fluid dynamics.

But the present author invented a new concept in the coordinate system where all abscissas and ordinates are positive which is known as ‘Bhattacharyya’s Coordinates’ in the year 2021 [XXV].

The author invented a new concept in number theory as the ‘Theory of Dynamics of Numbers’ in the year 2022 where numbers are dynamic [XXVI].

The author also invented a new concept in “Theory of Quadratic Equation – 1’ [XXVII] and II [XXVIII] where the author has become successful to find the solution of any quadratic equation in real number even if the discriminant of the quadratic equation $b^2 - 4ac < 0$ with the help of the theory of dynamics of numbers in real numbers without using complex numbers.

With these three new concepts namely the Theory of Dynamics of Numbers’, ‘Theory of Quadratic Equation – I and II’ and ‘Bhattacharyya’s Coordinates’ the author has found the solution of the quadratic equation, $x^2 + 1 = 0$ in real numbers only.

By introducing the novel concepts the author has become successful to establish the fundamental relations between the stream function (Ψ) and velocity potential (φ) in fluid dynamics using real numbers only whereas Cauchy – Riemann had achieved the relations using imaginary numbers.

III. Formulation of the problem and its solution.

III.i <u>Some Definitions</u>

(A) <u>Count up straight line</u> :

If a straight line is drawn by taking points moving away from the origin or a fixed point O to another point P, then the straight line OP is called count up OP.

It is symbolically represented as $\overset{\uparrow}{\overline{OP}}$. $\overset{\uparrow}{\overline{OP}}$ means the distance between the points O and P and the direction will be from O to P in fig.. 1.

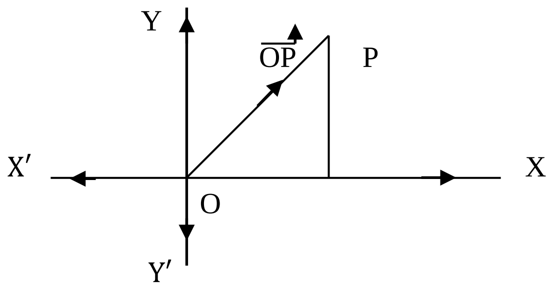

Fig. 1.

(B) **<u>Countdown straight line</u> :**

If a straight line is drawn by taking points moving towards the origin or a fixed point O from another point P, then the straight line OP is called countdown OP.

It is symbolically represented as $\overset{\downarrow}{\overline{OP}}$. $\overset{\downarrow}{\overline{OP}}$ means the distance between the points O and P and the direction will be from the point P to the O as in fig. 2.

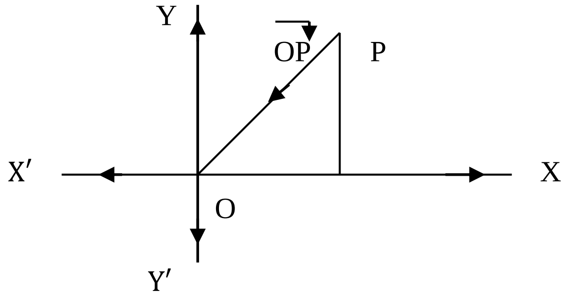

Fig. 2.

Symbols : ‘ $\overset{\uparrow}{\overline{\quad}}$ ’ means a count up straight line that is a bar with an upward arrow.
‘ $\overset{\downarrow}{\overline{\quad}}$ ’ means a countdown straight line that is a bar with downward arrow.

Note that $\overset{\downarrow}{\overline{OP}}$ is vertically opposite to $\overset{\uparrow}{\overline{OP}}$ having equal distance and $\overset{\downarrow}{\overline{OP}} = -\overset{\uparrow}{\overline{OP}}$.

Note : (1) Significance of the forward arrow (→) and backward arrow (←) over the head of a line means the direction of the line only.

(2) If there is no symbol or a bar only over the head of a line such as OP or $\overline{OP}$ means a line with the distance between two points O and P only.

III.ii. Motion in two – dimension :

The motion of a fluid is said to be two-dimensional when a fluid moves in such a way that at any instant the flow pattern in a certain plane (say XOY) is the same as that in all other parallel planes within the fluid. For maintaining physical reality we assume that the fluid in two-dimensional motion is confined between two planes parallel to the plane of motion and at a unit distance apart.

Method to find stream function or current function :

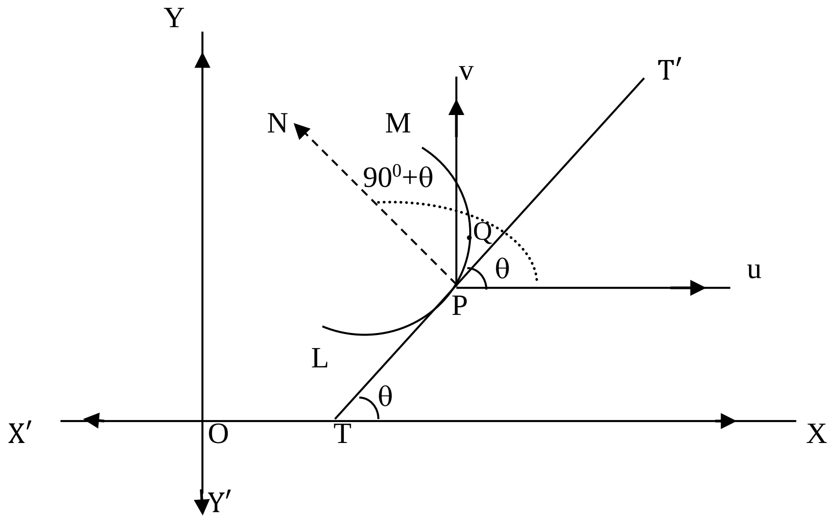

Fig. 3

Let LM be any curve in the X – Y plane. Let P be any arbitrary point on LM such that arc LM = s and let Q be a neighbouring point on LM such that arc LQ = s + δs.

Let TPT′ be tangent at the point P on the curve LM.

Let $\angle$ PTX = θ. So, The angle between that tangent at P and x axis is θ.

Let u = (x, y) and v = (x, y) be the velocity components at P.

Let PN be the inward drawn normal at P of the curve LM. So, PN is perpendicular on TPT′ at P.

Let

$$z^2 = x^2 + y^2 \, . \tag{1}$$

Then the differential equation of equation (1) will be

$$zdz = xdx + ydy \tag{2}$$

Suppose,

$$z^2 = -1 \tag{3}$$

$$\therefore \; z^2 + 1 = 0 \tag{4}$$

According to the theory of the dynamics of numbers, equation (4) takes the form

$$z^2 \; + \; 1 = 0$$

According to 3rd law of the theory of dynamics of numbers

$$z^2 = 1$$

$$\text{or, } z = 1$$

$$\therefore \; z \; = \; 1 \; = \; -1$$

So, the solution of equation (4) will be

$$z = -1 \tag{5}$$

We have

$$z = \sqrt{x^2 + y^2}$$

$$\therefore \partial z = \partial \sqrt{x^2 + y^2}$$

$$\partial\,(-1) = \frac{1}{2} \cdot \frac{2x\,\partial x}{\sqrt{x^2 + y^2}} + \frac{1}{2} \cdot \frac{2y\,\partial y}{\sqrt{x^2 + y^2}}$$

$$\text{or, } 0 = \frac{x}{\sqrt{x^2 + y^2}}\,\partial x + \frac{y}{\sqrt{x^2 + y^2}}\,\partial y$$

Again we have

$\frac{x}{\sqrt{x^2+y^2}} = \text{Cos}\,(90^0+\theta) = -\text{Sin}\theta$ and $\frac{y}{\sqrt{x^2+y^2}} = \text{Sin}\,(90^0+\theta) = \text{Cos}\,\theta$ [$\because$z = – 1 and considering x, y w.r.t. u, v and normal PN of the curve]

$$0 = -\,\partial x\,\text{Sin}\theta + \partial y\,\text{Cos}\,\theta$$

So, we have

$$\partial x\,\text{Sin}\,\theta = \partial y\,\text{Cos}\,\theta$$

Again we have

$$\frac{dy}{dx} = \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta} = \text{Tan}\,\theta = \frac{v}{u}$$

[$\because$ angle between u and v is 90^0 and $\text{Tan}\,\theta = \frac{v\,\text{Sin}\,90}{u+v\,\text{Cos}\,90} = \frac{v}{u}$]

or,

$$\frac{dx}{u} = \frac{dy}{v} \tag{6}$$

or,

$$vdx - udy = 0 \tag{7}$$

We know the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8a}$$

or,

$$\frac{\partial v}{\partial y} = \frac{\partial\,(-u)}{\partial x} \tag{8b}$$

Equations (8a) and (8b) show that L. H. S, of equation (7) must be exact differential dΨ (say). Thus we have

$$vdx - udy = d\Psi = \frac{\partial\Psi}{\partial x}\,dx + \frac{\partial\Psi}{\partial y}\,dy \tag{9}$$

so that,

$$u = -\,\frac{\partial\Psi}{\partial y} \text{ and } v = \frac{\partial\Psi}{\partial x} \tag{10}$$

This function Ψ is called the stream function. By using equations (7) and (9) the streamlines are given by dΨ = 0, i.e. by equation Ψ = c, where c is an arbitrary constant. Hence the stream function is constant along a streamline. The stream function or current function always exists in all types of two-dimensional motion whether rotational or irrational.

Note that from fig. 3, we can easily deduce that

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} \tag{11}$$

from the relation

$$ydx - xdy = 0$$

Let the velocity components of P are u (x, y, z), v (x, y, z), and w (x, y, z).

Then we get

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \tag{12}$$

Since,

$$(dz)^2 = (dx)^2 + (dy)^2 \text{ and } w^2 = u^2 + v^2$$

Equation (12) is known as the equation of streamline or line of flow.

IV. Method to find the Velocity Potential or Velocity Function :

Let us suppose that the fluid velocity at time t is q = (u, v, w). Also suppose that at the considered instant t, there exists a scalar function φ (x, y, z, t) uniform throughout the entire field of flow such that

$$-\, d\varphi = udx + vdy + wdz \tag{1}$$

or,

$$-\, d\varphi = -\left(\frac{\partial\varphi}{\partial x}\, dx + \frac{\partial\varphi}{\partial y}\, dy + \frac{\partial\varphi}{\partial z}\, dz\right) \tag{2}$$

Then the expression on the R. H. S. of equation (1) is an exact differential and we get

$$u = -\frac{\partial\varphi}{\partial x}\, , v = -\frac{\partial\varphi}{\partial y} \text{ and } w = -\frac{\partial\varphi}{\partial z} \tag{3}$$

So,

$$q = -\nabla\, \varphi = -\text{grad}\ \varphi \tag{4}$$

φ is called the velocity potential. The negative sign in equation (4) is a convension. It implies that the flow takes place from higher potentials to lower potentials.

To hold equation (4), the necessary and sufficient condition is

$$\nabla \mathrm{X}\, q = 0 \qquad \text{or} \quad \text{Curl } q = 0 \tag{5}$$

or,

$$i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + j\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + k\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \tag{6}$$

Now, the surface, φ (x, y, z, t) = constant (7)

are called equipotential. The streamlines

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \tag{8}$$

are cut at right angles by the surface given by the differential equation

$$udx + vdy + wdz = 0 \tag{9}$$

and the condition for the existence of such orthogonal surfaces is the condition that equation (9) may possess a solution of the form (7) at the considered instant t, the condition is.

$$u\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + v\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + w\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \tag{10}$$

when the velocity potential exists, equation (3) holds.

Then

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = -\frac{\partial^2 \varphi}{\partial y\, \partial z} + \frac{\partial^2 \varphi}{\partial z\, \partial y} = 0 \tag{11}$$

Therefore,

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \tag{12}$$

Similarly

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \quad \text{and} \quad \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \tag{13}$$

Using equations (12) and (13), we find that condition (10) is satisfied. Hence the surface exists which cut the streamlines orthogonally. We also conclude that at all points of the field of flow, the equipotential is cut orthogonally by the streamlines.

V. Fundamental relation between φ and Ψ in two-dimensional fluid motions.

Suppose that $z^2 = x^2 + y^2$ and $w^2 = \varphi^2 + \Psi^2$ and $w^2 = -1$ subject to prove that w is a real number where x, y, and φ, Ψ are all real numbers.

Now,

$$w^2 + 1 = 0 \tag{1}$$

According to the 3rd law of the theory of dynamics of numbers, equation (1) takes the form

$$\overset{\downarrow}{w^2} + \overset{\uparrow}{1} = 0$$

According to 3rd law of the theory of dynamics of numbers

$w^2 = 1$

or, $w = 1$

$\therefore\ w = 1 = -1$

$\therefore\ w = -1$ is the solution of equation (1)

$w = -1$ means the distance of w is one unit but the direction of w is vertically opposite to w.

Hence, $w = -1$ is also a real number science 1 is a real number having its real existence.

Suppose that

$$w^2 = \varphi^2 + \Psi^2 \tag{2}$$

or,

$$w = \sqrt{\varphi^2 + \Psi^2} \tag{3}$$

We have

$$\frac{dy}{dx} = \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta} = \text{Tan}\,\theta = \frac{\Psi}{\varphi}$$

So,

$$\frac{dx}{\varphi} = \frac{dy}{\Psi}$$

We know that

$$(dx)^2 + (dy)^2 = (dz)^2 \tag{5}$$

and using the equations (2), (3) and (5), we get

$$\frac{dx}{\varphi} = \frac{dy}{\Psi} = \frac{dz}{w} \tag{6}$$

Equation (6) shows that φ and Ψ and their first-order derivatives are continuous within a given region i.e., at any point of the region specified by z. From equation (6) we may say that the necessary and sufficient conditions are satisfied for the existence of an irrational motion in two dimensions so that the velocity potential exists such that

$$u = -\frac{\partial\varphi}{\partial x} \tag{7a}$$

and

$$v = -\frac{\partial\varphi}{\partial y} \tag{7b}$$

and also in two-dimensional flow, the stream function exists such that

$$u = -\frac{\partial \Psi}{\partial y} \tag{8a}$$

and

$$v = \frac{\partial \Psi}{\partial x} \tag{8b}$$

Therefore, from equations (7a) and (8a) and also from equations (7b) and (8b) we get

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \Psi}{\partial y} \tag{9a}$$

and

$$\frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x} \tag{9b}$$

The equations (9a) and (9b) are fundamental relations between stream function (Ψ) and velocity potential (φ) in two-dimensional irrotational fluid motions.

Problem – 1

Prove or verify the fundamental relations between stream function and velocity potential in real variables in two-dimensional fluid motion given by $\varphi^2 + \Psi^2 = -4$.

Solution :

Suppose,

$$w^2 = \varphi^2 + \Psi^2 = -4 \tag{1}$$

Now,

$$w^2 = -4 \tag{2}$$

or,

$$w^2 + 4 = 0 \tag{3}$$

According to the theory of dynamics of numbers, equation (3) takes the form

$$w^2 + 4 = 0 \tag{4}$$

According To the 3rd law of the theory of dynamics of numbers we have

$w^2 = 4$

or, $w = 2$

So the solution to equation (4) will be

$w = 2 = -2$

Therefore, the solution to equation (2) is

$$w = -2 \tag{5}$$

We know that

$$\frac{\mathrm{dx}}{\varphi} = \frac{\mathrm{dy}}{\Psi} = \frac{\mathrm{dz}}{\mathrm{w}} = \frac{\mathrm{dz}}{-2} \tag{6}$$

$$\therefore \frac{\varphi}{-2} = \frac{\mathrm{dx}}{\mathrm{dz}} \tag{7}$$

Also, we know that

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{dz}{\sqrt{x^2+y^2}} \tag{8}$$

$$\therefore \frac{\mathrm{dx}}{\mathrm{dz}} = \frac{\mathrm{x}}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}} \tag{9}$$

$$\therefore \varphi = -\frac{2\mathrm{x}}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}} \tag{10}$$

Again from equation (6) we have

$$\frac{\Psi}{-2} = \frac{\mathrm{dy}}{\mathrm{dz}} \tag{11}$$

and, from equation (8) we have,

$$\frac{\mathrm{dy}}{\mathrm{dz}} = \frac{\mathrm{y}}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}}$$

$$\therefore \Psi = -\frac{2\mathrm{y}}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}} \tag{12}$$

From equation (10) we have

$$\varphi = -\frac{2\mathrm{x}}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}}$$

$$\therefore \frac{\partial\varphi}{\partial\mathrm{x}} = -\left[\sqrt{\mathrm{x}^2+\mathrm{y}^2}\,\frac{\mathrm{d(2x)}}{\mathrm{dx}} - 2\mathrm{x}.\frac{\mathrm{d}\sqrt{\mathrm{x}^2+\mathrm{y}^2}}{\mathrm{dx}}\right] / \left(\sqrt{\mathrm{x}^2+\mathrm{y}^2}\right)^2$$

$$= -\left[\sqrt{\mathrm{x}^2+\mathrm{y}^2}\,.2 - 2\mathrm{x}.\frac{1}{2}\,(\mathrm{x}^2+\mathrm{y}^2)^{-\frac{1}{2}}\{2\mathrm{x}+2\mathrm{y}\frac{\mathrm{dy}}{\mathrm{dx}}\}\right] / (\mathrm{x}^2+\mathrm{y}^2)$$

$$= -2\left[\sqrt{\mathrm{x}^2+\mathrm{y}^2} - \frac{x}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}}\,.\left\{x + y.\frac{y}{x}\right\}\right] / (\mathrm{x}^2+\mathrm{y}^2)$$

$$= -2\left[\sqrt{\mathrm{x}^2+\mathrm{y}^2} - \frac{x^2+y^2}{\sqrt{\mathrm{x}^2+\mathrm{y}^2}}\right] / (\mathrm{x}^2+\mathrm{y}^2)$$

$$= -2\frac{\left[\sqrt{\mathrm{x}^2+\mathrm{y}^2} - \sqrt{\mathrm{x}^2+\mathrm{y}^2}\right]}{\mathrm{x}^2+\mathrm{y}^2}$$

$$= \frac{0}{\mathrm{x}^2+\mathrm{y}^2} = 0$$

$$\therefore \frac{\partial\varphi}{\partial x} = 0 \tag{13}$$

or,

$$\mathrm{d}\,\varphi = 0 \tag{14}$$

On integration we have,

$$\varphi = C_1, \text{ constant} \tag{15}$$

Similarly, from equation (12) we have

$$\Psi = \frac{-2y}{\sqrt{x^2+y^2}}$$

$$\therefore \frac{d\Psi}{dy} = -\left[\sqrt{x^2+y^2}.2 - 2y\left\{\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\left(2x\frac{dx}{dy}+2y\right)\right\}\right]/(\sqrt{x^2+y^2})^2$$

$$= -2\left[\sqrt{x^2+y^2} - \frac{y}{\sqrt{x^2+y^2}}\left(x.\frac{x}{y}+y\right)\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2} - \frac{y}{\sqrt{x^2+y^2}}.\frac{x^2+y^2}{y}\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\right]/(x^2+y^2)$$

$$= \frac{0}{x^2+y^2} = 0$$

$$\therefore \frac{\partial\Psi}{\partial y} = 0 \tag{16}$$

Or,

$$d\Psi = 0 \tag{17}$$

On integration we have

$$\Psi = C_2, \text{ constant} \tag{18}$$

Therefore, from equations (13) and (16) we have

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\Psi}{\partial y} \tag{19}$$

Again from equations (10) and (12) we have

$$\varphi = -\frac{2x}{\sqrt{x^2+y^2}} \quad \text{and} \quad \Psi = -\frac{2y}{\sqrt{x^2+y^2}}$$

Then,

$$\frac{\partial\varphi}{\partial y} = -\left[\sqrt{x^2+y^2}.\frac{d\,(2x)}{dy} - 2x.\frac{d}{dy}\left(\sqrt{x^2+y^2}\right)\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2}.\frac{d\,x}{dy} - x.\frac{1}{2}\,(x^2+y^2)^{-\frac{1}{2}}\{2x\frac{dx}{dy}+2y\}\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2}.\frac{x}{y} - \frac{x}{\sqrt{x^2+y^2}}\left(x.\frac{x}{y}+y\right)\right]/(x^2+y^2)$$

$$= -2\left[\frac{x}{y}.\sqrt{x^2+y^2} - \frac{x}{y}.\frac{x^2+y^2}{\sqrt{x^2+y^2}}\right]/(x^2+y^2)$$

$$= -\frac{x}{y}.2\left[\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\right]/(x^2+y^2)$$

$$\therefore \frac{y}{x}.\frac{\partial\varphi}{\partial y} = \frac{0}{x^2+y^2} = 0$$

or,

$$\frac{dy}{dx}.\frac{\partial\varphi}{\partial y} = 0 \qquad \left[\because \frac{y}{x} = \frac{dy}{dx}\right] \tag{20}$$

Again,

$$\frac{\partial\Psi}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{2y}{\sqrt{x^2+y^2}}\right]$$

$$= -\left[\sqrt{x^2+y^2}.\frac{d(2y)}{dx} - 2y.\frac{d}{dx}\left(\sqrt{x^2+y^2}\right)\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2}.\frac{dy}{dx} - y.\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\{2x+2y\frac{dy}{dx}\}\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2}.\frac{y}{x} - \frac{y}{\sqrt{x^2+y^2}}\left\{x+y.\frac{y}{x}\right\}\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2}.\frac{y}{x} - \frac{y}{\sqrt{x^2+y^2}}\left\{\frac{x^2+y^2}{x}\right\}\right]/(x^2+y^2)$$

$$= -2\left[\sqrt{x^2+y^2}.\frac{y}{x} - \frac{y}{x}.\sqrt{x^2+y^2}\right]/(x^2+y^2)$$

$$= -\frac{y}{x}.2\left[\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\right]/(x^2+y^2)$$

$$\therefore \frac{x}{y}\frac{\partial\Psi}{\partial x} = \frac{0}{x^2+y^2} = 0$$

or,

$$\frac{x}{y}\frac{\partial\Psi}{\partial x} = 0$$

or,

$$\frac{dx}{dy}\frac{\partial\Psi}{\partial x} = 0 \qquad \left[\because \frac{x}{y} = \frac{dx}{dy}\right] \tag{21}$$

From equation (20) and (21) we have

$$\frac{dy}{dx}\frac{\partial\varphi}{\partial y} = \frac{dx}{dy}.\frac{\partial\Psi}{\partial x} \tag{22}$$

Since Ψ is an exact differential,

$$\frac{\partial\Psi}{\partial x}dx + \frac{\partial\Psi}{\partial y}dy = 0 \tag{23}$$

and we know

$$u = -\frac{\partial \varphi}{\partial x} \text{ and } v = -\frac{\partial \varphi}{\partial y}$$

and vdx – udy = 0

$$\text{So, } -\frac{\partial \varphi}{\partial y}\, dx + \frac{\partial \varphi}{\partial x}\, dy = 0 \tag{24}$$

from equation (23) we have

$$\frac{dx}{dy} - \frac{\partial \Psi}{\partial y} \Big/ \frac{\partial \Psi}{\partial x} \tag{25}$$

and from equation (24) we have

$$\frac{dy}{dx} = \frac{\partial \varphi}{\partial y} \Big/ \frac{\partial \varphi}{\partial x} \tag{26}$$

Now, putting the values of $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in equation (22) we have

$$\frac{\frac{\partial \varphi}{\partial y}}{\frac{\partial \varphi}{\partial x}} \cdot \frac{\partial \varphi}{\partial y} = -\frac{\frac{\partial \Psi}{\partial y}}{\frac{\partial \Psi}{\partial x}} \cdot \frac{\partial \Psi}{\partial x}$$

or,

$$\left(\frac{\partial \varphi}{\partial y} \cdot \frac{\partial \Psi}{\partial x}\right) \cdot \frac{\partial \varphi}{\partial y} = -\left(\frac{\partial \Psi}{\partial y} \cdot \frac{\partial \varphi}{\partial x}\right) \cdot \frac{\partial \Psi}{\partial x}$$

or,

$$-1 . \frac{\partial \varphi}{\partial y} = - . (-1) . \frac{\partial \Psi}{\partial x} \quad [\because \text{the product of two perpendicular slopes} = -1]$$

or,

$$\frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x}$$

Hence the proof

VI. Conclusion

Fundamental relations between streamline (Ψ) and velocity potential (φ) in two-dimensional irrotational fluid motions can be obtained without using the concept of complex variables. This becomes possible after the invention of three new concepts in mathematics namely (1) Theory of Dynamics of Numbers, (2) Theory Quadratic Equations – I and II (3) Rectangular Bhattacharyya's Coordinates by the same author. These new concepts need a wide application in any branch of mathematics, science, and technology.

Conflict of Interest:

The authors declare that no conflict of interest to report the present study.

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