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AN OPENING OF A NEW HORIZON IN THE THEORY OF QUADRATIC EQUATION : PURE AND PSEUDO QUADRATIC EQUATION – A NEW CONCEPT

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Abstract

In this paper, the author has opened a new horizon in the theory of quadratic equations. The author proved that the value of x which satisfies the quadratic equation cannot be the only criteria to designate as the root or roots of an equation. The author has developed a new mathematical concept of the dimension of a number. By introducing the concept of the dimension of number the author structured the general form of a quadratic equation into two forms: 1) Pure quadratic equation and 2) Pseudo quadratic equation. First of all the author defined the pure and pseudo quadratic equations. In the case of pure quadratic equation $ax^2 + bx + c = 0$ *, the root of the equation will be a two-dimensional number having one root only while in the case of pseudo quadratic equation* $ax^2 + bx + c = 0,$ *the root of the equation will be a one-dimensional number having two roots only. The author proved that all pseudo quadratic equation is factorizable but all factorizable quadratic equation is not a pseudo quadratic equation. The author begs to differ from the conventional theorem: "A quadratic equation has two and only two roots."*

By introducing the concept that any quadratic surd is a two-dimensional number, the author developed a new theorem: "In a quadratic equation with rational coefficients, irrational roots cannot occur in conjugate pairs" and proved it.

Any form of quadratic equation $ax^2 + bx + c = 0,$ *can be solved by the application of the 'Theory of Dynamics of Numbers' even if the discriminant* $b^2 - 4ac < 0$ *in real number only without introducing the concept of an imaginary number. Therefore, the question of imaginary roots does not arise in the method of solution of any quadratic equation.*

Keywords : Dimension of Numbers, Dynamics of Numbers, Quadratic Equation, Rectangular Bhattacharra's Coordinates, significance of roots of a Quadratic Equation

I. Introduction :

There is a great interest to find the roots of a quadratic equation in the history of the method of solution of a quadratic equation from past to present. According to the conventional method at present any quadratic equation possesses two roots of different types : (i) Both rational (Positive, Negative) numbers or (one positive and the other negative) numbers. (ii) Conjugate pairs of irrational numbers (iii) Conjugate pairs of imaginary numbers.

The object of the present paper is to find the root or roots of any quadratic equation $ax^2 + bx + c = 0,$ by introducing the newly invented mathematical concept of "Theory of Dynamics of Numbers" [Prabir Chandra Bhattacharyya, : *J. Mech. Cont. & Math. Sci., Vol. – 17, No. – 1, January (2022). Pp. 37 – 53*]

There are three laws of the Theory of Dynamics of Numbers

1) 0 (zero) is defined as starting point of any number. There is an infinite number of directions through which the numbers can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers.

2) The count up numbers are always greater than or equal to the count down numbers. The count up numbers can move independently but the motion of count down numbers is dependent on the motion of count up numbers. The motion of the count down numbers exists if and only if there are motions of count up numbers.

3) For every equation, the count up numbers are always equal to the count down numbers.

A new mathematical concept has been developed by the author to find the root of a quadratic equation where the root depends on the inherent nature of the unknown quantity (say x) of the quadratic equation. The author has defined a new mathematical concept of the dimension of a number. On the basis of the dimension of the unknown quantity (say x) of the quadratic equation $ax^2 + bx + c = 0,$ the author defined and structured two types of a quadratic equation (1) Pure quadratic equation (2) Pseudo quadratic equation. The author also finds the method of solution of both types of quadratic equations. In the case of the pure quadratic equation, the unknown quantity (say x) is a two-dimensional number having one root only and in the case of a pseudo quadratic equation, the unknown quantity (say x) is a one-dimensional number having two roots only.

Further, the author solved any form quadratic equation $ax^2 + bx + c = 0$ by introducing the new mathematical concept of the "Theory of Dynamics of Numbers" even if the discriminant $b^2 - 4ac < 0$ in real numbers without using imaginary numbers.

However, the method of solution of a quadratic equation has not been investigated by any author previously with a similar approach.

Finally, the author stated and proved some new theories in the quadratic equation by applying the new concept of the dimension of numbers and the Theory of Dynamics of Numbers.

II. Literature Review

The concept of formation of a quadratic equation had been originally developed on the basis of the concept to find one unknown quantity of a rectangular area having length, breadth, and area when any two quantities are known out of three quantities of the structure. For the first in Berlin Papyrus (CA 2160 – 1700 BC), in Egypt, a solution of the quadratic equation had been found [Smith, 1953, p. 443]. In the Indus civilization (3000 BC – 500 AD) the presence of development in the field of mathematics was found in the construction of buildings followed by standardized measurements of bricks in a ratio of 4:2:1. Since the script of Harappans has not been yet deciphered there is no written evidence of development in the field of mathematics at the time of Indus civilization. Standard weights and length measurement scales were known to the Harappan civilization in the decimal system [Thapar, R., 2000]. Advanced civilization is possible if and only if there was a strong mathematical background in precision. Therefore, Indian mathematics on Babylonians cannot be ruled out even in absence of evidence. Babylonian mathematicians constructed a set of quadratic problems and solved the quadratic equations from Babylonian clay Tablets (2000 – 1700 BC). The basic method of solving this problem is computing the square (Katz, 2007), Euclid (325 – 270 BC), a Greek mathematician solved the quadratic problem of Babylonian geometrically (Gandz, 1937). In the 9th century, Al-Khwarizmi tried to solve the abstract problem and applied the algorithm to solve the quadratic equation algebraically. Al-Karki, Savadorsa, Ibn Erza, and Immanual Bonfils contributed their studies from Babylonian and Egyptian (Gandz, 1937). At that time the mathematician of Babylon, Egypt, Greek, and Arabic countries had no perception of negative roots in quadratic equations. Indian mathematician Bhaskara – II (1114 – 1185) introduced both positive and negative roots of the quadratic equation [Katz, 1998, p. 226 – 227].

In "Nine chapters on Mathematical Art" ($\cong 100\ BC$) written by Jin Shang Sunachu we can find the formation of a quadratic equation as

$$x^2 + (b-a)x = \frac{1}{2}\left[c^2 - (b - a^2)\right].$$ [Ling, w f Needham, J. 1955].

Aryabhata (476 – 550 AD), an Indian Mathematician had used the quadratic equation but he had not given a formal solution anywhere. Sanskrit Sutra (10, Sutra 25) states the rule on how to calculate the interest on the principal. English version of the Sanskrit Sutra states that 'Multiply Amount (A), Time (t) and Principal (p) and add half of the Principal square.' [Dutta, B. B. 1929].

Sridhara Acharya (870 – 930), a Bengali, Hindu Pandit and Mathematician of ancient India was the first person who developed an algorithm to solve quadratic equations in Sanskrit verse [B. B. Dutta, 1929]. B. B. Dutta translated the Sanskrit verse into English which is as follows :

"Multiply both sides (of an equation) by a known quantity equal to four times the coefficient of the square of the unknown, add both sides the known quantity equal to the square of the (original) coefficient of the unknown; then extract the root."

Now, the Sanskrit verse takes the form

$$ax^2 + bx = c$$

Multiply both sides by 4a and then add b^2 to both sides we get

$$4a^2x^2 + 4abx + b^2 = b^2 + 4ac$$

Now, we can extract the value of the unknown quantity x as

$$x = \frac{-b+\sqrt{b^2+4ac}}{2a}$$

The negative root of the quadratic equation had not been considered by Sridhara but Bhaskara – II had considered two roots of the quadratic equation following the same approach and he considered $\pm$ sign with square root [Smith 1951, P. 159]. The name of other mathematicians in ancient India who contributed to developing the method of solution of the quadratic equation is Brahmagupta, Mahavira, Prothudakasvami, Padmanabha, Sripati, and Narayana.

Girolama Cardano (1501 – 1576), was an Italian Mathematician who had given a complex solution for the quadratic equation by first using an imaginary number without using the word "imaginary" in the year 1545.

In 1637, René Descartes published a book "La Geometric" which contained special cases of the quadratic formula in the form known today [Cooley, 1993, P. 95 – 96]. Descartes discovered a complete solution for both positive, and negative imaginary roots of the quadratic equation [Katz, 1998, P. 448].

The author published a paper "A Novel Concept in Theory of Quadratic Equation" in March 2022 [Bhattacharyya, Prabir Chandra. : "A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION". *J. Mech. Cont. & Math. Sci., Vol.-17, No.-3, March (2022) pp 41-63*] on the basis of the published paper by the same author "An Introduction to Theory Dynamic of Numbers" in January 2022 [Bhattacharyya, Prabir Chandra.: "AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS: A NEW CONCEPT". *J. Mech. Cont. & Math. Sci., Vol.-17, No.-1, January (2022). pp 37-53*]. The author showed that a quadratic equation $ax^2 + bx + c = 0$, has one root in real number only based on an unknown quantity (say x). Also, the author becomes successful to solve a quadratic equation $ax^2 + bx + c = 0$ even if the discriminant $b^2 - 4ac < 0$ in real numbers without introducing the concept of an imaginary number.

In the fifth century Aryabhata, an Indian mathematician invented 0 (zero) as a placeholder in numerals but the present author is the first person who defined 0 (zero) as the starting point of any number by introducing the 'Theory of Dynamics of

Numbers'. The doctrine of 'Sunyata' or 'Emptiness' is one of the profound contributions of philosophy from India to the world.

III. Formulation of the problem and its solution

Some definitions :

Pure Quadratic Equation: An equation in one unknown quantity (say *x*) in the form $ax^2 + bx + c = 0$ is called a Pure quadratic equation when the character of the structure of second-degree expression $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ is two-dimensional. Here a ≠ 0 is called the coefficient of x^2, b, the coefficient of *x,* and c, the constant term.

The numerical value of the inherent nature of *x* which satisfies the Pure quadratic equation $ax^2 + bx + c = 0$, is called the root of the quadratic equation. The inherent nature of *x* can be determined uniquely from the nature of the constant term of the quadratic equation $ax^2 + bx + c = 0$ provided the structure of second-degree expression $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$, must be in two dimensions. Note that the Pure quadratic equation has one and only one root. For example, $x^2 - x - 3 = 0$ is a Pure quadratic equation.

Pseudo Quadratic Equation: An equation in one unknown quantity (say *x*) in the form $ax^2 + bx + c = 0$ is called a Pseudo quadratic equation when the character of the structure of second-degree expression $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$, can be converted into one-dimensional two-linear expressions by factorization such that $ax^2 + bx + c = (px + q)(rx + s) = 0$. Here, a ≠ 0 is called the coefficient of x^2, b the coefficient of *x* and c the constant term in the Pseudo Quadratic Equation $ax^2 + bx + c = 0$

Also, $p \neq 0$ is called the coefficient of *x* and q the constant term of the first one - dimensional linear expression of the linear equation $px + q = 0$.

Again, r ≠ 0 is called the coefficient of *x* and s, the constant term of the second one-dimensional linear expression of the linear equation, $rx + s = 0$

Therefore, a Pseudo quadratic equation can be written as

$$ax^2 + bx + c = (px + q)\,(rx + s) = 0$$

Clearly, $px + q = 0$ and $rx + s = 0$ are one-dimensional two-linear equations.

The numerical value of inherent nature of an unknown quantity (say *x*) which satisfies the equation $px + q = 0$, will be one of the roots of the Pseudo quadratic equation $ax^2 + bx + c = 0$.

Again, the numerical value of the inherent nature of an unknown quantity (say *x*) which satisfies the equation $rx + s = 0$, will be another root of the Pseudo quadratic equation $ax^2 + bx + c = 0$.

Note that the Pseudo Quadratic Equation $ax^2 + bx + c = 0$ has two roots only. For example, $x^2 + 3x + 2 = 0$ is a Pseudo quadratic equation.

Adfected Quadratic Equation: An equation involving a variable x in the second degree as well as the first degree is called Adfected Quadratic Equation.

Adfected Quadratic Equation may be a Pure Quadratic Equation or a Pseudo Quadratic Equation.

Dimension of a Number: The inherent characteristics of the structure of a rational or irrational number from which the number is generated is called the Dimension of a number.

Quadratic Surd: A square root of the positive real quantity is called a Quadratic Surd if its value cannot be exactly determined although its value can be determined to any degree of accuracy.

For example :

$$\sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt{7}\ \ldots\ldots\ldots\ldots\ldots \text{ etc.}$$

$$-\sqrt{2},\ -\sqrt{3},\ -\sqrt{5},\ -\sqrt{7}\ \ldots\ldots\ldots\ldots \text{ etc.}$$

According to the "Theory of Dynamics Of Numbers" 0 (zero) is defined as the starting point of any rational or irrational number, 0 (zero) means the absence of any number. The square root of 0 (zero) is defined as equal to 0 (zero), i.e., $\sqrt{0} = 0$.

A quadratic surd number that is moving away from the origin, 0 (zero) is called count up quadratic surd number.

For example :

$$\overrightarrow{\sqrt{2}} = +\sqrt{2},\ \overrightarrow{\sqrt{3}} = +\sqrt{3}.\ \overrightarrow{\sqrt{5}} = +\sqrt{5},\ \overrightarrow{\sqrt{7}} = +\sqrt{7}\ \ldots\ldots.. \text{ etc.}$$

and

A quadratic surd number that is moving towards the origin, 0 (zero) is called count down quadratic surd number.

For example

$$\overleftarrow{\sqrt{2}} = -\sqrt{2},\ \overleftarrow{\sqrt{3}} = -\sqrt{3}.\ \overleftarrow{\sqrt{5}} = -\overrightarrow{\sqrt{5}},\ \overleftarrow{\sqrt{7}} = -\sqrt{7}$$

To find the condition of the dimension of an unknown quantity (say x) in a quadratic equation $ax^2 + bx + c = 0$

Let us consider the general form of a quadratic equation $ax^2 + bx + c = 0$ in one unknown quantity (say x) where a ≠ 0, is called the coefficient of x^2, b is the coefficient of x, c is the constant term and a, b, and c are rational numbers.

I. (A) The unknown quantity (say x) of the quadratic equation $ax^2 + bx + c = 0$ will be two-dimensional number if any one of the following conditions is satisfied :

1) b = 0
2) $\sqrt{b^2 - 4ac}$ is a quadratic surd number
3) $b < \sqrt{b^2 - 4ac}$, a rational number
4) $b^2 - 4ac < 0$.

Note : In the above-mentioned cases the quadratic equation $ax^2 + bx + c = 0$ is called a Pure quadratic equation even if the quadratic expression $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ is factorizable into two factors.

I. (B) The unknown quantity (say x) of the quadratic equation $ax^2 + bx + c = 0$ will be one dimensional if the equation satisfies all the following conditions :

1) The quadratic expression $ax^2 + bx + c$ of the quadratic equation is factorizable into two linear factors.

2) Factorizable quadratic equation $ax^2 + bx + c = 0$ must not be satisfied by any one of the conditions stated in I (A).

Note : In that case, the quadratic equation $ax^2 + bx + c = 0$ is called a Pseudo quadratic equation.

The number of root or roots of a quadratic equation $ax^2 + bx + c = 0$, according to the new mathematical concept in the theory of quadratic equations.

I. The value of x which satisfies the quadratic equation can not be the only criteria to designate as the root or roots of the equation. First of all, we have to identify whether the equation is a Pure quadratic equation or a Pseudo quadratic equation.

II. A Pure quadratic equation has one and only one root even if the expression $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ is factorizable into two factors.

III. In a Pseudo quadratic equation the quadratic expression $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ is always factorizable into two linear factors and the equation has two roots only in a rational number.

IV. Pairs of conjugate roots of irrational numbers cannot occur in any quadratic equation $ax^2 + bx + c = 0$ as we find in the convensional method of solution.

V. Pairs of conjugate roots of imaginary number cannot occur in any quadratic equation $ax^2 + bx + c = 0$ as we find it in the convensional method of solution.

Problem 1 :

Prove that $\sqrt{2}$ is a two-dimensional count up quadratic surd number.

Proof :

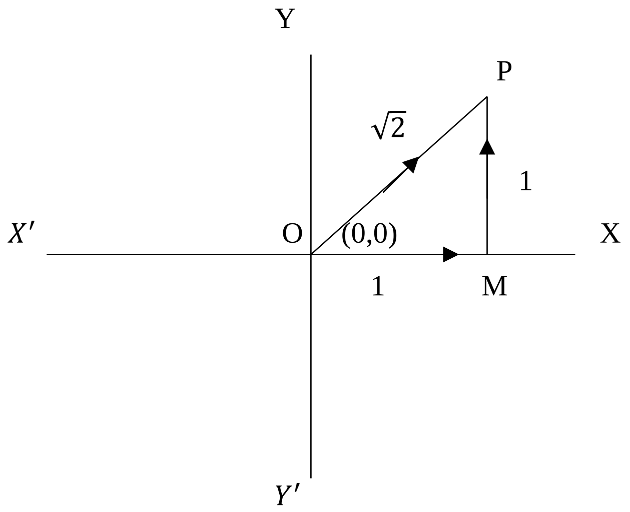

Fig. 1

Let $\overrightarrow{OM} = \vec{1} = +1 \ and \ \overrightarrow{MP} = \vec{1} = +1$

We know that

$$\overrightarrow{OP^2} = \overrightarrow{OM^2} + \overrightarrow{MP^2} = (1)^2 + (1)^2 = 2$$

$$\therefore \overrightarrow{OP} = +\sqrt{2}$$

Let $\overrightarrow{OP} = \vec{X} = +x, thus\ x = \sqrt{2}$

From Fig. 1 it is clear that point P is moving away from 0 (zero). So $\overrightarrow{OP} = x = \sqrt{2}$ is a count up number, Also it is evident that the quadratic surd number $\sqrt{2}$ is generated from the characteristics of the structure of the two-dimensional number.

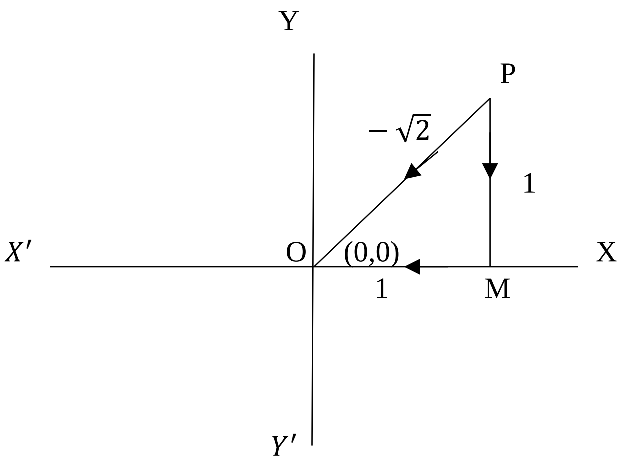

Fig. 2

Now, from Fig. 2, we have

$$\overrightarrow{PO^2} = \overleftarrow{OP^2}$$

$$\therefore \overrightarrow{PO} = \overleftarrow{OP} = \overleftarrow{\sqrt{2}} = -\sqrt{2}$$ is also a two-dimensional number.

Note : Countdown $\sqrt{2} = -\sqrt{2}$, is equal to count up $\sqrt{2}$ with rotation of 180^0 in the anti-clockwise direction of $\overrightarrow{OP}$ at the point P

Problem 2 :

Prove that if $b < \frac{\sqrt{b^2-4ac}}{2a}$ then $ax^2 + bx + c = 0$ is a Pure Quadratic Equation.

Proof : If we consider

$$ax^2 + bx + c = 0 \tag{1}$$

is a Pure quadratic equation and a, b, and c are positive rational numbers.

Then

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \tag{2}$$

Now, if we consider b is a negative rational number

Then

$$x = \frac{-(-b)+\sqrt{(-b)^2-4ac}}{2a} = \frac{b+\sqrt{b^2-4ac}}{2a} \tag{3}$$

from equation (3) we have

$$b > 0 \text{ and } \sqrt{b^2-4ac} > 0 \text{ then } x > 0$$

The value of $x = \frac{-b+\sqrt{b^2-4ac}}{2a} > 0$ when b is a negative rational number.

$$\therefore -b + \sqrt{b^2-4ac} > 0$$

$$\Rightarrow b - \sqrt{b^2-4ac} < 0$$

$$\Rightarrow b < \sqrt{b^2-4ac},$$ a rational number.

Conversely, we can say if $b < \sqrt{b^2-4ac}$ then the equation $ax^2 + bx + c = 0$ is a Pure Quadratic Equation.

A quadratic equation has generally two roots of an equation according to convensional method of solution from past to present. But Bhaskara – II (1114 – 1185 AD) an ancient Indian Mathematician showed with an example that these two roots of a quadratic equation are not always acceptable [B. B. Dutta and A. N. Singh, History of Hindu Mathematics, Part – II, P. 71 – 72].

Problem 3: [Problem of Bhaskara -- II]

The fifth part of a troop of monkeys, leaving out three, squared, has entered a cave, one is seen to have climbed on a branch of a tree. Tell how many monkeys are they?

Solution: In this, the value of troop is x; now the question takes the form :

$$\left(\frac{1}{5}\,x - 3\right)^2 + 1 = x$$

$$\Rightarrow x^2 - 55\,x + 250 = 0$$

$$\Rightarrow (x - 50)\,(x - 5) = 0$$

In this case, two values of x are 50 and 5. The second value, 5 should not be acceptable as it is inapplicable. Why? If the troop consists of 5 monkeys, its fifth part will be 1 (one) monkey. How can then leave out 3 monkeys? Again, how can the reminder be said to have entered the cave?

Bhaskara – II commented that it seems to have a wider significance but he could not state the significance. The author has investigated the said problem and becomes successful to find the reason behind it.

The author solved the problem as follows :

$$x^2 - 55x + 250 = 0 \quad \ldots\ldots (1)$$

Comparing equation (1) with the general form of quadratic equation

$ax^2 + bx + c = 0$, here a = 1, b = – 55, and c = 250

$$\therefore \sqrt{b^2 - 4ac} = \sqrt{(-55)^2 - 4.1.250} = \sqrt{2025} = 45, \text{ a rational number}$$

$$\therefore\ b < \sqrt{b^2 - 4ac}, \text{ a rational number.}$$

Therefore, equation (1) is a Pure Quadratic Equation having one root only through equation (1) is factorizable into two factors.

Now, from equation (1) we have

$$x^2 - 55x = -250$$

$$\Rightarrow \left(x - \frac{55}{2}\right)^2 = \left(\frac{55}{2}\right)^2 - 250$$

$$\Rightarrow \left(x - \frac{55}{2}\right)^2 = \frac{2025}{4} = \left(\frac{45}{2}\right)^2$$

$$\Rightarrow x - \frac{55}{2} = \frac{45}{2}$$

$$\Rightarrow x = \frac{45}{2} + \frac{55}{2} = \frac{100}{2} = 50$$

Therefore, the number of monkeys in a troop is 50 only.

Significance : The value x which satisfies the quadratic equation cannot be the only criteria to designate as the root or roots of the equation. Here, equation (1) is a Pure Quadratic Equation. Hence, equation (1) has one and only one root.

Problem 4 :

Find the length and breadth of a square where the area is 100 square meter.

Solution : Let us consider the square land OPQR whose length OP = x meter, and breadth OR = x meter and its area OPQR == 100 square meter.

According to the problem, we have

$x^2 = 100$ ………. (1)

$\Rightarrow x^2 - 100 = 0$ ………. (2)

$\Rightarrow (x + 10)\ (x - 10) = 0$

$\therefore\ x = -10$ or $x = +10$

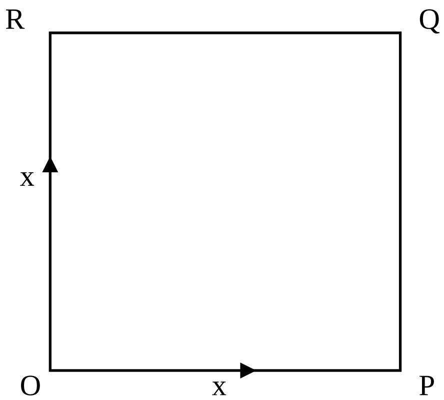

Fig. 3

It will not be possible for us to construct a square land with the length = – 10m and breadth = – 10 m, though it is computationally true from equation (2).

We have to introspect why it has happened ?

Now, if we compare equation (2) with the general form of quadratic equation $ax^2 + bx + c = 0$, we have, a = 1, b = 0 and c = – 100

Since, b = 0, equation (2) is a Pure Quadratic Equation.

Now, let us solve equation (2) by using the Theory of Dynamics of Numbers

$x^2 - 100 = 0$

Since, – 100 < 0, the inherent nature of x is a two-dimensional count up x number.

So, equation (2) takes the form

$$\overrightarrow{x^2} + \overleftarrow{100} = 0$$

According to the third law of the Theory of Dynamics of Numbers

$$x^2 = 100$$

$$\Rightarrow x = 10$$

Here, the inherent nature of x is count up $x = + x$

$x = + 10$ only, is the solution of the equation (2)

Therefore, the length and breadth of the square land = 10 meter only.

Observation :

(1) The value x which satisfies the quadratic equation cannot be the only criteria to designate as the root or roots of the equation. Here, the equation $x^2 - 100 = 0$, is a pure quadratic equation

(2) In that case length and breadth is equal to -10 meter does not arise

(3) All Pseudo Quadratic equation is factorizable but all factorizable quadratic equation is not Pseudo Quadratic Equation.

Problem 5 :

Prove that $x^2 - 2 = 0$ is a Pure Quadratic Equation.

Proof : Let us consider $\overrightarrow{OP} = \vec{x}$

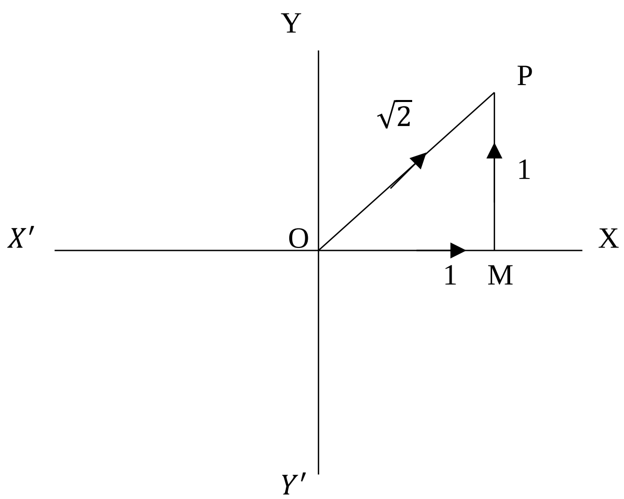

Fig. 4

We know,

$$\overrightarrow{OP^2} = \overrightarrow{OM^2} + \overrightarrow{MP^2} = \overrightarrow{1^2} + \overrightarrow{1^2} = \overrightarrow{2}$$

Therefore

$$\overrightarrow{OP^2} = \overrightarrow{x^2} = \overrightarrow{2}$$

$$\therefore x = +\sqrt{2} \text{ only}$$

From Fig. 4, it is evident that the inherent characteristic of the structure of unknown variable x is two-dimensional since $\sqrt{2}$ is a quadratic surd number.

Hence, $x^2 - 2 = 0$ is a Pure Quadratic Equation having one root only.

Note : if $x = \sqrt{2}$, be the root of an equation then the Pure Quadratic Equation will be $x^2 - 2 = 0$. In that way, we can find the quadratic equation if the root of the equation is given in a quadratic surd number.

Problem 6 :

Solve :

$$x^2 + 2 = 0$$

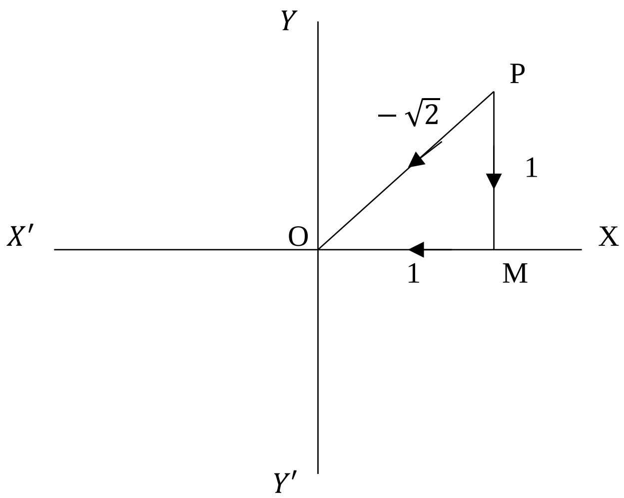

Fig. 5

Solution :

From Fig. 5, we can construct a Pure Quadratic Equation

$$x^2 + 2 = 0 \;\ldots\ldots\ldots\; (1)$$

Since + 2 > 0, the inherent nature of x is countdown x. So, equation (1) takes the form

$$\overleftarrow{x^2} + \overrightarrow{2} = 0 \;\ldots\ldots\; (2)$$

According to 3 rd law of the Theory of Dynamics of Numbers

We have,

$$x^2 = 2$$

$$\Rightarrow x = 2$$

$\therefore$ The unknown quantity $x = \sqrt{2}$ will be in countdown motion of x.

Therefore, $\overleftarrow{x} = \overleftarrow{\sqrt{2}}$, is the solution of the quadratic equation (1). The equation (1) represents a Pure Quadratic Equation.

Note : If $x = -\sqrt{2}$ be the root of an equation then the equation will be $x^2 + 2 = 0$. In that way, we can find the equation if the root is given in a negative quadratic surd number.

Now, if we solve the above equation (1) by convensional method :

$$x^2 + 2 = 0 \;\ldots\ldots\; (1)$$

$$\Rightarrow x^2 = -2$$

$$\Rightarrow x = \pm\sqrt{-2} = \pm i\sqrt{2}, where\ i = \sqrt{-1}$$

Observations :

1) We can easily find the numerical value of x by using the theory of dynamics of numbers. It should be noted that $\overrightarrow{PO}$ is in the vertically opposite direction of $\overrightarrow{OP}$, i.e. 180^0 anti-clockwise rotation to $\overrightarrow{OP}$ at the point P. Therefore, $-\sqrt{2}$ is with 180^0 anti-clockwise rotation to $+\sqrt{2}$, a real number.

2) $x^2 - 2 = 0$ and $x^2 + 2 = 0$ are two different Pure Quadratic Equations and the root of the equations are $x = +\sqrt{2}$ and $x = -\sqrt{2}$, respectively. Therefore, $x = +\sqrt{2}$ and $x = -\sqrt{2}$ cannot occur as the roots of a one and the same quadratic equation.

Note that $x = +\sqrt{2}$ and $x = -\sqrt{2}$ may occur as a conjugate pair of the root of a Pseudo Bi – quadratic equation $(x^2 - 2)(x^2 + 2) = 0$ or $x^4 - 4 = 0$

3) It is not possible to find the numerical value of x by using the concept of imaginary numbers.

Theorem: In a quadratic equation with rational coefficients irrational roots cannot occur in conjugate pairs :

Proof: Consider the quadratic equation of the general form :

$$ax^2 + bx + c = 0 \;\ldots\ldots\ldots\; (1)$$

where the coefficients a, b, and c are rational numbers.

Let, $p + \sqrt{q}$ (where p is a rational number and $\sqrt{q}$ is a quadratic surd number) be a root of the equation (1)

Therefore, we can write

$$x = p + \sqrt{q}, \quad q > 0 \text{ …….. (2)}$$

$$\Rightarrow x - p = \sqrt{q} > 0 \text{ ……… (3)}$$

Let,

$$x - p = y$$

$$y = \sqrt{q} > 0$$

Therefore, y is a count up number.

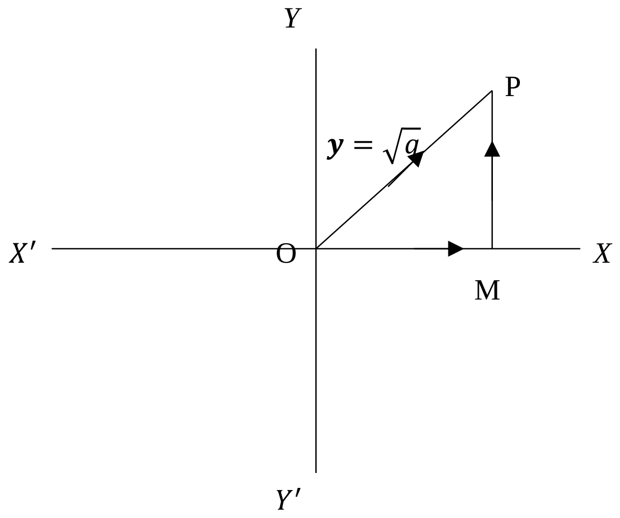

Fig. 6

From Fig. 6, we have

$$\overrightarrow{OP} = \vec{y} = \overrightarrow{\sqrt{q}}$$

$$\therefore y^2 = q$$

$$\Rightarrow y^2 - q = 0 \text{ ……… (4)}$$

Equation (4) is a two-dimensional Pure Quadratic Equation.

Now, putting the value of $y = x - p$ in equation (4) we get,

$$(x - p)^2 - q = 0$$

$$\Rightarrow x^2 - 2px + p^2 - q = 0 \text{ ……….. (5)}$$

Now, comparing equation (5) with equation (1) we can find that

a = 1, b = – 2 p and c = p^2 – q .

So, the required Pure Quadratic Equation having root $p + \sqrt{q}$ is

$$x^2 - 2px + p^2 - q = 0$$

Again, let us consider $p - \sqrt{q}$ (where p is a rational number and $\sqrt{q}$ is a quadratic surd number) be the another root of the equation (1)

Let,

$$x = p - \sqrt{q}, \quad q > 0 \text{ …….. (6)}$$

$$\therefore x - p = -\sqrt{q} < 0 \text{ ……… (7)}$$

Let, $x - p = y$

$\therefore$ y < 0

So, it is clear that *y* is a countdown number.

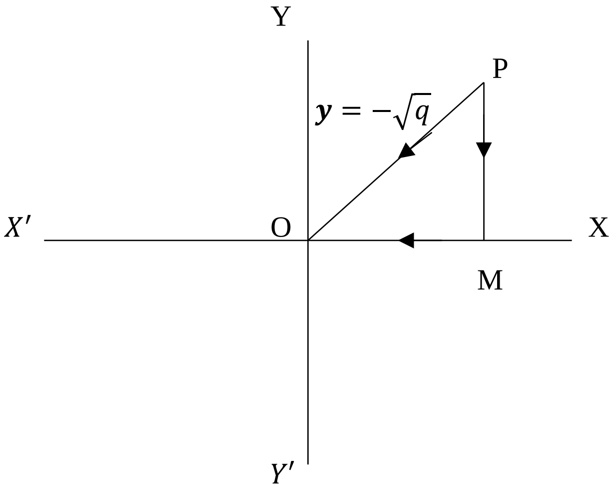

Fig. 7

From Fig. 7, we have

$$\overrightarrow{PO^2} = \overleftarrow{OP^2} = \overleftarrow{y^2}$$

$$\therefore \overleftarrow{y^2} = \overleftarrow{(-\sqrt{q}\,)^2}$$

$$\Rightarrow -y^2 = -q$$

$$\Rightarrow -y^2 + q = 0 \text{ ……… (8)}$$

Now, putting the value of y in equation (8), we get

$$-(x-p)^2 + q = 0$$

$$\Rightarrow -x^2 + 2px - p^2 + q = 0 \text{ ……….. (9)}$$

Now, comparing equation (9) with equation (1) we get

$$a = -1, b = 2p \; and \; c = -p^2 + q.$$

So, the required Pure Quadratic Equation having its root $p - \sqrt{q}$ is

$$-x^2 + 2px - p^2 + q = 0.$$

Here, we find that the coefficients of equation (5) and equation (9) are not one and the same concerning the general form of the quadratic equation $ax^2 + bx + c = 0$. Therefore equation (5) and equation (9) do not represent one and the same quadratic equation.

We observed that the equation (5) having its root, $p + \sqrt{q}$ and the equation (9) having its root $p - \sqrt{q}$ do not belong to one and the same quadratic equation.

Therefore, irrational roots cannot occur in conjugate pairs in a quadratic equation.

Note : (1) The quadratic expression of equation (5) will represent graphically a concave upward parabola because $a = 1 > 0$ whereas the quadratic expression of equation (9) will represent graphically a concave downward parabola because $a = -1 < 0$.

(2) So, the two quadratic equations $x^2 - 2px + p^2 - q = 0$ and $-(x^2 - 2px + p^2 - q) = 0$ are not one and the same quadratic equation. Therefore, we cannot write as convensional method that

$$-(x^2 - 2px + p^2 - q) = 0$$

$$\Rightarrow x^2 - 2px + p^2 - q = 0$$

because the negative sign (–) before the quadratic expression $x^2 - 2px + p^2 - q$ bears great significance which is mentioned in Note : (1).

Problem 7 :

Prove that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ cannot occur in conjugate pairs of the general form :

$$ax^2 + bx + c = 0$$

when coefficient a, b, c are rational numbers.

Proof :

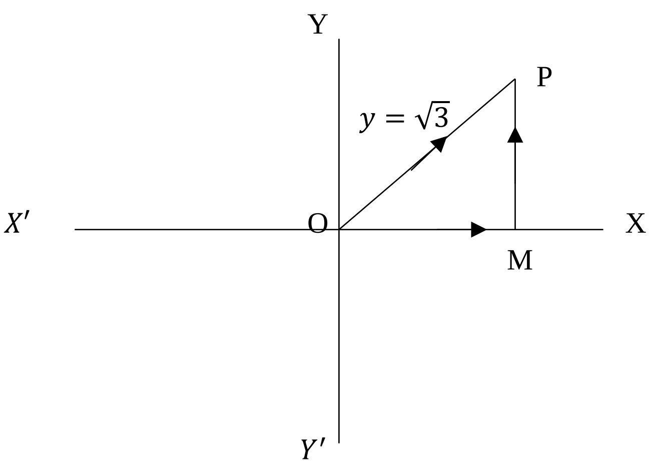

Fig. 8

Let us consider the general form of the quadratic equation

$$ax^2 + bx + c = 0 \ \ldots\ldots. \ (1)$$

when coefficients a, b, and c are rational numbers.

Let, $2 + \sqrt{3}$ be a root of the equation (1)

Therefore, we can write

$$x = 2 + \sqrt{3} \ \ldots\ldots\ldots.. \ (2)$$

$$\Rightarrow x - 2 = \sqrt{3} \ \ldots\ldots.. \ (3)$$

Let, $x - 2 = y = \sqrt{3} > 0$

So, y is a count up number.

From Fig. 8, we have

$$\overrightarrow{OP} = \vec{y} = \overrightarrow{\sqrt{3}}$$

$$\Rightarrow y^2 = 3$$

$$\Rightarrow y^2 - 3 = 0 \ \ldots\ldots\ldots\ldots\ldots \ (4)$$

Equation (4) is a two-dimensional Pure Quadratic Equation.

Now, putting the value of $y = x - 2$ in equation (4) we get

$$(x - 2)^2 - 3 = 0$$

$$\Rightarrow x^2 - 4x + 4 - 3 = 0$$

$$\Rightarrow x^2 - 4x + 1 = 0 \ \ldots\ldots\ldots.. \ (5)$$

Now comparing equation (5) with equation (1), we get

$$a = 1, b = -4, c = 1$$

Since, $\sqrt{b^2 - 4ac} = \sqrt{(-4)^2 - 4.1.1} = \sqrt{16 - 4} = \sqrt{12}$, a quadratic surd.

So, equation (5) satisfies the condition of a Pure Quadratic Equation.

Therefore, the required Pure quadratic Equation having root $2 + \sqrt{3}$ is

$x^2 - 4x + 1 = 0$

Now, let us consider $2 - \sqrt{3}$ is the another root of the equation (1)

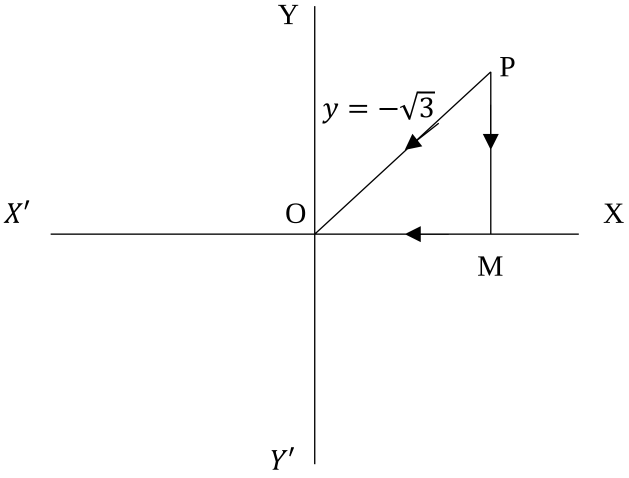

Fig. 9

Let, $x = 2 - \sqrt{3}$ ……….. (6)

$\Rightarrow x - 2 = -\sqrt{3}$

Let, $x - 2 = y$ …………. (7)

Then $y = -\sqrt{3}$ $\quad\therefore y < 0$

Therefore, it is clear that y is a countdown number.

From Fig. 9, we have

$$\overrightarrow{PO^2} = \overleftarrow{OP^2} = \overleftarrow{y^2}$$

$$\therefore \overleftarrow{y^2} = \overleftarrow{(-\sqrt{3})^2} = \overleftarrow{3}$$

$$\Rightarrow -y^2 = -3$$

$$\Rightarrow -y^2 + 3 = 0 \quad \text{…………. (8)}$$

Equation (8) is a two-dimensional quadratic equation

Now, putting the value of $y = x - 2$ in equation (8) we get

$$-(x-2)^2 + 3 = 0$$

$$\Rightarrow -x^2 + 4x - 4 + 3 = 0$$

$$\Rightarrow -x^2 + 4x - 1 = 0 \quad \text{……….. (9)}$$

Now, comparing equation (9) with equation (1) we get

$$a = -1, b = 4, c = -1$$

Since, $b^2 - 4ac = (+4)^2 - 4.(-1).(-1) = 16 - 4 = 12$

$$\therefore \sqrt{b^2 - 4ac} = \sqrt{12}$$

Therefore, equation (9) satisfies the condition of a Pure Quadratic Equation.

So, the required Pure Quadratic Equation. Having root $2-\sqrt{3}$ is

$$-x^2+4x-1=0$$

Here, we find that the coefficients of equation (5) and equation (9) are not one and the same concerning the general form of the quadratic equation $ax^2+bx+c=0$. Therefore equation (5) and equation (9) do not represent one and the same quadratic equation.

We observed that the equation (5) having root $2+\sqrt{3}$ and the equation (9) having root $2-\sqrt{3}$ are not one and the same quadratic equation.

Therefore, conjugate pairs of root $2+\sqrt{3}$ and $2-\sqrt{3}$ can not occur in one and the same Pure Quadratic Equation.

Note : (1) The quadratic expression of equation (5) will represent graphically a concave upward parabola because $a = 1 > 0$ whereas the quadratic expression of equation (9) will represent graphically a concave downward parabola because $a = -1 < 0$.

(2) Conjugate pairs of roots $2+\sqrt{3}$ and $2-\sqrt{3}$ may occur in the Pseudo Bi – Quadratic Equation

$$(x^2-4x+1)(-x^2+4x-1)=0$$

Problem 8 :

Solve :

$$x^2 - x - 3 = 0$$

Solution :

$$x^2 - x - 3 = 0 \text{ .... } (1)$$

Now, the discriminant of equation (1) is

$\sqrt{b^2-4ac} = \sqrt{(-1)^2-4.1.-3} = \sqrt{1+12} = \sqrt{13}$, a quadratic surd number.

Since the square root of the discriminant of equation (1) is a quadratic surd, the equation (1) is a Pure quadratic equation.

Since, $-3 < 0$, according to the theory of dynamics of numbers the equation takes the form

$$\overrightarrow{x^2-x}+\overleftarrow{3}=0 \text{ .... } (2)$$

According to 3 rd law of the theory of dynamics of numbers

$$x^2 - x = 3$$

$$\Rightarrow x^2 - 2.\frac{1}{2}.x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 3$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 = 3 + \left(\frac{1}{2}\right)^2 = \frac{13}{4}$$

$$\Rightarrow x - \frac{1}{2} = \frac{\sqrt{13}}{2}$$

$$\Rightarrow x = \frac{1}{2} + \frac{\sqrt{13}}{2} = \frac{1+\sqrt{13}}{2}$$

Solution of the equation (1) is count up $x = \overrightarrow{x} = +x$

$$\therefore x = \frac{1+\sqrt{13}}{2}$$

Note: We can find one and only one root in a Pure quadratic equation.

Problem 9 :

Solve :

$$x^2 + x + 4 = 0$$

Solution :

$$x^2 + x + 4 = 0 \ldots\ldots (1)$$

Since, 4 > 0, the inherent nature of one unknown quantity of *x* in equation (1) is a two-dimensional countdown *x* number. Also, the discriminant of equation (1) is – 15, i.e., less than 0 (zero). Hence, equation (1) is a Pure quadratic equation.

Now, according to the theory of dynamics of numbers equation (1) takes the form

$$\overleftarrow{x^2 + x} + \overrightarrow{4} = 0$$

According to 3 rd law of the theory of dynamics of numbers

$$x^2 + x = 4$$

$$\Rightarrow x^2 + 2.\frac{1}{2}.x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 4$$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 = 4 + \frac{1}{4} = \frac{17}{4}$$

$$\Rightarrow x + \frac{1}{2} = \frac{\sqrt{17}}{2}$$

$$\Rightarrow x = -\frac{1}{2} + \frac{\sqrt{17}}{2} = \frac{-1+\sqrt{17}}{2}$$

According to the inherent nature of x, x is a countdown $x = \overleftarrow{x} = -x$

So, the solution to equation (1) will be

$$\overleftarrow{x} = -x = -\frac{-1+\sqrt{17}}{2}$$

Now, let us solve equation (1) by convensional method

$$x^2 + x + 4 = 0$$

$$\therefore\ x = \frac{-1 \pm \sqrt{(1)^2 - 4.1.4}}{2.1} = \frac{-1 \pm \sqrt{-15}}{2} = \frac{-1 \pm i\sqrt{15}}{2}$$

Observation :

I. We can find the solution to equation (1) in the numerical value of x which is $\overleftarrow{x} = -\frac{-1+\sqrt{17}}{2}$ by using the theory of dynamics of numbers though the discriminant of the equation (1) is – 15, i.e. less than 0 (zero).

II. Since we do not know the numerical value of $i = \sqrt{-1}$, we are unable to find the solution of equation (1) in the numerical value of x by convensional method.

Problem 10 :

Solve :

$$x^2 + 5x + 6 = 0$$

Solution :

$$x^2 + 5x + 6 = 0 \ \ldots\ldots\ldots \ (1)$$

In comparisons with the general form of quadratic equation $ax^2 + bx + c = 0$, here a = 1, b = 5, c = 6

In equation (1) we find that (i) $b > 0$ (ii) $\sqrt{b^2 - 4ac} = \sqrt{5^2 - 4.1.6} = \sqrt{1} = 1$, a rational number. equation (1) is factorizable and equation (1) does not satisfy any one of the conditions of the Pure quadratic equation.

Hence equation (1) is a Pseudo quadratic equation.

Since, 6 > 0, the inherent nature of x is a countdown number according to the theory of dynamics of numbers.

Now,

$$x^2 + 5x + 6 = 0$$

$$\Rightarrow x^2 + 2x + 3x + 6 = 0$$

$$\Rightarrow x(x+2) + 3\,(x+2) = 0$$

$$\Rightarrow (x+2)(x+3) = 0$$

$$\therefore x + 2 = 0 \ or \ x + 3 = 0$$

$$\therefore\ x = -2 \ or \ x = -3$$

Therefore, – 2 and – 3 are the roots of the equation (1)

IV. Conclusion

In the fifth century Aryabhata, an Indian Mathematician is the first person who invented 0 (zero) as a placeholder and in the algorithm for finding square roots in his Sanskrit treatises whereas the present author is the first person who defined 0 (zero) as the starting point of any number. The doctrine of 'Sunyata' or 'Emptiness' is one of the profound contributions of philosophy from India to the world.

The author proved that the value of x which satisfies the quadratic equation cannot be the only criteria to designate as the root or roots of an equation with examples.

A new mathematical concept of the dimension of a number has been developed by the author. Introducing this new concept, the author formed the structure of the general form of a quadratic equation $ax^2 + bx + c = 0$, into two forms: 1) Pure Quadratic Equation (2) Pseudo Quadratic Equation. The author becomes successful to prove that the root of a Pure quadratic equation has one and only one root which is a two-dimensional number but the root of a Pseudo quadratic equation has two roots only which are one-dimensional numbers. The author also proved that all Pseudo quadratic equation $ax^2 + bx + c = 0$, is factorizable but all factorizable quadratic equation is not Pseudo quadratic equation.

The author proved that any quadratic surd number is a two-dimensional number. The author differs from the convensional theorem in the theory of quadratic equations : "A quadratic equation has two and only two roots."

In light of this new concept the author stated and proved a new theorem "In a quadratic equation with rational coefficients irrational roots cannot occur in conjugate pairs."

A quadratic equation $ax^2 + bx + c = 0$ belonging to any form whether it is a Pure quadratic equation or a Pseudo quadratic equation, can be solved by introducing the concept of the "Theory of Dynamics of Numbers" even if the discriminant $b^2 – 4ac < 0$, in real numbers only without executing the concept of imaginary numbers. Therefore, there cannot exist any imaginary root in any form of quadratic equation, $ax^2 + bx + c = 0$.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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