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HYDROMAGNETIC UNSTEADY FREE CONVECTION FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE

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Abstract

The unsteady free convection flow of an electrically conducting fluid past an impulsively started vertical plate acted on by a uniform transverse magnetic field has been considered. The solutions are obtained analytically and their natures are shown graphically for different values of the Hartmann number.

Keyword and phrases : *convection flow, magnetic flow, vertical plate, Hartmann number.*

সংক্ষিপ্তসার

একটি সম - তির্যক চৌম্বক ক্ষেত্র দ্বারা ক্রিয়াশীল আকস্মিক খাতে শুরু হওয়া উলম্ব প্লেটে বিদ্যুৎ পোষিত পরিবাহী প্রবাহী পদার্থের অতিক্রমনের পরিবর্তী মুক্ত পরিচালন প্রবাহকে বিবেচনা করা হয়েছে। ইহাদের সমাধান বিশ্লেষণী পদ্ধতি নির্ণয় করা হয়েছে এবং হার্টম্যান সংখ্যার বিভিন্ন মানের জন্য লেখচিত্রেরসাহায্যে ইহাদের প্রকৃতিকে দেখানো হয়েছে।

1. Introduction :

The flow past an impulsively started semi-infinite plate was studied by Stewartson [1,2], Hall[3], Tani and Yu [4] and others. Illingworth [5] solved the Stokes' problem for a compressible fluid by the method of successive approximation while Soundalgeker [6] studied the problem for a vertical plate taking into account the presence of the free convection currents due to the temperature difference of the plate and fluid. Soundalgeker [7] also generalized the same problem considering the flow past an impulsively started vertical plate resulting from the buoyancy forces which arise from the combination of temperature and species concentration.

In the present paper our object is to consider the unsteady free convection flow of an electrically conducting fluid past an infinite vertical plate under the action of a uniform transverse magnetic field. It is assumed that the flow past an impulsively started infinite plate. The analytical expressions for the velocity, temperature and skin-friction are obtained and the nature of the velocity distribution are shown graphically for different values of the Hartmann number.

2. Formulation of the problem

Let us consider the unsteady free convection flow of a viscous incompressible conducting fluid past an infinite vertical plate. The x'-axis is taken along the plate and y'-axis is taken normal to the plate. At time $t' > 0$, the plate is assumed to be moving impulsively in the upward direction with constant velocity U_0 and the plate temperature is also raised to T'_ω. Also it is assumed that a uniform magnetic field B_0 is applied along the direction of the x'-axis.

Then the flow is governed by the system of equations :

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u, \tag{1}$$

$$\rho C'_P \frac{\partial T'}{\partial y'} = K \frac{\partial^2 T'}{\partial y'2}. \tag{2}$$

The initial and boundary conditions are

$$\begin{aligned} &u' = 0, T' = T'_\infty, \text{ for } y' \geq 0, \; t' \leq 0, \\ &\left.\begin{aligned} &u' = 0, T' = T'_\omega, \text{ at } y' = 0, \\ &u' = 0, T' = T'_\infty, \text{ as } y' \to \infty, \end{aligned}\right\} \quad t' > 0. \end{aligned} \tag{3}$$

Here u' is the velocity in x'-direction, t' the time, g the acceleration due to gravity, β the coefficient of volume expansion, T' the temperature of the fluid, T'_∞ the temperature of the fluid far away from the plate, T'_ω the plate temperature, ν the kinematic viscosity, ρ' the density, C'_p the specific heat at constant pressure, K the thermal conductivity.

On introducing the following non-dimensional quantities

$$u = u'/U_0, \theta = \frac{T' - T'_\infty}{T_\omega - T'_\infty}, y = y'U_0/\nu, t = \frac{U_0^2 t'}{\nu} \tag{4}$$

$$G_r = \frac{\nu g\beta(T'_w - T'_\infty)}{U_0^3} = \text{Grashoff number},\ P = \frac{\mu C_p}{k} = \text{Prandtl number},$$

$$M = \frac{B_0 \nu}{U_0}\sqrt{\frac{\sigma}{\mu}} = \text{Hartmann number},$$

in equations (1), (2), we get

$$\frac{\partial u}{\partial t} = G_r \theta + \frac{\partial^2 u}{\partial y^2} - M^2 u, \tag{5}$$

$$P\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

and the corresponding initial and boundary conditions are

$$\begin{aligned} &u = 0,\ \theta = 0, \quad \text{for all } y \geq 0,\ t \leq 0, \\ &\left.\begin{aligned} &u = 1,\ \theta = 1, \ \text{at}\ y = 0 \\ &u = 0,\ \theta = 0, \ \text{as}\ y \to \infty \end{aligned}\right\} t > 0. \end{aligned} \tag{7}$$

3. Solutions

Using Laplace-Transform technique, the solutions of the equations (5) and (6) subject to the boundary conditions (7) are given by

$$\theta = \text{erfc}\,(\eta\sqrt{\rho}), \tag{8}$$

$$\begin{aligned} u = 1/2\left(1 - \frac{G_r}{M^2}\right) & e^{M^2 t}\{e^{-2M\eta\sqrt{t}}\,erfc(\eta - M\sqrt{t}) \\ & + e^{2M\eta\sqrt{t}}\,erfc(\eta + M\sqrt{t}) + \frac{G_r}{M^2}\,erfc(\eta\sqrt{P}) \\ & + \frac{G_r}{M^2}[e^{M'^2 t}\{e^{-2M'\eta\sqrt{t}}\,erfc(\eta - M'\sqrt{t}) \\ & + e^{2M'\eta\sqrt{t}}\,erfc(\eta + M'\sqrt{t})\} - e^{\alpha t}\{e^{-2\eta\sqrt{P\alpha t}} \\ & erfc(\eta\sqrt{P} - \sqrt{\alpha t}) + e^{2\eta\sqrt{P\alpha t}}\,erfc(\eta\sqrt{P} + \sqrt{\alpha t})\}], \end{aligned} \tag{9}$$

where

$$\eta = y/2\sqrt{t},\ M'^2 = \frac{M^2 P}{P-1} \text{ and } \alpha = \frac{M^2}{P-1}$$

The skin-friction is given by

$$\tau = \tau / pU_0^2 = -\frac{du}{d\eta}\Bigg|_{\eta=0} \tag{10}$$

Substituting (9) into (10) we get

$$\begin{aligned}\tau = {} & 2\left(1-\frac{G_r}{M^2}\right)e^{M^2 t}\left\{M\sqrt{t}\ erfc(M\sqrt{t})+\frac{1}{\sqrt{\pi}}e^{-M^2 t}\right\}+\frac{2G_r}{M^2}\sqrt{\frac{P}{\pi}} \\ & +\frac{4G_r}{M^2}\left[e^{M'^2 t}\left\{M'\sqrt{t}\ erf(M'\sqrt{t})+\frac{1}{\sqrt{\pi}}e^{-M'^2 t}\right\}-e^{\alpha t}\left\{\sqrt{P}\alpha t\ erf(\sqrt{\alpha t})+\sqrt{\frac{P}{\pi}}e^{-\alpha t}\right\}\right]\end{aligned} \tag{11}$$

4. Numerical discussions

The numerical values of the velocity are shown in figure 1 for different values of the Hartmann number M. For numerical calculation, it is assumed that P=2.0, t=0.2, G_r=1.0. From the Figure, we observe that the velocity decreases as M increases.

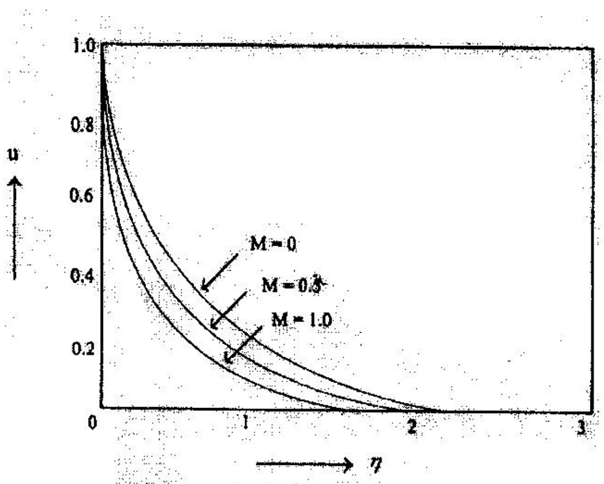

Fig. 1

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