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STEADY FLOW OF A MICROPOLAR FLUID THROUGH COAXIAL CIRCULAR CYLINDERS UNDER CONSTANT PRESSURE CRADIENT

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Abstract

The aim of this paper is to investigate the problem of steady flow of micropolar fluid in an annulus bounded by two co-axial circular cylinders of radii a and b, b being greater than a. The annular flow takes place under the action of constant pressure gradient. The velocity and microrotation component as well as the rate of discharge of the fluid through the annulus and time time of efflux have been derived analytically in closed forms. Numerical calculations have been given to find out the velocity in viscous fluid and micropolar fluid and a percentage decrease in micropolar fluid over viscous fluid corresponding to this flow have been compared. The microrotation has also been calculated numerically. The rate of discharge for viscous flow is greater than the corresponding rate of discharge in micropolar fluid flow and a percentage decrease is also calculated.It is clear from the numerical calculations that the fluid velocity is always less in micropolar fluid than in viscous fluid. Also the rate of discharge in micropolar fluid is considerably less than that of viscous fluid. In fact, all important results are less in micropolar fluid than the viscous fluid.

Keyword and phrases : *micropolar fluid, cylinder, steady flow, circular pipe.*

সংক্ষিপ্তসার

a এবং b ($b>a$) ব্যসার্ধ বিশিষ্ট দুটি সমাক্ষ বৃত্তীয় বেলনদ্বারা আবদ্ধ বলয়াকারে সূক্ষ্ম মেরু প্রবাহী পদার্থের সুস্থীর প্রবাহের সমস্যার অনুসন্ধান করাই এই গবেষণা পত্রের লক্ষ্য। ধ্রুবক চাপ নতিমাত্রা ক্রিয়ার ফলে বলয়াকার প্রবাহ ঘটে। গতিবেগ এবং সূক্ষ্ম - আবর্তনের উপাংশ তৎসহ বলয়ের মধ্য দিয়ে প্রবাহী পদার্থের নির্গমনের হার এবং নিরন্তর পরিবর্তনশীল প্রবাহ সময়বদ্ধ আকারে বিশ্লেষণী পদ্ধতিতে নির্নয় করা হয়েছে। সান্দ্র-প্রবাহী পদার্থ এবং সূক্ষ্ম - মেরু প্রবাহী পদার্থে গতিবেগ সাংখ্যমানে নির্ণয় করা হয়েছে এবং সান্দ্র-প্রবাহী পদার্থের প্রবাহের পরিপ্রেক্ষিতে সূক্ষ্ম - মেরু প্রবাহী পদার্থের প্রবাহের শতকরা হার যে কমে যায় তা দেখানো হয়েছে এবং তুলনা করা হয়েছে। সূক্ষ-আবর্তনের সাংখ্যমান নির্ণয় করা হয়েছে। সান্দ্র - প্রবাহী পদার্থের প্রবাহের নির্গমন হার সূক্ষ-মেরু প্রবাহী পদার্থের প্রবাহের নির্গমন হার অপেক্ষা বেশী এবং ইহার শতকরা হার কত কম তাহাও নির্ণয় করা হয়েছে । সাংখ্য গণনার মাধ্যমে ইহা স্পষ্ট যে সূক্ষ - মেরু প্রবাহী পদার্থের গতিবেগ সান্দ্র - প্রবাহী পদার্থের গতিবেগ অপেক্ষা সর্বদা কম।সূক্ষ-মেরু প্রবাহী পদার্থের নির্গমন হার সান্দ্র-প্রবাহী পদার্থের নির্গমন হার অপেক্ষা যথেষ্ট কম। বস্তুতঃ সব গুরুত্বপূর্ণ ফলাফলের ক্ষেত্রেই সূক্ষ - মেরু প্রবাহী পদার্থের ফলাফল সান্দ্র - প্রবাহী পদার্থের ফলাফল অপেক্ষা কম।

1. Introduction.

In recent years a new class of fluids executing microscopic effects arising from the local structure and micromotion of the fluid elements has been considered by many researcher [1,2,3,4,5]. In this class the fluid elements are influenced by the spin inertia and for this result it supports stress moments and body moments. As the constitutive equations and the field equations are very complicated and cumbersome to handle, the mathematical problem is not easily amenable to solution as in the corresponding classical problem. A micropolar fluid, in general, exhibits the microrotational effects and microrotational inertia and therefore it supports couplestress and body couples.

In reality this type of fluids are sometimes considered to be more useful than the classical fluids. Classical mechanics of fluids will not be very appropriate under such circumstances where microscopic effects are to be considered. For example, Colloidal fluids, liquid crystals, fluids with additives and suspensions fall into this category. Moreover, if we consider the physical structure of the fluid, it is adequately accomodated by the fluids consisting of bar-like lelments, diatomic gases and animal blood. Clearly, to investigate the flow problem of such type of fluids the classical continuum theory breaks down. Under such circumstances it is necessary to consider a mathematical structure of fluid indentical to the micropolar fluid as considered by Eringen *[1,2,3,4,5]*.

Recently, the unsteady flow of a micropolar fluid past a rotating cylinder has been considered by Sengupta and Pal *[6]*. They *[7]* also considered the problem of transient flow of micropolar fluid through a rectangular channel. Ghosh and Sengupta *[8]* studied the steady motion of a micropolar fluid through a cylindrical pipe. They *[9]* also investigated asymptotic suction problem in the unsteady flow of micropolar liquids. Sengupta and Pal *[10]* considered the problem of steady flow of a micropolar fluid in an annulus bounded by two co-axial circular cylinders. The velocity and microrotation components are obtained in closed form and then stresses and couple stresses are derived.

Following the concept of micropolar fluid as proposed by Eringen *[1,2,5,]* and also following the line of mathematical development of such type of problems as considered above, the authors of the present paper has investigated steady flow of a micropolar fluid through a co-axial circular cylinder with radii a and b, b being greater than a and under constant pressure gradient. A numerical investigation of the titled problem has been made in different directions. Percentage decrease of the velocity and rate of discharge from classical viscous fluid micropolar fluid has been exhibited numerically.

2. Basic equations of motion

In the absence of external body force and body couples, the equations of motion for a micropolar fluid through co-axial circular cylinders bounded by radii a and b (b>a) are

$$(\lambda+2\mu+x)\nabla\nabla.\vec{v}-(\mu+x)\nabla\times\nabla\times\vec{v}+x\nabla\times\vec{\sigma}-\nabla p=\rho v_k \tag{1}$$

and

$$(\alpha+\beta+\delta)\nabla\nabla.\vec{\sigma}-\delta\nabla\times\nabla\times\vec{\sigma}+x\nabla\times\vec{v}-2x\vec{\sigma}=\rho j\sigma_k \tag{2}$$

where $\vec{v}$ is the velocity, $\vec{\sigma}$ is the microrotation vector and p is the pressure. The quantities $\lambda,\mu,x,\alpha,\beta,\sigma$ and j are constants depending upon the characteristic of the particular fluid and

$$v_k=\frac{\partial v_k}{\partial t}+v_{k\ell}v_\ell;\sigma_k=\frac{\partial\sigma_k}{\partial t}+\sigma_{k\ell}\sigma_\ell.$$

The fluid being assumed to be incompressible and homogeneous with constant density p, the equation of continuity is

$$\nabla\cdot\vec{v}=0 \tag{3}$$

3. Statement of the problem and its solution.

Here we wish to find the solution of the equations (1) and (2) when the fluid is flowing steadily through co-axial circular cylinders bounded by radii *a* and *b* (*b*>*a*), the pressure gradient being regarded as constant.

We choose cylindrical co-ordinates (r,θ,z), the z-axis being taken along the axis of the co-axial cylinders.For a steady flow parallel to the axis we seek to determine the velocity and microrotation components as

$$\left.\begin{aligned} &v_r=v_\theta=o \qquad v_z=\omega(r)\\ &\sigma_r=\sigma_z=0, \quad \sigma_\theta=\sigma(r)\end{aligned}\right\} \tag{4}$$

with $p_r=p_q=0$ and $v_{k,k}=0$.

Putting $\frac{\partial p}{\partial z}=-p_1$, where p_1 is the constant pressure gradient, we have from (1) and (2)

$$(\mu+x)(r\omega')'+x(r\sigma)'=-rp_1 \tag{5}$$

$$\delta\left(\sigma'+\frac{1}{r}\sigma\right)'-x\omega'-2x\sigma=0 \tag{6}$$

where the superposed primes indicate the differentiation with respect to r.

From (5) we get

$$\omega' = \frac{1}{\omega + x}\left(-\frac{1}{2}rP_1 - x\sigma\right) + \frac{C_1}{r} \tag{7}$$

where C_1 is an arbitary constant.

Putting this value of ω' in (6) we obtain

$$\sigma'' + \frac{\sigma'}{r} - \left(k^2 + \frac{1}{r^2}\right)\sigma = -rP_2 - \frac{x}{\delta}\cdot\frac{C_1}{r} \tag{8}$$

where

$$k = \left(\frac{2\mu + x}{\mu + x}.\frac{x}{\delta}\right)^{\frac{1}{2}}, P_2 = \frac{xP_1}{2(\mu + x)\delta}$$

The general solution of (8) is

$$\sigma = AI_1(kr) + BK_1(kr) + \frac{P_2 r}{k^2} + \frac{x}{k^2\delta}\cdot\frac{C_1}{r} \tag{9}$$

where I_1 is the modified Bessel function of the first kind of first order and K_1 is the modified Bessel function of second kind of first order and A, B are arbitrary constants.

Putting this value of σ in (7) and integrating we find

$$\omega = \frac{x}{k(\mu + x)}\left[-AI_0(kr) + BK_0(kr)\right] - \frac{P_1 r^2}{2(2\mu + x)} + \frac{2\mu}{2\mu + x}\cdot C_1 \log r + C \tag{10}$$

where I_0,K_0 modifiéd Bessel function of order zero and second kind respectively and C is an arbitrary constant.

The walls of the circular cylinders being sufficiently rough to prevent any slipping, the boundary conditions are

$$\omega(a) = \omega(b) = 0 \tag{11}$$

$$\sigma(a) = \sigma(b) = 0 \tag{12}$$

Using (11) and (12) we get from (9) and (10)

$$\mathrm{AI_1}\ (\mathrm{k}a) + \mathrm{BK_1}\ (\mathrm{k}a) + \frac{x}{k^2\delta}\cdot\frac{C_1}{a} = -\frac{P_2 a}{k^2}$$

$$\mathrm{AI_1}\ (\mathrm{k}b) + \mathrm{BK_1}\ (\mathrm{k}b) + \frac{x}{k^2\delta}\cdot\frac{C_1}{b} = -\frac{P_2 b}{k^2}$$

$$\frac{-x}{k(\mu + x)}I_0(ka).A + \frac{x}{k(\mu + x)}.k_0(ka).B + \frac{1\mu \log a}{2\mu + x}.C_1 + C = \frac{P_1 a^2}{2(2\mu + x)}$$

$$\frac{-x}{k(\mu+x)}I_0(kb).A+\frac{x}{k(\mu+x)}.k_0(kb).B+\frac{1\mu\log b}{2\mu+x}.C_1+C=\frac{P_1b^2}{2(2\mu+x)}$$

In order to find the constants A,B,C and C_1, we write the above four relations in the form

$$\alpha_{11}A+\alpha_{12}B+\alpha_{13}C_1+\alpha_{14}C=\beta_1 \tag{13}$$

$$\alpha_{21}A+\alpha_{22}B+\alpha_{23}C_1+\alpha_{24}C=\beta_2 \tag{14}$$

$$\alpha_{31}A+\alpha_{32}B+\alpha_{33}C_1+\alpha_{34}C=\beta_3 \tag{15}$$

$$\alpha_{41}A+\alpha_{42}B+\alpha_{43}C_1+\alpha_{44}C=\beta_4 \tag{16}$$

where

$$\alpha_{11}=I_1(ka),\alpha_{12}=k_1(ka),\alpha_{13}=\frac{x}{k^2\delta a},\alpha_{14}=0,\beta_1=\frac{-P_2a}{k^2};$$

$$\alpha_{21}=I_1(kb),\alpha_{12}=k_1(kb),\alpha_{23}=\frac{x}{k^2\delta b},\alpha_{24}=0,\beta_2=\frac{-P_2b}{k^2};$$

$$\alpha_{31}=\frac{-x}{k(\mu+x)}I_0(ka),\alpha_{32}=\frac{x}{k(\mu+x)}k_0(ka),\alpha_{33}=\frac{2\mu\log a}{2\mu+x},\alpha_{34}=1,\beta_3=\frac{-P_1a^2}{2(2\mu+x)};$$

$$\alpha_{41}=\frac{-x}{k(\mu+x)}I_0(kb),\alpha_{42}=\frac{x}{k(\mu+x)}k_0(kb),\alpha_{33}=\frac{2\mu\log b}{2\mu+x},\alpha_{44}=1,\beta_3=\frac{-P_1b^2}{2(2\mu+x)}.$$

From (13),(14),(15) and (16), we have by Cramer's rule

$$A=\frac{\Delta_1}{\Delta},B=\frac{\Delta_2}{\Delta},C_1=\frac{\Delta_3}{\Delta},C=\frac{\Delta_4}{\Delta},$$

where $\Delta=|\alpha_{ij}|$; i,j = 1,2,3,4

and Δ_i is obtained from Δ by replacing I-th column by $[\beta_1\beta_2\beta_3\beta_4]^T$.

Hence we finally obtain from (9) and (10)

$$\sigma(r)=\frac{1}{\Delta}\left[\Delta_1I_1(kr)+\Delta_2k_1(kr)+\frac{\Delta}{k^2}\cdot p_2r+\frac{x\Delta_3}{k^2\delta}\cdot\frac{1}{r}\right] \tag{17}$$

$$\omega(r)=\frac{1}{\Delta}\left[\frac{x}{k(\mu+x)}\{-\Delta_1I_0(kr)+\Delta_2k_0(kr)\}-\frac{P_1r^2.\Delta}{2(2\mu+x)}+\frac{2\mu\Delta_3}{2\mu+x}\log r+\Delta_4\right] \tag{18}$$

The rate of discharge of the fluid is given by

$$Q = \int_0^{2\pi} \int_a^b \omega(r).d\theta dr$$

$$= \frac{2\pi}{\Delta}[\frac{-x}{k^2(\mu+x)}\{\Delta_1[bI_1(kb)-aI_1(ka)]+\Delta_2[bk_1(kb)-ak_1(ka)]\}$$

$$-\frac{P_1\Delta}{8(2\mu+x)}(b^4-a^4)+\frac{\mu\Delta_3}{2\mu+x}\left\{(b^2\log b-a^2\log a)-\frac{1}{2}(b^2-a^2)\right\}+\frac{1}{2}\Delta_4(b^2a^2)] \quad (19)$$

Let t be the time of efflux and V be the volume of liquid. then, $t = \frac{V}{Q}$ where Q is given by (19).

4. Numerical results

Now we calculate numerically the values of the velocity ω and microrotation σ of micropolar fluid for various distance of the fluid elements taking $P_1 = x = 1$, $\delta = 1.9$, $\mu = 9$ and $a = 1$, $b = 5$. the corresponding velocities word σ for viscous fluid are calculated and these are exhibited in the following table.

Table – I

R	W	ω	Percentage of decrease	σ
1.5	0.1332308	0.128734	3.3531286	0.1148912
2	0.2037843	0.1939937	4.80443937	0.1556432
2.5	0.2337157	0.219508	6.0790525	0.1671549
3	0.2328485	0.2159035	7.277264	0.1636742
3.5	0.2064236	0.1889052	8.4866265	0.1489642
4	0.1575687	0.1421385	9.7926809	0.1214605
4.5	0.0883016	0.0783324	11.289943	0.0753788

We also calculate numerically the value of the rate of discharge Q of micropolar fluid when $P_1 = x = 1$, $\delta = 1.9$, $\mu = 9$ and $a = 1$, $b = 5$. The corresponding rate of discharge q for viscous fluid is also calculated and these values are shown in the following table.

Table – II

Q	Q	Percentage of decrease
8.7332011	6.8332181	21.75586

5. Discussions

Clearly, from the numerical investigation of the problem we find that the velocity of the micropolar fluid as well as the rate of discharge are always decreased from the classical viscous fluid. The decrease in velocity in micropolar fluid from the classical viscous fluid gradually increase as the fluid elements recedes more and more from the axis of the cylinder. It is exhibited in the table. Moreover, it is that the microrotation component gradually increase upto a certain distance from the axis of the cylinder and thereafter it gradually decreases.

References

1) A.C. Eringen, *Intern. J. Engg. Sci.*, 2, (1964), 205

2) A.C. Eringen, *Proc. XI Intern. Congress of Appl. Mech.* Springer Verlag. (1965)

3) A.C. Eringen, *Proc. 5 th Symposium of Naval Hydrodynamics*, Bergen, Sept. 10. (1964).

4) A.C. Eringen, *Nonlinear theory of continuous Media.* Macgraw-Hill. (1962)

5) A.C. Eringen, *J.Math. Mech.* **16** , (1966).

6) P.R. Sengupta and S.K. Paul, *J.Math. Phy. Sci.*, **22**, (1988), 4.

7) S.K. Paul and P.R. Sengupta, (1967) - *Rev. Roum. Sci. Techn. Mech. Appl.*, **32**, (1967) 2.

8) P.C. Ghosh and P.R. Sengupta, (1963) – *North Bengal Univ. Review*, (Sci. & Techno.), 4, (1963) 2

9) P.R. Sengupta and P.C. Ghosh, *Journal of Technology*, **XXVIII**, (1982), 1.

10) P.R. Sengupta and S.K. Paul , *Journal of Technology*, **XXVIII**, (1982), 2.