

SURFACE WAVES IN VISCO-ELASTIC INITIALLY STRESSED SOLIDS

Sudipta Sengupta & Indrajit Roy

Indian Institute of Mechanics of Continua and Mathematical Sciences.

P – 9/1, L.I.C. Township, Madhyamgram, Kolkata- 700129, W.B.,India.

E-mail: sudipta_sen_gupta@excite.com

E-mail: debraj0017@rediffmail.com

(Received on September 9,2005)

Abstract.

The objective of this investigation is to study general surface waves and Rayleigh, Love and Stoneley waves as particular cases in visco-elastic solids under initial stress of hydrostatic tension or compression. Firstly, the general theory of surface waves in visco-elastic solids under initial stress has been formulated.. The visco-elasticity of the solid medium involving time rate of stress and strain is considered to be of first order, The general n -th order visco-elasticity of similar type is very cumbersome to handle.. The investigated problems and the wave-velocity equations are in fair agreement with the corresponding results of the classical problems in absence of viscosity and initial stress.

সংক্ষিপ্তসার

উদ্ভূতকৈ টান অথবা সংচাপের প্রারম্ভিক পীড়নের অধীন সান্দ্র - স্থিতিস্থাপক কঠিন বস্তুতে সাধারণ তল - তরঙ্গ, র‍্যালি, লাভ এবং স্টোনলি তরঙ্গের অনুসন্ধানই এই গবেষণার লক্ষ্য। প্রথমতঃ প্রারম্ভিক পীড়নের অধীন সান্দ্র-স্থিতিস্থাপক কঠিন বস্তুতে তল - তরঙ্গের সাধারণ তত্ত্ব সূত্রাকারে প্রকাশ করা হয়েছে। এরপর ভুকম্পন তরঙ্গের তল-তরঙ্গ যেমন - ব্যালে, লাভ, স্টোনলি তরঙ্গের মত বিশেষ তল-তরঙ্গের ক্ষেত্রে অনুসন্ধানের কাজে এই সাধারণ তত্ত্বকে ব্যবহার করা হয়েছে। পীড়ণ ও বিকৃতির সময়-হার কঠিন বস্তুর মাধ্যমের সান্দ্র-স্থিতিস্থাপকতাকে প্রথম ক্রম হিসাবে বিবেচনা করা হয়েছে। অনুরূপ ধরনের সাধারণ n -তম ক্রমের সান্দ্র - স্থিতিস্থাপকতাকে কাজ করা যথেষ্ট দুর্বল। যা'হোক লেখকদ্বয় পরবর্তী কাজে ভুকম্পন বিদ্যার ক্ষেত্রে অনুরূপ বিশ্লেষণ পরিবেশন করার সামগ্রিক প্রয়াস করবেন। সান্দ্রতা এবং প্রারম্ভিক পীড়ণ অনুপস্থিত হলে অনুসন্ধানকৃত সমস্যা এবং তরঙ্গ বেগ সমীকরণ সুবিদিত সমস্যার অনুরূপ ফলা-ফলের সঙ্গে সম্পূর্ণভাবে সঙ্গতিপূর্ণ।

Keyword and phrases : surface wave, visco-elastic solid, initial stress.

সংক্ষিপ্তসার

উদ্ভূতকৈ টান অথবা সংচাপের প্রারম্ভিক পীড়নের অধীন সান্দ্র - স্থিতিস্থাপক কঠিন বস্তুতে সাধারণ তল - তরঙ্গ, র‍্যালি, লাভ এবং স্টোনলি তরঙ্গের অনুসন্ধানই এই গবেষণার লক্ষ্য। প্রথমতঃ প্রারম্ভিক পীড়নের অধীন সান্দ্র-স্থিতিস্থাপক কঠিন বস্তুতে তল - তরঙ্গের সাধারণ তত্ত্ব সূত্রাকারে প্রকাশ করা হয়েছে। এরপর ভুকম্পন তরঙ্গের তল-তরঙ্গ যেমন - ব্যালে, লাভ, স্টোনলি তরঙ্গের মত বিশেষ তল-তরঙ্গের ক্ষেত্রে অনুসন্ধানের কাজে এই সাধারণ তত্ত্বকে ব্যবহার করা হয়েছে। পীড়ণ ও বিকৃতির সময়-হার কঠিন বস্তুর মাধ্যমের সান্দ্র-স্থিতিস্থাপকতাকে প্রথম ক্রম হিসাবে বিবেচনা করা হয়েছে। অনুরূপ ধরনের সাধারণ n -তম ক্রমের সান্দ্র - স্থিতিস্থাপকতাকে কাজ করা যথেষ্ট দুর্বল। যা'হোক লেখকদ্বয় পরবর্তী কাজে ভুকম্পন বিদ্যার ক্ষেত্রে অনুরূপ বিশ্লেষণ পরিবেশন করার সামগ্রিক প্রয়াস করবেন। সান্দ্রতা এবং প্রারম্ভিক পীড়ণ অনুপস্থিত হলে অনুসন্ধানকৃত সমস্যা এবং তরঙ্গ বেগ সমীকরণ সুবিদিত সমস্যার অনুরূপ ফলা-ফলের সঙ্গে সম্পূর্ণভাবে সঙ্গতিপূর্ণ।

1. Introduction.

The Cauchy theory of initial stress has been presented in the book of Love [1]. If a sheet of metal is rolled up into a cylinder and the edges welded together, the cylinder so formed is in a state of initial stress. The unstressed state can not be attained without cutting the cylinder open and the cylinder is always in a state of initial stress. Similarly, if a body is in equilibrium under the mutual gravitation of its parts, it is in a state of initial stress; if the body is large, the stress is enormous. We may consider the earth to be in a state of initial stress under its own gravitation. The initial stress is developed by slow degrees of creep and it is hydrostatic in nature. Moreover, the earth is under the action of large masses over its surface; it also creates initial stress. Under such circumstances if waves are propagated over the surface of the earth it may be influenced by the initial stress. In this paper this initial stress has been taken into

consideration in studying the surface waves. The theory of initial stress has been considered by Biot [2] in a different manner which is highly applicable and admits wider applications and it contains a good number of references [3-6] of the works in mechanics of continua in dealing with various problems of highly applicable in nature. Yu and Tang [7] investigated magneto-elastic waves in initially stressed conductors. Dey and De [21] studied velocity of shear waves in an initially stressed incompressible anisotropic medium. The mathematical modeling of visco-elasticity has been presented in the monographs of Flugge [8], Bland [9] and the work of Hunter [10].

In discussing earthquake waves on the surface of the earth, Jeffreys [11-13] studied surface waves, very befitting to the actual situation of earthquakes. It is presented to a limited extent in his book 'The Earth' [14]. He also studied the elastic waves in a stratified medium. Prior to this, Rayleigh [15] studied the elastic waves propagated along the plane surface of the solid. Stoneley [16-18] studied elastic waves at the surface of separation of two solids and Love waves in a triple surface layer. Moreover, he studied Rayleigh waves in a medium with two surface layers. Rajneesh and Deswal [20] investigated surface wave propagation through a cylindrical bore in a micropolar generalized thermoelastic medium without energy dissipation. Acharya and Mandal [22] studied propagation of Rayleigh surface waves with small wavelengths in nonlocal visco-elastic solids. Pal, Acharya & Sengupta [23] investigated effect of surface stresses on surface waves in elastic solids.

Following these concepts and the theory given by Yu and Tang [7], the present authors have investigated different types of surface waves in visco-elastic medium under the influence of initial stress. The wave velocity equations obtained in each case is in agreement with the corresponding classical results in absence of viscosity and initial stress.

2. The problems and basic equations

The equations of motion for a perfectly elastic solid under initial stress (hydrostatic tension or compression) are [7]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = -p_0 \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (i, j = 1, 2, 3) \quad (1)$$

where p_0 is the hydrostatic tension when $p_0 < 0$ or compression when $p_0 > 0$, τ_{ij} is the stress tensor over the initial stress, u_i is the displacement vector with respect to coordinate axes ox_1, ox_2, ox_3 at time t , and ρ is the density of the material.

Let us consider that M_1 and M_2 be two electrically conducting charge free isotropic, homogeneous, visco-elastic, semi-infinite solid media welded in contact under an initial hydrostatic tension or compression. We further assume that the medium still remains isotropic and homogeneous under the action of initial stress. We consider a system of orthogonal Cartesian axes $0x_1x_2x_3$, where the origin O is on the interface of the two media and ox_3 is normal to the interface (fig.1).

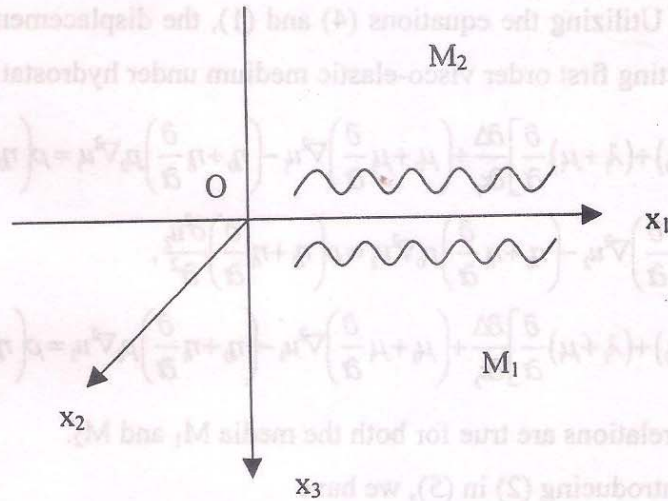


Figure 1. Interface Geometry.

We now consider the possibility of a type of wave traveling along the positive direction of x_1 axis in such a manner that the disturbance produced is largely confined to the neighbourhood of the boundary of the media and at any instant all particles on any line parallel to x_2 axis have equal displacements. According to the first assumption we can conclude that the wave is a surface one and from the second assumption it is easy to understand that all partial derivatives with respect to x_2 coordinate are zero. Now, by using the formulae $\mathbf{u} = \text{grad}\phi + \text{curl}\psi$, the displacement components u_1 and u_3 at any point can be expressed in the following form:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}, \quad (2)$$

so that

$$\tilde{N}^2 f = D \cdot \tilde{N}^2 y = \frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}, N^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, D = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \quad (3)$$

where ϕ and ψ are functions of coordinates x_1, x_3 , and time t , which are, in fact, displacements potentials.

Now, the first order stress-strain relations of an isotropic visco-elastic medium is [19]

$$\left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) \tau_{ij} = \left(\lambda_0 + \lambda_1 \frac{\partial}{\partial t} \right) \Delta \delta_{ij} + 2 \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) e_{ij} \quad (4)$$

in which η_0, λ_0, μ_0 are elastic constants, η_1, λ_1, μ_1 are constants due to viscosity, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ is the strain tensor, δ_{ij} is the Kronecker symbol and Δ is the dilatation.

Utilizing the equations (4) and (1), the displacement equations of motion for a conducting first order visco-elastic medium under hydrostatic stress can be written as

$$\left. \begin{aligned} \left[(\lambda_0 + \mu_0) + (\lambda_1 + \mu_1) \frac{\partial}{\partial t} \right] \frac{\partial \Delta}{\partial x_1} + \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \nabla^2 u_1 - \left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) p_0 \nabla^2 u_1 &= \rho \left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 u_1}{\partial t^2}, \\ \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \nabla^2 u_2 - \left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) p_0 \nabla^2 u_2 &= \rho \left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 u_2}{\partial t^2}, \\ \left[(\lambda_0 + \mu_0) + (\lambda_1 + \mu_1) \frac{\partial}{\partial t} \right] \frac{\partial \Delta}{\partial x_3} + \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \nabla^2 u_3 - \left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) p_0 \nabla^2 u_3 &= \rho \left(\eta_0 + \eta_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 u_3}{\partial t^2}, \end{aligned} \right\} \quad (5)$$

These relations are true for both the media M_1 and M_2 .

Now introducing (2) in (5), we have

$$\left. \begin{aligned} \frac{\partial^2 \phi}{\partial t^2} &= \left[\frac{V_{1p}^2 + V_{2p}^2}{L} \frac{\partial}{\partial t} - \frac{p_0}{\rho} \right] \nabla^2 \phi, \\ \frac{\partial^2 \psi}{\partial t^2} &= \left[\frac{V_{1s}^2 + V_{2s}^2}{L} \frac{\partial}{\partial t} - \frac{p_0}{\rho} \right] \nabla^2 \psi, \\ \frac{\partial^2 u_2}{\partial t^2} &= \left[\frac{V_{1s}^2 + V_{2s}^2}{L} \frac{\partial}{\partial t} - \frac{p_0}{\rho} \right] \nabla^2 u_2 \end{aligned} \right\} \quad (6)$$

$$\text{where } V_{1p}^2 = \frac{\lambda_0 + 2\mu_0}{\rho}, V_{2p}^2 = \frac{\lambda_1 + 2\mu_1}{\rho}, V_{1s}^2 = \frac{\mu_0}{\rho}, V_{2s}^2 = \frac{\mu_1}{\rho}, L = \eta_0 + \eta_1 \frac{\partial}{\partial t}.$$

Equations (6) are applicable to both M_1 and M_2 . Here M_2 is identified by the properties $\rho', \eta'_0, \eta'_1, \lambda'_0, \lambda'_1, \mu'_0, \mu'_1$.

Now in this problem we apply the following boundary conditions of continuity across the interface of the media M_1 and M_2 :

- (i) The displacement components u_i at the interface must be continuous.
- (ii) The stress components τ_{31}, τ_{32} and τ_{33} which are given as

$$\left. \begin{aligned} L\tau_{31} &= \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \left(2 \frac{\partial^2 \phi}{\partial x_1 \partial x_3} + \frac{\partial^2 \psi}{\partial x_1^2} - \frac{\partial^2 \psi}{\partial x_3^2} \right), \\ L\tau_{32} &= \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \frac{\partial u_2}{\partial x_3}, \\ L\tau_{33} &= \left(\lambda_0 + \lambda_1 \frac{\partial}{\partial t} \right) \nabla^2 \phi + 2 \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 \psi}{\partial x_1 \partial x_3} + \frac{\partial^2 \phi}{\partial x_3^2} \right) \end{aligned} \right\} \quad (7)$$

must be continuous across the interface.

3. Solutions

For investigating the equations (6), we consider the harmonic solutions of the form

$$(\phi, \psi, u_2) = [\hat{\phi}(x_3), \hat{\psi}(x_3), \hat{u}_2(x_3)] e^{i(\eta x_1 - \omega t)} \quad (8)$$

for the medium M_1 . For the medium M_2 we can assume similar solutions taking

$\hat{\phi}', \hat{\psi}', \hat{u}_2'$ in place of $\hat{\phi}, \hat{\psi}, \hat{u}_2$.

Introducing (8) in (6) we have

$$\left. \begin{aligned} \frac{d^2 \hat{\phi}}{dx_3^2} - \left[\eta^2 - \frac{\omega^2 \eta_k^*}{V_{kp}^2 - \eta_k^* \frac{p_0}{\rho}} \right] \hat{\phi} &= 0, \\ \frac{d^2 \hat{\psi}}{dx_3^2} - \left[\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \eta_k^* \frac{p_0}{\rho}} \right] \hat{\psi} &= 0, \\ \frac{d^2 \hat{u}_2}{dx_3^2} - \left[\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \eta_k^* \frac{p_0}{\rho}} \right] \hat{u}_2 &= 0 \end{aligned} \right\} \quad (9)$$

where $\eta_k^* = (\eta_0 - i\omega\eta_1)$, $V_{kp}^2 = V_{1p}^2 - i\omega V_{2p}^2$, $V_{ks}^2 = V_{1s}^2 - i\omega V_{2s}^2$.

Similar relations can be obtained for M_2 by using the dashed variables such as $\hat{\phi}', \hat{\psi}', \hat{u}_2', \eta_0', \eta_1', \lambda_0', \lambda_1', \mu_0', \mu_1', \rho'$ and $V_{1p}', V_{2p}', V_{1s}', V_{2s}', V_{kp}'^2, V_{ks}'^2, \eta_k'^*$.

The solutions of equations (9) are of exponential types. Now, as ϕ, ψ and u_2 describe surface waves, they must become vanishingly small as $x_3 \rightarrow \infty$. So, for the medium M_1 the solutions of equations (6) may be considered in the form of

$$\left. \begin{aligned} \phi &= \left[A e^{-x_3(\eta^2 - \zeta_1^2)^{\frac{1}{2}}} + B e^{-x_3(\eta^2 - \zeta_2^2)^{\frac{1}{2}}} \right] e^{i(\eta x_1 - \omega t)}, \\ \psi &= B_1 e^{\left\{ -x_3(\eta^2 - \zeta_2^2)^{\frac{1}{2}} + i(\eta x_1 - \omega t) \right\}}, \\ u_2 &= C e^{\left\{ -x_3 \left[\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \eta_k^* \frac{p_0}{\rho}} \right]^{\frac{1}{2}} + i(\eta x_1 - \omega t) \right\}} \end{aligned} \right\} \quad (10)$$

and for the medium M_2 the solutions of (6) can be taken as

$$\left. \begin{aligned} \phi' &= \left[A' e^{x_3(\eta'^2 - \zeta_1'^2)^{\frac{1}{2}}} + B' e^{x_3(\eta'^2 - \zeta_2'^2)^{\frac{1}{2}}} \right] e^{i(\eta' x_1 - \omega t)}, \\ \psi' &= B_1' e^{x_3(\eta'^2 - \zeta_2'^2)^{\frac{1}{2}} + i(\eta' x_1 - \omega t)}, \\ u_2' &= C' e^{x_3 \left[\eta'^2 - \frac{\omega^2 \eta_k'^*}{V_{ks}'^2 - \eta_k'^* \frac{p_0}{\rho}} \right]^{\frac{1}{2}} + i(\eta' x_1 - \omega t)} \end{aligned} \right\} \quad (11)$$

where,

$$\left. \begin{aligned} \zeta_1^2 &= \frac{\omega^2 \eta_k^*}{V_{kp}^2 - \eta_k^* \frac{p_0}{\rho}}, \quad \zeta_1'^2 = \frac{\omega^2 \eta_k'^*}{V_{kp}'^2 - \eta_k'^* \frac{p_0}{\rho}}, \\ \zeta_2^2 &= \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \eta_k^* \frac{p_0}{\rho}}, \quad \zeta_2'^2 = \frac{\omega^2 \eta_k'^*}{V_{ks}'^2 - \eta_k'^* \frac{p_0}{\rho}}, \\ \text{and } Q_1 &= \sqrt{1 - \frac{\zeta_1^2}{\eta^2}}, \quad Q_2 = \sqrt{1 - \frac{\zeta_2^2}{\eta^2}}, \\ Q_1' &= \sqrt{1 - \frac{\zeta_1'^2}{\eta'^2}}, \quad Q_2' = \sqrt{1 - \frac{\zeta_2'^2}{\eta'^2}} \end{aligned} \right\} \quad (12)$$

While evaluating the quantities like $(\eta^2 - \zeta^2)^{\frac{1}{2}}$, the root with positive real part is taken into consideration in each case.

Now applying the boundary condition (7), we have

$$A - iQ_2 B_1 = A' + iQ_2' B_1' \quad (13a)$$

$$C = C' \quad (13b)$$

$$iQ_1 A + B_1 = -iQ_1' A' + B_1' \quad (13c)$$

$$\rho \frac{V_{ks}^2}{\eta_k^*} [2iQ_1 A + (1 + Q_2^2) B_1] = \rho \frac{V_{ks'}^2}{\eta_k^*} [-2iQ_1' A' + (1 + Q_2'^2) B_1'] \quad (13d)$$

$$-\rho \frac{V_{ks}^2}{\eta_k^*} \left\{ \eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \eta_k^* \frac{p_0}{\rho}} \right\}^{\frac{1}{2}} C = \rho \frac{V_{ks'}^2}{\eta_k^*} \left\{ \eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks'}^2 - \eta_k^* \frac{p_0}{\rho}} \right\}^{\frac{1}{2}} C' \quad (13e)$$

$$\frac{\rho}{\eta_k^*} \left[\{V_{kp}^2 (Q_1^2 - 1) + 2V_{ks}^2\} A - 2iQ_2 V_{ks}^2 B_1 \right] = \frac{\rho}{\eta_k^*} \left[\{V_{kp'}^2 (Q_1'^2 - 1) + 2V_{ks'}^2\} A' + 2iQ_2' V_{ks'}^2 B_1' \right] \quad (13f)$$

From (13b) and (13e) we have $C = C' = 0$. Hence we conclude that there is no propagation of the displacement u_2 . Therefore, the wave velocity equation is obtained from (13a), (13c), (13d) and (13f) by eliminating the constants A, B_1, A', B_1' in the determinant form as

$$|M_{ij}| = 0, (i, j = 1, 2, 3, 4) \quad (14)$$

in which,

$$M_{11} = 1, M_{12} = -iQ_2, M_{13} = -1, M_{14} = -iQ_2', M_{21} = iQ_1, M_{22} = 1, M_{23} = iQ_1', M_{24} = -1,$$

$$M_{31} = \rho \frac{V_{ks}^2}{\eta_k^*} 2iQ_1, M_{32} = \rho \frac{V_{ks}^2}{\eta_k^*} (1 + Q_2^2), M_{33} = \rho \frac{V_{ks'}^2}{\eta_k^*} 2iQ_1', M_{34} = -\rho \frac{V_{ks'}^2}{\eta_k^*} (1 + Q_2'^2),$$

$$M_{41} = \frac{\rho}{\eta_k^*} \{V_{kp}^2 (Q_1^2 - 1) + 2V_{ks}^2\}, M_{42} = -\frac{\rho}{\eta_k^*} 2iQ_2 V_{ks}^2, M_{43} = -\frac{\rho}{\eta_k^*} \{V_{kp'}^2 (Q_1'^2 - 1) + 2V_{ks'}^2\},$$

$$M_{44} = -\frac{\rho}{\eta_k^*} 2iQ_2' V_{ks'}^2.$$

Equation (14) gives the wave velocity along the common boundary of the two media in presence of initial stress in the nature of hydrostatic tension (or compression) and of first order viscosity including strain rate and stress rate.

4. Particular cases

A) Rayleigh Waves: In the particular case of Rayleigh waves the interface M_1 is considered as free surface and the medium M_2 is treated as vacuum. Then from (13d) and (13f) we have

$$2iQ_1 A + (1 + Q_2^2) B_1 = 0 \quad (15)$$

$$\{V_{kp}^2 (Q_1^2 - 1) + 2V_{ks}^2\} A - 2iQ_2 V_{ks}^2 B_1 = 0 \quad (16)$$

For the indispensable constants A, B_1 from equations (15) and (16) to assume non-zero values, we have

$$|M'_{ij}| = 0, (i, j = 1, 2) \quad (17)$$

where, $M'_{11} = 2iQ_1$, $M'_{12} = 1 + Q_2^2$, $M'_{21} = V_{kp}^2 (Q_1^2 - 1) + 2V_{ks}^2$, $M'_{22} = -2iQ_2 V_{ks}^2$.

Equation (17) represents the visco-elastic Rayleigh wave velocity equation under the initial stress in the nature of hydrostatic tension (or compression) in a medium including strain rate and stress rate.

Now, in the absence of viscous effect the equation (17) reduces to

$$\begin{vmatrix} 2iP_1 & 1 + P_2^2 \\ V_{1p}^2 (P_1^2 - 1) + 2V_{1s}^2 & -2iP_2 V_{1s}^2 \end{vmatrix} = 0 \quad (18)$$

$$\text{where, } P_1^2 = 1 - \frac{\omega^2}{\left(V_{1p}^2 - \frac{p_0}{\rho}\right)\eta^2}, \quad P_2^2 = 1 - \frac{\omega^2}{\left(V_{1s}^2 - \frac{p_0}{\rho}\right)\eta^2}$$

Equation (18) represents the elastic Rayleigh wave velocity equation under the influence of initial stress, hydrostatic tension (or compression) in nature.

Further, in absence of initial stress (i.e. $p_0 = 0$) we obtain from equation (18) the Rayleigh wave velocity for the elastic medium as

$$4\left(1 - \frac{c^2}{V_{1p}^2}\right)^{1/2} \left(1 - \frac{c^2}{V_{1s}^2}\right)^{1/2} = \left(2 - \frac{c^2}{V_{1s}^2}\right)^2 \quad (19)$$

where, $c^2 = \frac{\omega^2}{\eta^2}$. This equation is in complete agreement with the classical result B.

b) Love Waves: In case of Love wave u_2 will be the only component of displacement vector \mathbf{u} to play the role. Here we consider that the medium M_2 is bounded by two horizontal plane surfaces at a finite distance H apart and the upper plane surface is free while the medium M_1 is extended to an infinitely great distance (figure 2).

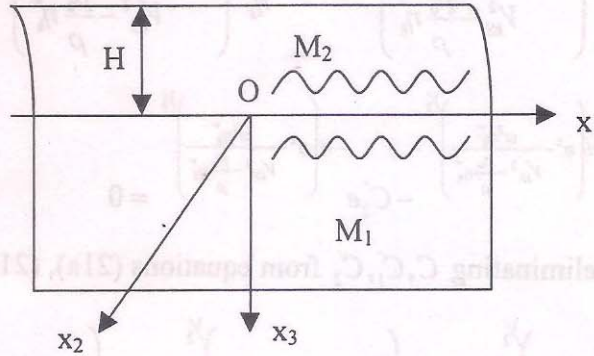


Figure 2: Formulation of Love wave.

In this case, the striking fact is that the displacement in medium M_2 may no longer diminish with distance from the boundary of two media M_1 and M_2 so that for M_2 we can preserve the full solution as

$$u_2' = \left[C_1' e^{x_3 \left(\eta^2 - \frac{\omega^2 \eta_k^{**}}{V_{ks}^{'2} - \frac{p_0}{\rho} \eta_k^{**}} \right)^{1/2}} + C_2' e^{-x_3 \left(\eta^2 - \frac{\omega^2 \eta_k^{**}}{V_{ks}^{'2} - \frac{p_0}{\rho} \eta_k^{**}} \right)^{1/2}} \right] e^{i(\eta x_1 - \omega t)} \quad (20)$$

Here, the restriction that the real part of the expression $\left(\eta^2 - \frac{\omega^2 \eta_k^{**}}{V_{ks}^{'2} - \frac{p_0}{\rho} \eta_k^{**}} \right)^{1/2}$ be positive is

not necessary.

For all times the boundary conditions for the present case are:

- I. u_2 and τ_{32} are continuous at $x_3 = 0$

II. $\tau_{32} = 0$ at $x_3 = -H$, for all times and places.

Applying these boundary conditions we have

$$C = C_1' + C_2' \quad (21a)$$

$$-\rho \frac{V_{ks}^2}{\eta_k^*} \left(\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} \right)^{1/2} C = \rho \frac{V_{ks}^2}{\eta_k^*} \left(\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} \right)^{1/2} (C_1' - C_2') \quad (21b)$$

$$C_1' e^{-H \left(\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} \right)^{1/2}} - C_2' e^{-H \left(\eta^2 - \frac{\omega^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} \right)^{1/2}} = 0 \quad (21c)$$

Now, eliminating C, C_1', C_2' from equations (21a), (21b) and (21c), we have

$$\rho \frac{V_{ks}^2}{\eta_k^*} \left(1 - \frac{c^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} \right)^{1/2} - \rho \frac{V_{ks}^2}{\eta_k^*} \left(\frac{c^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} - 1 \right)^{1/2} \tan \eta H \left(\frac{c^2 \eta_k^*}{V_{ks}^2 - \frac{p_0}{\rho} \eta_k^*} - 1 \right)^{1/2} = 0 \quad (22)$$

where $c = \frac{\omega}{\eta}$

Equation (22) represents the required wave velocity equations for Love wave in a visco-elastic solid medium under an initial hydrostatic tension or compression. It is clear that Love waves depend upon viscous field and initial stress of hydrostatic tension or compression.

C) Stoneley Waves: According to the classical theory, Stoneley waves, generalized form of Rayleigh waves, propagate in the vicinity of the interface of two semi-infinite solid media M_1 and M_2 . So, from our general case, Stoneley wave propagating along with common boundary of M_1 and M_2 can be determined by the roots of the wave velocity equation (14). Again, in the absence of viscosity, initial stress and strain rate and stress rate, this equation surely reduces to the classical result of Stoneley.

Acknowledgement:

The authors remained indebted to Late Prof. Dr. P. R. Sengupta, MSc, PhD, DSc, F.A.Sc.T, F.I.M.A, (UK) F.N.A.Sc. for his guidance in preparation of this paper.

References

- 1) A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity, Cam. Univ. Press, Fourth Ed. (1953)
- 2) M.A. Biot, , Mechanics of Incremental Deformations, John Wiley & Sons, Inc., New York. (1965)
- 3) M.A. Biot, Theory of Elasticity with Large Displacements and Rotations, John Wiley & Sons, Inc., New York, Chapman & Hall Ltd., London. (1939),
- 4) M.A. Biot, Non-linear theory of elasticity and the linearised case for a body under initial stress Philosophical Magazine, 27, (1939), 468
- 5) M.A. Biot, , *Jour.Appl.Phys.*, 10, (1939), 860
- 6) M.A. Biot, *Jour.Appl.Phys.* 2, 1940, 552.
- 7) C.P.Yu, and S. Tang, ZAMP, 17, 1966, 766.
- 8) W.Flugge, , Visco-elasticity, Blaisdell Publishing Co., London. (1967)
- 9) D.R.Bland, The Theory of Linear Visco-elasticity, Pergamon Press, London. (1960)
- 10) S.C. Hunter, Visco-elastic waves, Progress in Solid Mechanics(eds), I.N.Sneddon and R.Hill, North Interscience,Amsterdam, New York. (1960)
- 11) Sir H.Jeffreys, *Mon. Nat. R. Astr. Soc. Geophys. Suppl* 1 (1925) 282
- 12) Sir H.Jeffreys, *Mon. Nat. R. Astr. Soc. Geophys. Suppl* 3 (1935) 253
- 13) Sir H.Jeffreys, *Mon. Nat. R. Astr. Soc. Suppl.* 7, (1957), 332.
- 14) Sir H.Jeffreys, The Earth, Cambridge University Press, Fourth Edition. (1959).
- 15) L Rayleigh, (Struff, J.W.) *Proc. Lond. Math. Soc.* 17, (1885), 4.
- 16) R. Stoneley, *Proc.Roy Soc.A* - 106, (1924), 416.
- 17) R. Stoneley, *Mon. Nat. R. Astr. Soc.Geophys. Suppl.* 4, (1937) 43.
- 18) Stoneley,R.(1955), Rayleigh waves in a medium with two surface layers,Mon.Nat.R. Astr. Soc. Geophys. Suppl. 6, (1955) 610,7, (1955) 7.
- 19) W.Voigt, Theoretische Studien uber die Elasticitats Verhalttniss der Krystalle, Abh.Ges. Wiss. Gottingen 34. (1887)
- 20) Rajneesh Kumar & Sunita Deshwal (2003), *Proc. Nat. Acad. Sci., India*, 73 (A) 2003.
- 21) S. Dey, & P.K. De, (1999), *Sadhana*, 24,(1999) 215.
- 22) D.P. Acharya, & Asit Mondal, *Sadhana*, 27, (2002) 605.
- 23) P.K. Pal, D. Acharya, & P.R. Sengupta, *Sadhana*, 22, (1997) 659.