THERMAL STRESSES AND NONLINEAR THERMAL DEFORMATION ANALYSIS OF SHALLOW SHELL PANEL

Bipi Karmakar*, P. Biswas**, R. Kahali*** and S. Karanjai****

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Abstract:

Using the Galerkin's procedure, the problem of thermal stresses and nonlinear thermal deformation has been analysed for a shallow shell panel. The variation of the central deflection for a square panel has been shown in tabular form.

Keyword and phrases: thermal stress, thermal deformation, shallow shell panel.

সংক্ষিপ্তসার'

অগভীর খোলক প্যানেলের জন্য গেলারকিনের (Galerkin) পদ্ধতি প্রয়োগ করে তাপজ পীড়ণ এবং অ-রৈখিক তাপজ বিকৃতিকে বিশ্লেষণ করা হয়েছে। বর্গাকার প্যানেলের ক্ষেত্রে কেন্দ্রীয় বিক্ষেপের পরিবর্তনশীলতাকে তালিকাকারে দেখানো হয়েছে।

1. Introduction

Modern aerospace strutures such as high-speed spacecrafts, missiles and engineering and nuclear structures are often subjected to thermal loads and reveal a cleary nonlinear response. In such situations the associated strains and stresses are usually determined from von Karman coupled nonlinear partial differential equations extended to thermal loading in terms of transverse displacement and stress function. Although citations of several authors may be made who have employed the method, only a few is mentioned here which contains several cross-references [1-3].

The purpose of the present paper is to further generalize theequations for the case of a simply supported rectangular panel under thermal loading. Application of Galerkin's procedure ultimately leads to a cubic equation involving several parameters.

2. Basic governing equations

Following Donnel [4] Timosshenko-Kriger [5] and with usual notations, basic governing equations for transverse displacement (w) and stress function (F) can be derived in the forms:

$$D\nabla^4 w + \frac{\alpha_1 E}{1 - \nu} (\nabla^2 M_T) = \frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2}$$
 (1)

$$\nabla^4 F = Eh \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] - \alpha_t E \left(\nabla^2 N_T \right) - \frac{Eh}{R} \frac{\partial^2 w}{\partial x^2}$$
 (2)

where M_T and N_T are given by Nowacki [6]

3. Rectangular panel simply-supported at the edges.

We approach the particular problem concerning a rectangular panel simply-supported at the edges, the origin being located at one corner of the shell in the middle surface. Let a, b, be the length and peripheral width of the shell and are taken as the x and y axes, z - axis being normally downwards. The distribution of temperature in the direction of z - axis is taken as linear in the form [6]

$$T(x, y, z) = \tau_0(x, y) + z\tau(x, y)$$
(3)

where
$$\tau_0(x,y) = \frac{T_1 + T_2}{2}, \tau(x,y) = \frac{T_1 - T_2}{h}$$
 (4)

and
$$T_1 = T(x, y, +\frac{h}{2}), T_2 = T(x, y, -\frac{h}{2})$$
 (5)

Since M_T is constant, one can express it in the form of the Fourier series

$$M_T = \sum_{m=1,3}^{\alpha} \sum_{n=1,3}^{\alpha} a_{nm} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$
 (6)

where
$$a_{mn} = \frac{16M_T}{mm\pi^2}$$
. (7)

The deflection w is assumed in the form

$$w = w_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{h}$$
 (8)

which satisfies the following boundary conditions for simply-supported edge:

$$w = 0 = \frac{\partial^2 w}{\partial x^2} + \frac{M_T}{D(1 - v)} \quad at \ x = 0, a$$

$$w = 0 = \frac{\partial^2 w}{\partial y^2} + \frac{M_T}{D(1 - v)} \quad at \ x = 0, b$$
(9)

Since N_T is constant and appears in the boundary conditions for inplane displacements, we take ∇^2 $(N_T) = 0$ in equation (1) from which the stress function is obtained in the from

$$F(x,y) = A\frac{x^2}{2} + B\frac{y^2}{2} + \frac{Ehw_0^2}{32} \left(\frac{a^2}{b^2} \cos \frac{2\pi x}{a} + \frac{b^2}{a^2} \cos \frac{2\pi y}{b}\right) + \frac{Ehw_0}{Ra^2\pi^2 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b},$$
(10)

where A and B are arbitary constants to be determined from inplane boundary conditions.

In accordance with the conditions occurring in airplane structures, the shell is considered rigidly framed, all edges remaining unaltered after deformation. The elongations of the shell in the directions of x and y are independent of y and x respectively. By [ref. 5, p-426] one gets the constants A and B as:

$$A = \frac{Ehw_0^2 \pi^2}{8(1-v^2)} \left[\frac{1}{b^2} + \frac{v}{a^2} \right] - \frac{E\alpha_i N_T}{1-v} ,$$

$$B = \frac{Ehw_0^2 \pi^2}{8(1-v^2)} \left[\frac{1}{b^2} + \frac{v}{a^2} \right] - \frac{E\alpha_i N_T}{1-v} .$$
(11)

Applying Galerkin's procedure in equation (1) a cubic equation is obtained for central deflections in the following non-dimensional form:

$$C_{1} \left(\frac{w_{0}}{h}\right)^{3} - c_{2} \left(\frac{w_{0}}{h}\right)^{2} + c_{3} \left(\frac{w_{0}}{h}\right) - C_{4} = 0$$
 (12)

where C₁, C₂, C₃ and C₄ are known constants.

Median surface membrane stresses N_{x} and N_{y} are given by

$$(N_x) = C_5 \left(\frac{w_0}{h}\right)^2 - C_6 \left(\frac{w_0}{h}\right) - \frac{\alpha_i (T_1 + T_2)}{2(1 - \nu)}$$

$$\frac{\frac{a}{2}, \frac{b}{2}}{Eh}$$

$$(13)$$

$$(N_{y}) = C_{1} \left(\frac{w_{0}}{h}\right)^{2} - C_{8} \left(\frac{w_{0}}{h}\right) - \frac{\alpha_{t}(T_{1} + T_{2})}{2(1 - \nu)}$$

$$\frac{\frac{a}{2}, \frac{b}{2}}{Eh}$$
(14)

where C₅, C₆, C₇ and C₈ are known constants.

4. Numerical Computations

Table – I exhibits the variations of non-dimensional central defiectious (w/h) for a square panel (b/a=1) and for b/a = 2 considering a/h = b/h = 10 and α_t = 11.9 x 10⁻⁶ (steel) per o/c

Table - 1

w/h	$(T_1 + T_2 = 10^0)$	$(T_1 + T_2 = 50^0)$	$(T_1 + T_2 = 100^0)$
	$(T_1 - T_2)/h$	$(T_1 - T_2)/h$	$(T_1 - T_2)/h$
0	0	0	0
.2	.008168(R/h=10)* .009248(R/h=20)* .008732(R/h=10)** .009838(R/h=20)**	.007734(R/h=10)* .008814(R/h=20)* .008293(R/h=10)** .009398(R/h=20)**	.007295(R/h=10)* .008375(R/h=20)* .007853(R/h=10)** .008598(R/h=20)* *
.4	.025690(R/h=10)* .026120(R/h=20)* .030582(R/h=10)** .031024(R/h=20)**	.024820(R/h=10)* .025250(R/h=20)* .029700(R/h=10)** .030140(R/h=20)**	.023940(R/h=10)* .024375(R/h=20)* .028820(R/h=10)** .029260(R/h=20)**

.6	.052654(R/h=10)* .053626(R/h=20)* .069162(R/h=10)** .070150(R/h=20)**	.051350(R/h=10)* .052320(R/h=20)* .067840(R/h=10)** .068830(R/h=20)**	.050035(R/h=10)* .051007(R/h=20)* .066520(R/h=10)** .067520(R/h=20)* *
.8	.090900(R/h=10)* .098630(R/h=20)* .036040(R/h=10)** .137810(R/h=20)**	.095160(R/h=10)* .096880(R/h=20)* .034280(R/h=10)** .136050(R/h=20)**	.093410(R/h=10)* .095140(R/h=20)* .132523(R/h=10)** .134290(R/h=20)**
10.	.115801(R/h=10)* .142830(R/h=20)* .191080(R/h=10)** .218710(R/h=20)**	.113660(R/h=10)* .140660(R/h=20)* .188880(R/h=10)** .216510(R/h=20)**	.111466(R/h=10)* .138460(R/h=20)* .186680(R/h=10)** .214310(R/h=20)**

^{* (}results a square panel when a/b = 1), ** (results for the panel when b/a = 2)

5. Observations

In brief, it is observed from the above table for numerical results that deflections increase with increase of temperature along the thickness of the panel and such increase is higher for shell geometries other than a square panel.

It is also observed that when $T_1 + T_2$ increases, deflection decreases with the increase of $(T_1 - T_2)/h$ for any shell geometry.

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