



## SIMULATION OF WAVE SOLUTIONS OF A MATHEMATICAL MODEL REPRESENTING ELECTRICAL ENGINEERING BY USING AN ANALYTICAL TECHNIQUE

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### Abstract

*The existing article examines the mathematical model (MM) representing electrical engineering (EE). We implement the unified technique (UT) to discover new wave solutions (WS) and to erect numerous kinds of solitary wave phenomena (SWP) for the studied model (SM). The SM is one of the models that have vital applications in the area of EE. The taken features provide a firm mathematical framework and may be necessary to the WSs. As an outcome, we get new kinds of WSs from. With 3-d, density, contour, and 2-d for different values of time parameters, mathematical effects explicitly manifest the suggested algorithm's full reliability and large display. We implement a few figures in 3-d, density, contour, and 2-d for diverse values of time parameters to express that these answers have the properties of soliton waves.*

**Keywords:** The UT method; MM; the modified Zakharov-Kuznetsov equation, EE, WSs.

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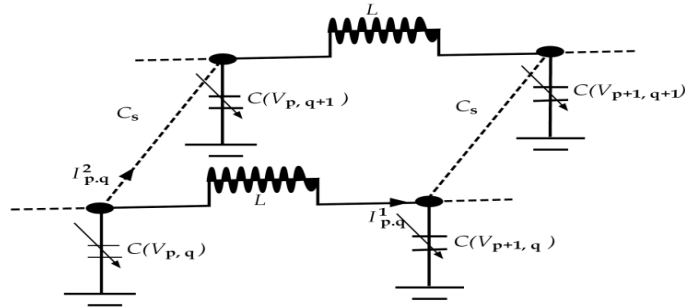
### I. Introduction

Newly, it is an interesting zone for scientists, researchers, mathematicians, and scholars to perform WSs to MMs with the partner of computational packages which are simply monotonous, uniform, and tedious mathematical computations. MMs show a fabulous performance to express the physical devices of natural aspects and dynamic rules in engineering, applied mathematics, physics, and numerous other experimental areas. Studying and investigating the computational solutions of these models is considered one of many researchers' basic interests. Consistent with these computational solutions, many engineers, mathematicians and physicians established a few approaches and still annoying to discover novel general approaches to get computational solutions of these models, for example, the  $(\Phi, \Psi)$ -expansion scheme [XXV, XX], hybrid B-spline collocation technique [XXXIV], Lie symmetry analysis [XXVIII], the modified Khater method [XXXV], the generalized explanatory rational task process [XXXI], Hirota direct method [XVII] the  $(1/G')$ -extension process

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[XXXII], rational ansatz method [XV], adapted (G'/G)-extension process [XVIX, XXIII, XXI, XXIV], the modified Sardar sub-equation method [XXXIX], the unified technique [VI, XXVI, XIV, XXVII, I], Adomian decomposition method [IX], Bernstein approximation method [XI], complex envelop antazs [XL], The first integral process [XVIII], new extended direct algebraic method [XXXVIII], enhanced improved simple equation process [XLII], modified exp-function method [XXIX, XXX], the bifurcation process [XXXVII], multiple Exp-function process [XVI], reproducing kernel process [II], Sine-Gordon expansion method [V], the GK method [XXII] and many more.

The purpose of this paper is to give the UT [VI] and the Hamiltonian system [VIII, VII] to determine ESs for a discrete nonlinear transmission line (NLTL) model [XII, XXXVI, III]. The above model is also recognized through the modified ZK equation that aids in explaining the device of diverse aspects [XLI, XXXIII, X, IV] in addition to defining the progress of weakly non-linear ion-acoustic waves in a plasma containing hot iso-thermal electrons and cold-ions in the attendance of a uniform magnetic field in the x-direction. MMs have been studied as fundamental in various applications. This model has been applied to express multiple physical phenomena, natural, engineering, and mechanical. That appears because it includes previously unknown multi-variable functions and their derivatives. Such as the electrical transmission (ET) lines, which are discussed as a helpful standard of structures for exploring non-linear excitations, perform inside non-linear media, as nominated in Figure 1.



**Fig. 1.** Linear illustration of the non-linear ET line.

The non-linear ET line is made based on periodically stacking with var-actors or through organizing inductors and var-actors in a 1D lattice. The non-linear network with a few couple non-linear LC with a dispersive ET line has contained this MM. Numerous alike dispersive ET lines are joined through capacitance  $C_s$  at each node, as signified in Figure 1, where a conductor L and a non-linear capacitor of capacitance  $C(V_{p,q})$  are in each line in the shunt branch.

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The MM which signifies the discrete NLT is conferred via the mZK equation that is conveyed via Duan [XIII] when he employed the Kirchhoff law on the MM, is delivered:

$$\frac{\partial^2 R_{p,q}}{\partial S^2} = \frac{1}{L} (V_{p+1,q} - 2V_{p,q} + V_{p-1,q}) + C_s \frac{\partial^2}{\partial S^2} (V_{p,q+1} - 2V_{p,q} + V_{p,q-1}), \quad (1)$$

where  $V_{p,q} = V_{p,q}(S)$  is the voltage wherein the non-linear charge is solved as

$$R_{p,q} = C_0 \left( V_{p,q} + \frac{\alpha_1}{2} V_{p,q}^2 + \frac{\alpha_2}{3} V_{p,q}^3 \right), \quad (2)$$

where  $\alpha_1, \alpha_2$  are constants. From (2) into (1), we have:

$$\begin{aligned} C_0 \frac{\partial^2}{\partial S^2} \left( V_{p,q} + \frac{\alpha_1}{2} V_{p,q}^2 + \frac{\alpha_2}{3} V_{p,q}^3 \right) \\ = \frac{1}{L} (V_{p+1,q} - 2V_{p,q} + V_{p-1,q}) + C_s \frac{\partial^2}{\partial S^2} (V_{p,q+1} - 2V_{p,q} + V_{p,q-1}) \end{aligned} \quad (3)$$

Exchanging  $V_{p,q}(S) = V(p, q, S)$ , we find:

$$C_0 \frac{\partial^2}{\partial S^2} \left( V + \frac{\alpha_1}{2} V^2 + \frac{\alpha_2}{3} V^3 \right) = \frac{1}{L} \frac{\partial^2}{\partial p^2} \left( V + \frac{1}{12} \frac{\partial^2}{\partial p^2} \right) + C_s \frac{\partial^4}{\partial S^2 \partial q^2} \left( V + \frac{1}{12} \frac{\partial^2 V}{\partial q^2} \right) \quad (4)$$

Sector 2 displays the UT. And the new WSs of MM representing EE are defined as implementing the UT in Sector 3. Section 4 offerings the numerical simulations (NS) of the acquired WSs. Finally, in Sector 5, the conclusion is nominated.

## II. The UT

We announce the general form MM:

$$\Xi(R(x, y, t), R_t(x, y, t), R_x(x, y, t), R_{xx}(x, y, t), R_{xxx}(x, y, t), R_y(x, y, t), R_{yy}(x, y, t), \dots) = 0, \quad (5)$$

Now we announce the technique of transformation:

$$R = R(x, y, t) = R(\Theta), \quad \Theta = k_1 x + k_2 y + k_3 t \quad (6)$$

From (6) into (5), we get:

$$\Gamma(R, k_3 R', k_1 R', \dots) = 0. \quad (7)$$

According to UT, we find:

$$R(\Theta) = E_0 + \sum_{i=1}^N (E_i \Omega^i + E_{-i} \Omega^{-i}), \quad i = 0, 1, 2, \dots, N \quad (8)$$

whose  $E_i$  and  $E_{-i}$  are quantities to be considered consequently on the situation that  $E_i$  and  $E_{-i}$  cannot be zero at a time. The task  $\Omega(\Theta)$  contains the Riccati equation

$$\Omega' = \Omega^2 + \lambda \quad (9)$$

whose solutions are specified as follows:

when  $\lambda < 0$ , we have:

$$\Omega(\Theta) = \begin{cases} \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \\ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \\ \frac{\sqrt{-\lambda} \{ \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta))} \\ \frac{\sqrt{-\lambda} \{ -\cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) + \sinh(2\sqrt{-\lambda}(\Theta + \Delta))} \end{cases}$$

when  $\lambda > 0$ , we have:

$$\Omega(\Theta) = \begin{cases} \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Theta + \Delta)) + \Pi} \\ \frac{-\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Theta + \Delta)) + \Pi} \\ \frac{\sqrt{\lambda} \{ i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) - i\Psi \}}{\Psi + \cos(2\sqrt{\lambda}(\Theta + \Delta)) - i \sin(2\sqrt{\lambda}(\Theta + \Delta))} \\ \frac{\sqrt{\lambda} \{ -i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) + i\Psi \}}{\Psi + \cos(2\sqrt{\lambda}(\Theta + \Delta)) + i \sin(2\sqrt{\lambda}(\Theta + \Delta))} \end{cases}$$

when  $\lambda = 0$ , we have:

$$\Omega(\Theta) = -\frac{1}{\Theta + \Delta}.$$

From (9), (8) and (7) and assembling all terms through the same order of  $\Omega$  together, the left-hand side of (7) is adapted into polynomial in  $\Omega$ . Associating each quantity of the polynomial to zero, we can acquire a set of algebraic equations which can be explained to discovery the values of the UT.

### III. The new WSs of MM representing EE are defined implementing the UT

Based on the reductive perturbation system, then (4) is transfer to the MM:

$$R_t + f_1 \phi R R_x + q R^2 R_x + d R_{xxx} + g R_{xyy} = 0, \quad (10)$$

where

$$y = \sqrt{\gamma} q, x = \sqrt{\gamma} (p - v_s S), t = \sqrt{\gamma} S, V(p, q, S) = \gamma R(x, y, t), v_s^2 = \frac{1}{LC_0}, f_1 = -\alpha_1 v_s, q = -\alpha_2 v_s, \\ d = \frac{1}{24 \alpha \alpha_1 L v_s}, g = \frac{\alpha_1}{288 L^2 v_s C_0^2}.$$

Substituting (6) into (10) and integrating, then we get:

$$6k_3 R + 3f_1 k_1 R^2 + 2q k_1 R^3 + 6k_1 (dk_1^2 + gk_2^2) R'' = 0. \quad (11)$$

According to the UT, we have:

$$R = E_{-1} \Omega^{-1} + E_0 \Omega^0 + E_1 \Omega^1 \quad (12)$$

Collecting the coefficient of  $\Omega$  and solving them to zero, then we find:

**Cluster I:**

$$E_0 = -\frac{f_1}{2q}, \quad E_1 = 0, \quad E_{-1} = \frac{(\pm\sqrt{-\lambda})f_1}{2q}, \quad k_2 = \pm\sqrt{\frac{-24d\lambda q k_1^2 + f_1^2}{24g\lambda q}}, \\ k_3 = \frac{f_1^2 k_1}{6q}.$$

Putting these values in (12), then we achieve:

$$R_{11}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right]^{-1}, \\ R_{12}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right]^{-1}, \\ R_{13}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{\sqrt{-\lambda} \{ \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta))} \right]^{-1}.$$

$$\begin{aligned}
 R_{14}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{\sqrt{-\lambda}\{\Psi - \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta))\}}{\Psi + \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) + \sinh(2\sqrt{-\lambda}(\Theta + \Delta))} \right]^{-1}. \\
 R_{15}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Theta + \Delta)) + \Pi} \right]^{-1}. \\
 R_{16}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{-\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Theta + \Delta)) + \Pi} \right]^{-1}. \\
 R_{17}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{\sqrt{\lambda}\{i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) - i\Psi\}}{\Psi + \cos(2\sqrt{\lambda}(\Theta + \Delta)) - i \sin(2\sqrt{\lambda}(\Theta + \Delta))} \right]^{-1}. \\
 R_{18}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{\sqrt{\lambda}\{i\Psi - i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta))\}}{\Psi + \cos(2\sqrt{\lambda}(\Theta + \Delta)) + i \sin(2\sqrt{\lambda}(\Theta + \Delta))} \right]^{-1}. \\
 R_{19}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{-\lambda})f_1}{2q} \left[ \frac{1}{(\Theta + \Delta)} \right]^{-1}.
 \end{aligned}$$

**Cluster II:**

$$\begin{aligned}
 E_0 &= -\frac{f_1}{2q}, \quad E_{-1} = 0, \quad E_1 = \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q}, \quad k_2 = \pm\sqrt{\frac{-24d\lambda q k_1^2 + f_1^2}{24g\lambda q}}, \\
 k_3 &= \frac{f_1^2 k_1}{6q}.
 \end{aligned}$$

Similarly, we get:

$$\begin{aligned}
 R_{21}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right\} \right]. \\
 R_{22}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right\} \right]. \\
 R_{23}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{\sqrt{-\lambda}\{\cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi\}}{\Psi + \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta))} \right\} \right].
 \end{aligned}$$

$$\begin{aligned}
 R_{24}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{\sqrt{-\lambda} \{ \Psi - \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) + \sinh(2\sqrt{-\lambda}(\Theta + \Delta))} \right\} \right]. \\
 R_{25}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Theta + \Delta)) + \Pi} \right\} \right]. \\
 R_{28}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{\sqrt{\lambda} \{ i\Psi - i\cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) \}}{\Psi + \cos(2\sqrt{\lambda}(\Theta + \Delta)) + i\sin(2\sqrt{\lambda}(\Theta + \Delta))} \right\} \right]. \\
 R_{29}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{4\lambda}})f_1}{q} \left[ \left\{ \frac{1}{(\Theta + \Delta)} \right\} \right].
 \end{aligned}$$

**Cluster III:**

$$\begin{aligned}
 E_0 &= -\frac{f_1}{2q}, \quad E_{-1} = \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q}, \quad E_1 = \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q}, \quad k_2 = \pm\sqrt{\frac{-48d\lambda q k_1^2 - f_1^2}{48g\lambda q}}, \\
 k_3 &= \frac{f_1^2 k_1}{6q}.
 \end{aligned}$$

Similarly, we get:

$$\begin{aligned}
 R_{31}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \left\{ \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right\} \\
 &\quad + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right\}^{-1}. \\
 R_{32}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \left\{ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right\} \\
 &\quad + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Pi} \right\}^{-1}.
 \end{aligned}$$

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$$R_{33}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{-\lambda} \{ \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) - \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\} \\ + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{\sqrt{-\lambda} \{ \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) - \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\}^{-1}.$$

$$R_{34}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{-\lambda} \{ -\cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) + \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\} \\ + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{\sqrt{-\lambda} \{ -\cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) + \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\}^{-1}.$$

$$R_{35}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\} \\ + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\}^{-1}.$$

$$R_{36}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{-\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + C)) + \Pi} \right\} \\ + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{-\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\}^{-1}.$$

$$R_{37}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{\lambda} \{ i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) - i\Psi \}}{\Psi + \cos(2\sqrt{\lambda}(\Phi + \Delta)) - i \sin(2\sqrt{\lambda}(\Phi + \Delta))} \right\} \\ + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{\sqrt{\lambda} \{ i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) - i\Psi \}}{\Psi + \cos(2\sqrt{\lambda}(\Phi + \Delta)) - i \sin(2\sqrt{\lambda}(\Phi + \Delta))} \right\}^{-1}.$$



$$R_{38}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{\lambda}\{-i\cos(2\sqrt{\lambda}(\Theta+\Delta)) + \sin(2\sqrt{\lambda}(\Theta+\Delta)) + i\Psi\}}{\Psi + \cos(2\sqrt{\lambda}(\Phi+\Delta)) + i\sin(2\sqrt{\lambda}(\Phi+\Delta))} \right\} \\ + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{\sqrt{\lambda}\{-i\cos(2\sqrt{\lambda}(\Theta+\Delta)) + \sin(2\sqrt{\lambda}(\Theta+\Delta)) + i\Psi\}}{\Psi + \cos(2\sqrt{\lambda}(\Phi+\Delta)) + i\sin(2\sqrt{\lambda}(\Phi+\Delta))} \right\}^{-1}.$$

$$R_{39}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{1}{8\lambda}})f_1}{q} \times \left\{ \frac{1}{\Phi+\Delta} \right\} + \frac{f_1}{8(\pm\sqrt{\frac{1}{8\lambda}})q} \times \left\{ \frac{1}{\Phi+\Delta} \right\}^{-1}.$$

**Cluster IV:**

$$E_0 = -\frac{f_1}{2q}, \quad E_{-1} = \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q}, \quad E_1 = \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q},$$

$$k_2 = \pm\sqrt{\frac{-96d\lambda q k_1^2 + f_1^2}{96g\lambda q}}, \quad k_3 = \frac{f_1^2 k_1}{6q}.$$

Similarly, we get:

$$R_{41}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta+\Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Phi+\Delta)) + \Pi} \right\} \\ + \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta+\Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Phi+\Delta)) + \Pi} \right\}^{-1}.$$

$$R_{42}(\Theta) = -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta+\Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Phi+\Delta)) + \Pi} \right\} \\ + \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{-\sqrt{-(\Psi^2 + \Pi^2)\lambda} - \Psi\sqrt{-\lambda} \cosh(2\sqrt{-\lambda}(\Theta+\Delta))}{\Psi \sinh(2\sqrt{-\lambda}(\Phi+\Delta)) + \Pi} \right\}^{-1}.$$

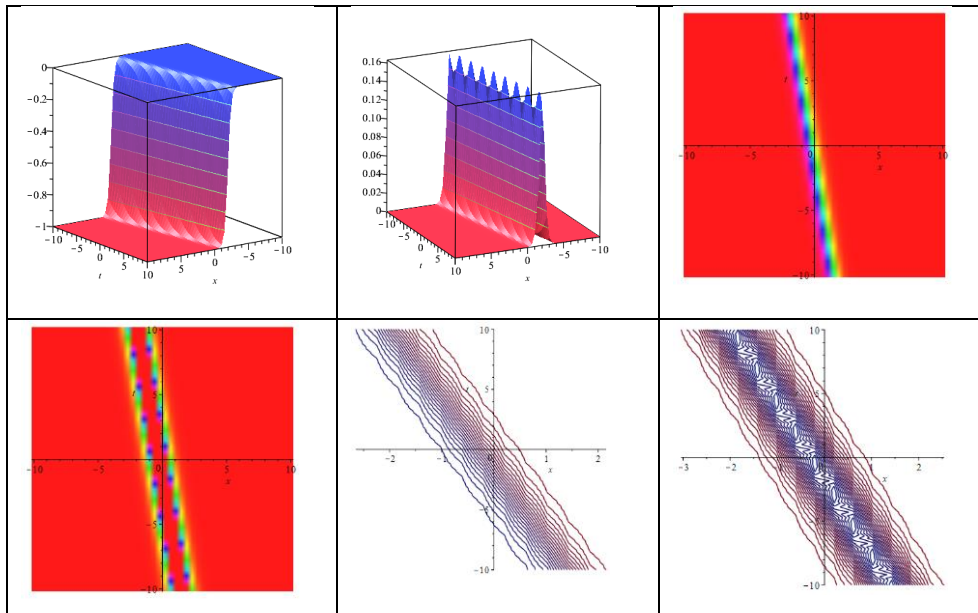
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$$\begin{aligned}
 R_{43}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{-\lambda} \{ \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) - \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\} \\
 &+ \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{\sqrt{-\lambda} \{ \cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) - \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) - \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\}^{-1}. \\
 R_{44}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{-\lambda} \{ -\cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) + \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\} \\
 &+ \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{\sqrt{-\lambda} \{ -\cosh(2\sqrt{-\lambda}(\Theta + \Delta)) - \sinh(2\sqrt{-\lambda}(\Theta + \Delta)) + \Psi \}}{\Psi + \cosh(2\sqrt{-\lambda}(\Phi + \Delta)) + \sinh(2\sqrt{-\lambda}(\Phi + \Delta))} \right\}^{-1}. \\
 R_{45}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\} \\
 &+ \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\}^{-1}. \\
 R_{46}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{-\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\} \\
 &+ \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{-\sqrt{(\Psi^2 - \Pi^2)\lambda} - \Psi\sqrt{\lambda} \cos(2\sqrt{-\lambda}(\Theta + \Delta))}{\Psi \sin(2\sqrt{\lambda}(\Phi + \Delta)) + \Pi} \right\}^{-1}. \\
 R_{47}(\Theta) &= -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{\lambda} \{ i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) - i\Psi \}}{\Psi + \cos(2\sqrt{\lambda}(\Phi + \Delta)) - i \sin(2\sqrt{\lambda}(\Phi + \Delta))} \right\} \\
 &+ \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{\sqrt{\lambda} \{ i \cos(2\sqrt{\lambda}(\Theta + \Delta)) + \sin(2\sqrt{\lambda}(\Theta + \Delta)) - i\Psi \}}{\Psi + \cos(2\sqrt{\lambda}(\Phi + \Delta)) - i \sin(2\sqrt{\lambda}(\Phi + \Delta))} \right\}^{-1}.
 \end{aligned}$$

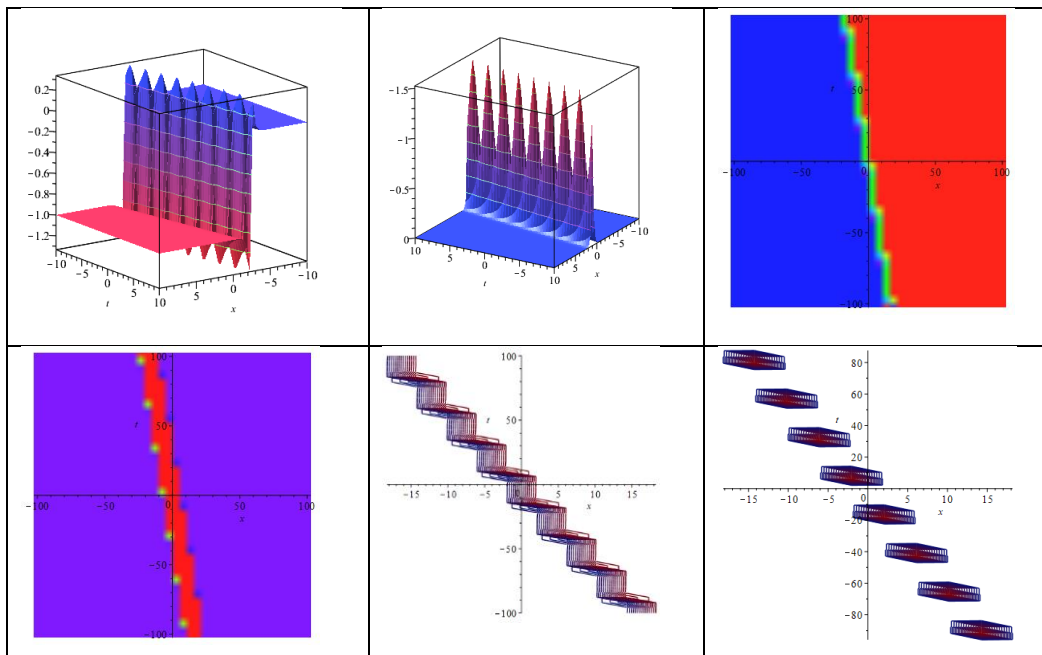
$$\begin{aligned}
 R_{48}(\Theta) = & -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{\sqrt{\lambda}\{-i\cos(2\sqrt{\lambda}(\Theta+\Delta)) + \sin(2\sqrt{\lambda}(\Theta+\Delta)) + i\Psi\}}{\Psi + \cos(2\sqrt{\lambda}(\Phi+\Delta)) + i\sin(2\sqrt{\lambda}(\Phi+\Delta))} \right\} \\
 & + \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{\sqrt{\lambda}\{-i\cos(2\sqrt{\lambda}(\Theta+\Delta)) + \sin(2\sqrt{\lambda}(\Theta+\Delta)) + i\Psi\}}{\Psi + \cos(2\sqrt{\lambda}(\Phi+\Delta)) + i\sin(2\sqrt{\lambda}(\Phi+\Delta))} \right\}^{-1}. \\
 R_{49}(\Theta) = & -\frac{f_1}{2q} + \frac{(\pm\sqrt{\frac{-1}{16\lambda}})f_1}{q} \times \left\{ \frac{1}{\Phi+\Delta} \right\} + \frac{f_1}{16(\pm\sqrt{\frac{-1}{16\lambda}})q} \times \left\{ \frac{1}{\Phi+\Delta} \right\}^{-1}.
 \end{aligned}$$

#### IV. NSs

We use the UM for the proposed model and then we obtained thirty-six solutions (sixteen hyperbolic WSs; sixteen trigonometric WSs; four rational WSs). Islam et al [28] applied the same model and then they obtained only twenty solutions. Upon evaluation of the two schemes, we see that the UM offers more solutions than Islam et al [28] scheme. The UM is implemented in the studied model for the first time. The obtained solutions of  $R_{11}(\Theta)$ ,  $R_{12}(\Theta)$ ,  $R_{13}(\Theta)$ ,  $R_{14}(\Theta)$ ,  $R_{21}(\Theta)$ ,  $R_{22}(\Theta)$ ,  $R_{23}(\Theta)$ ,  $R_{24}(\Theta)$ ,  $R_{31}(\Theta)$ ,  $R_{32}(\Theta)$ ,  $R_{33}(\Theta)$ ,  $R_{34}(\Theta)$ ,  $R_{41}(\Theta)$ ,  $R_{42}(\Theta)$ ,  $R_{43}(\Theta)$ ,  $R_{44}(\Theta)$  are hyperbolic function solutions, the solutions of  $R_{15}(\Theta)$ ,  $R_{16}(\Theta)$ ,  $R_{17}(\Theta)$ ,  $R_{18}(\Theta)$ ,  $R_{25}(\Theta)$ ,  $R_{26}(\Theta)$ ,  $R_{27}(\Theta)$ ,  $R_{28}(\Theta)$ ,  $R_{35}(\Theta)$ ,  $R_{36}(\Theta)$ ,  $R_{37}(\Theta)$ ,  $R_{38}(\Theta)$ ,  $R_{45}(\Theta)$ ,  $R_{46}(\Theta)$ ,  $R_{47}(\Theta)$ ,  $R_{48}(\Theta)$  are trigonometric function solutions and finally the solutions of  $R_{19}(\Theta)$ ,  $R_{29}(\Theta)$ ,  $R_{39}(\Theta)$ ,  $R_{49}(\Theta)$  are rational function solutions. Some of the obtained solutions ( $R_{11}(\Theta)$ ,  $R_{12}(\Theta)$ ,  $R_{15}(\Theta)$ ,  $R_{16}(\Theta)$ ,  $R_{29}(\Theta)$ ) provide numerical simulations (2-d, 3-d, density, and contour shapes) by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$  which are shown in Figures 2-11.

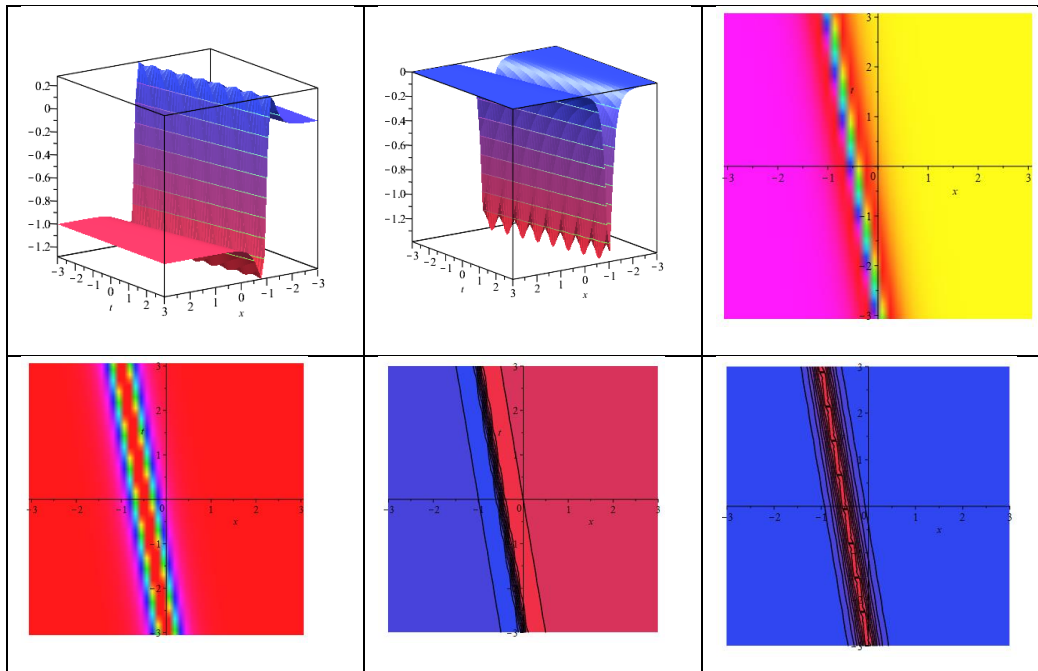


**Fig. 2:** 3-d, density and contour graph (Real and imaginary plots) of  $R_{11}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$

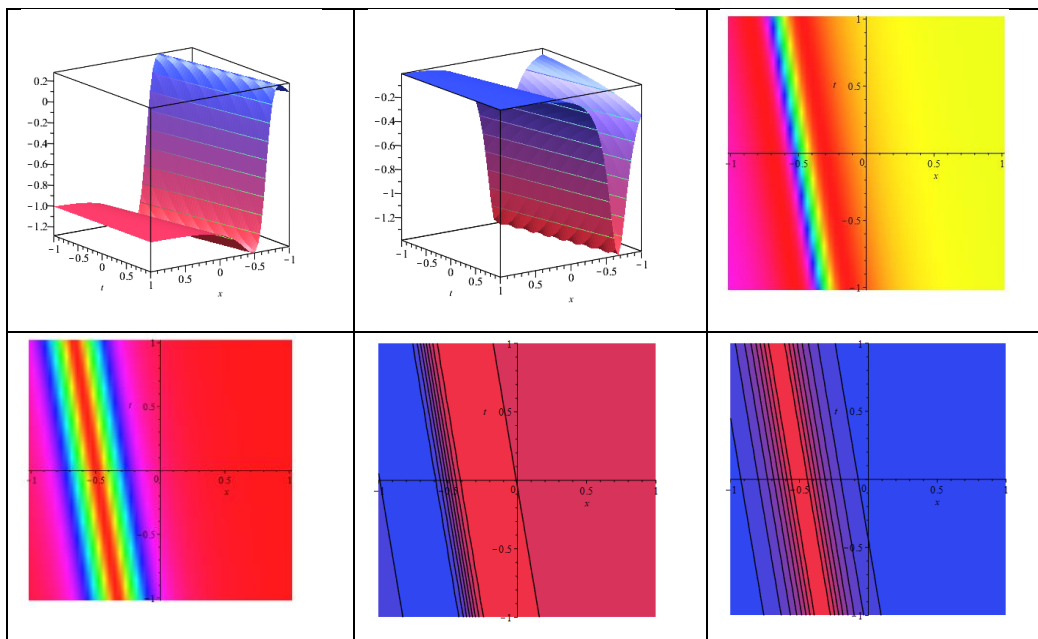


**Fig. 3:** 3-d, density and contour graph (Real and imaginary plots) of  $R_{12}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$

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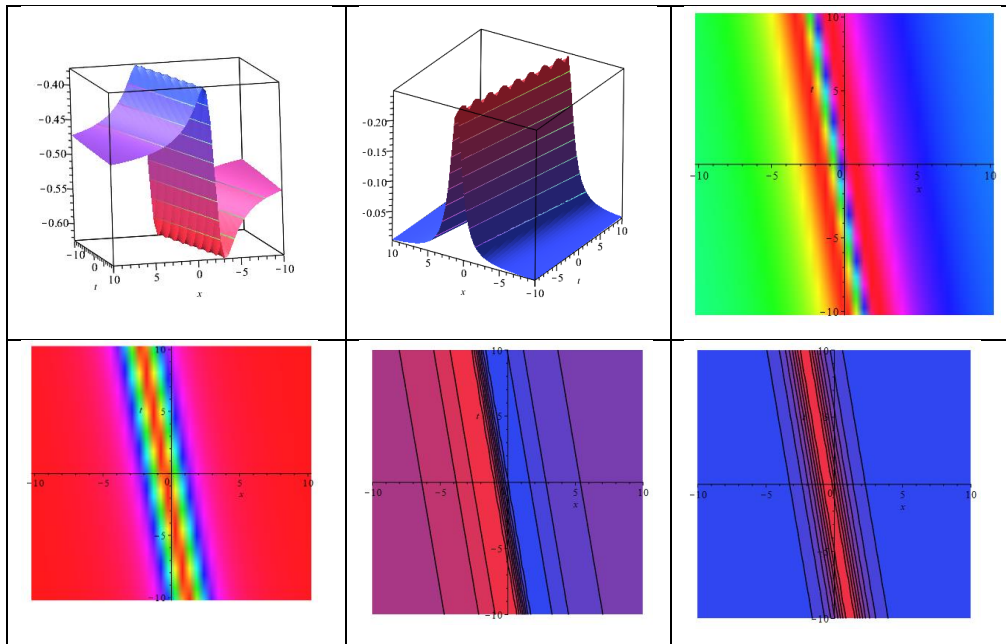


**Figure 4:** 3-d, density and contour graph (Real and imaginary plots) of  $R_{15}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$

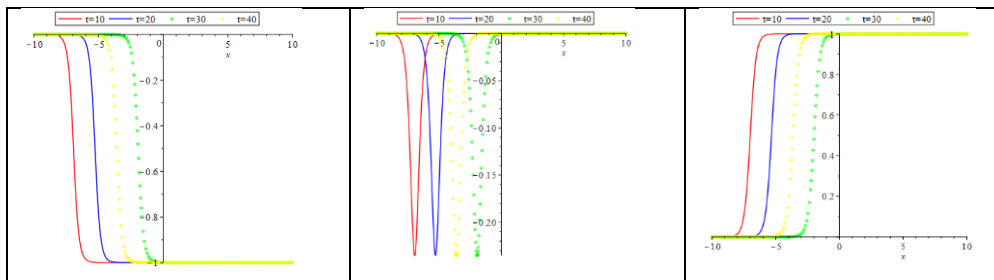


**Figure 5:** 3-d, density and contour graph (Real and imaginary plots) of  $R_{16}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$

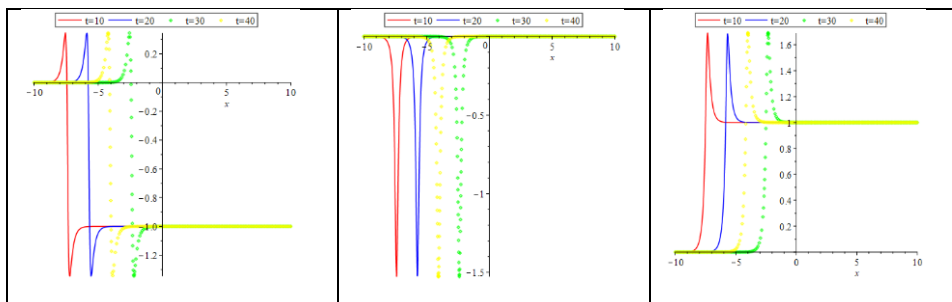
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**Figure 6:** 3-d, density and contour graph (Real and imaginary plots) of  $R_{29}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$

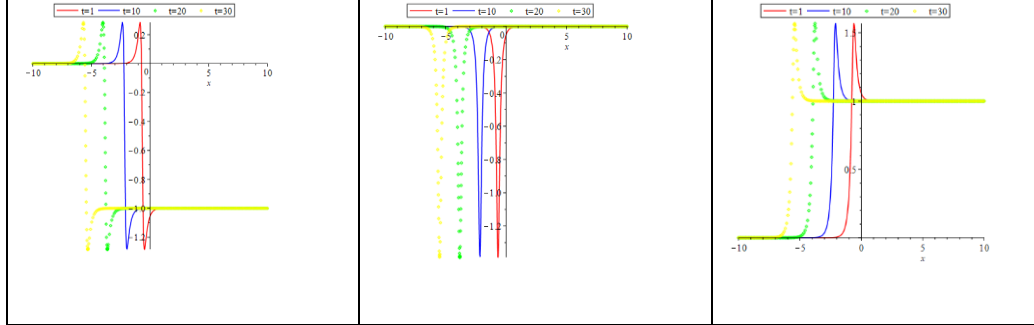


**Fig. 7.** 2-d graph (Real, imaginary and absolute plots) of  $R_{11}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$ .

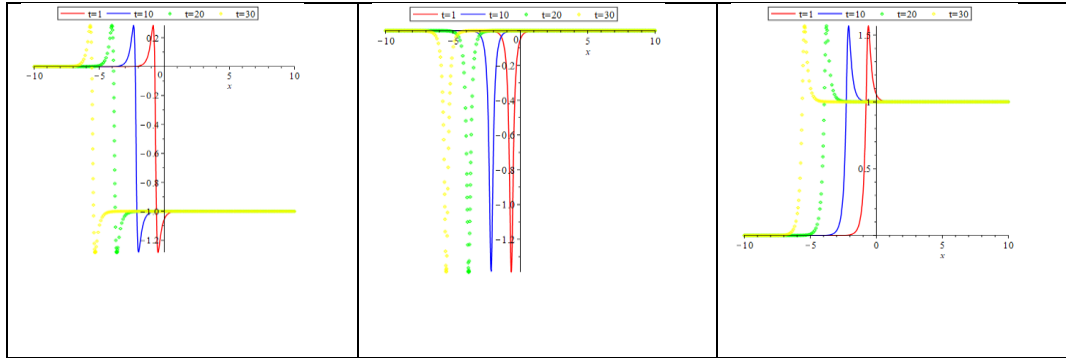


**Figure 8:** 2-d graph (Real, imaginary and absolute plots) of  $R_{12}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$ .

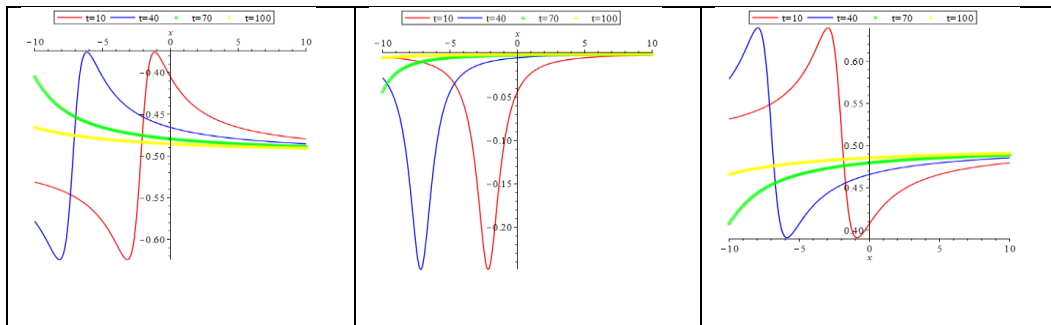
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**Figure 9:** 2-d graph (Real, imaginary and absolute plots) of  $R_{15}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$ .



**Figure 10:** 2-d graph (Real, imaginary and absolute plots) of  $R_{16}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$ .



**Figure 11:** 2-d graph (Real, imaginary, and absolute plots)  $R_{29}(\Theta)$  by considering  $\lambda = -1$ ,  $g = 1$ ,  $d = 1$ ,  $q = 1$ ,  $f_1 = 1$ ,  $k_1 = 2$ ,  $y = 10$ ,  $T = R = S = 1$ .

## V. Conclusion

The present article examines numerous kinds of SWPs (3D, density, contour, and 2D plots) of some obtained WSs of MMs arising in EE through the UT. The obtained features give a firm mathematical framework in EE and may be necessary to

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the WSs. In this work, we obtained thirty-six WSs from MMs arising in EE through the UT. Islam et al. [J. Appl. Comput. Mech. 7 (2), 715-726, (2021)] obtained only twenty WSs from the same equation. Comparing our obtained WSs and Islam et al. [J. Appl. Comput. Mech. 7 (2), 715-726, (2021)], the UT provides some WSs. Therefore, we conclude that the UT is a sample, robust for finding WWs of MMs arising in EE.

### **Conflict of Interest:**

The authors declare that no conflict of interest to report the present paper.

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