Ind. Jour. Mech. Cont. & Math. Sci., Vol.1 July (2006) Pages 53-60

# EFFECT OF RADIATION ON HYDROMAGNETIC VERTICAL CHANNEL FLOW WITH ZERO HEAT FLUX ON THE BOUNDARIES

D.C.Sanyal and R.Chattopadhyay
Department of Mathematics, University of Kalyani,
Kalyani – 741235, West Bengal, India.

**Abstract**

*The effect of radiation on combined free and forced convection flow of an electrically conducting viscous fluid through an open-ended vertical channel permeated by a uniform transverse magnetic field has been considered. The temperature on the walls has been supposed to vary linearly with distance and there is no heat flux on the boundaries. Assuming optically thin limit, the expressions for velocity, induced magnetic field, temperature and the non dimensional flow-rate are obtained and the influence of radiation on these quantities are observed either graphically or in tabulated forms.*

***Keyword and phrases :*** *viscus fluid, convection flow, magnetic fluid, heat flux, radiation.*

**সংক্ষিপ্তসার**

সম-তির্যক চৌম্বক ক্ষেত্রদ্বারা মুক্ত প্রান্ত উলম্ব চ্যানেলের মধ্য দিয়ে তড়িৎ পরিবহনকারী সান্দ্র-প্রবাহী পদার্থের উপর সংযুক্ত মুক্ত এবং বল প্রযুক্ত পরিচালন প্রবাহের বিকিরণের প্রভাব বিবেচনা করা হয়েছে। ধরে নেওয়া হয়েছে যে গাত্রের উষ্ণতা দূরত্বের সঙ্গে রেখিক ভেদে বিদ্যমান এবং প্রান্তগুলিতে কোন নিরন্তর পরিবর্তনশীল তাপ প্রবাহ নেই। আলোকতঃ সূক্ষ্ম সীমা ধরে নিয়ে গতিবেগ, আবিষ্ট চৌম্বক ক্ষেত্র, উষ্ণতা এবং মাত্রা বিহীন প্রবাহ-হার নির্ণয় করা হয়েছে এবং এই সকল বিষয়ের ক্ষেত্রে বিকিরণের প্রভাব লেখচিত্রের সাহায্যে অথবা তালিকাভূক্ত আকারে প্রকাশ করা হয়েছে।

## 1. Introduction

The problem of heat transfer in electrically conducting fluids has drawn the attention to the research workers due to its wide applications in diversified fields like in the design of pumps, shock tubes, magneto- hydrodynamic generators etc., a comprehensive review of which was given by Romig [1]. Siegel [2], Perlmutter and Siegel [3] and Alpher [4] considered the forced convection of an electrically conducting fluid flowing through a channel in the presence of a uniform transverse magnetic field. Gershuni and Zhukhovitsky [5] solved the problem of convective flow through a vertical channel taking the wall temperatures to be constant, whereas Yu [6] dealt with the same problem when the wall temperatures vary linearly with vertical distance in presence of a uniform transverse magnetic field.

But for space applications and higher operating temperatures, the effect of radiation to the above problems must be taken into account. Grief, Habib and Lin [7] considered the problem of fully developed laminar convection flow of a radiating gas in a vertical channel in optically thin limit. Viskanta [8] solved the problem of forced convection flow in a horizontal channel in presence of a uniform magnetic field and studied the influence of radiation on the temperature but not on the induced magnetic field. Gupta and Gupta [9] solved the radiation effect on the combined free and forced convection flow of an electrically conducting fluid inside on open-ended vertical channel and permeated by a uniform magnetic field. For the case of rarefied gases, Sanyal and Samanta [10] solved the problem allowing for a velocity slip at the boundary surface.

In the present paper we disscuss the effect of radiation on the combined free and forced convective hydromagnetic vertical channel flow with zero heat flux at the boundaries in the case of optically thin limit.

## 2. The problem and fundamental equations

Let us consider fully developed steady hydromagnetic laminar flow of an electrically conducting incompressible viscous fluid through an open-ended vertical channel in presence of a uniform magnetic field $B_0$ in the direction normal to the plates at a distance 2L apart. We take the origin at the centre of the channel, the z-axis along the vertical direction and the x-axis along the direction perpendicular to the plates. The uniform magnetic field $B_0$ acts in the direction of x-axis. In such a case the velocity and the induced magnetic field have only a component in the vertical direction which we denote by v and B respectively and all other physical quantities except temperature T and pressure p are functions of x alone [9]. We take the temperature inside the fluid as [9]

$$T = T^* (x) + Nz \qquad (1)$$

where N is the vertical temperature gradient.
The momentum equations in the x and z directions are

$$\frac{\partial p}{\partial x} + \frac{B}{\mu}\frac{dB}{dx} = 0 \qquad (2)$$

$$\nu\frac{d^2 v}{dx^2} + \frac{B_0}{\mu\rho}\frac{db}{dx} + g\beta(\Theta + N_z) - \frac{1}{p}\frac{\partial p}{\partial z} = 0 \qquad (3)$$

where

$$\Theta^* = T^*(x) - T\omega_0, \tag{4}$$

$\rho$ is the fluid density, $\mu$ is the magnetic permeability and g is acceleration due to gravity and $T\omega_0$ is the reference temperature. The equation of continuity is satisfied identically. The energy equation on neglecting the viscous and Ohmic dissipation is

$$Nv = \alpha \frac{d^2\Theta^*}{dx^2} - \frac{1}{\rho C_p}\frac{\partial q_R}{\partial x} \tag{5}$$

where $\alpha$ is the thermal diffusivity of the fluid, $C_p$ is the specific heat at constant pressure and $q_R$ is the radiative heat flux.

The magnetic induction equation is

$$\frac{d^2B}{dx^2} + \sigma\mu B_o \frac{dv}{dx} = 0, \tag{6}$$

$\sigma$ being the conductivity of the fluid.

The fluid does not absorb its own emitted radiation in the case of optically thin limit or, in other words, there is no self absorption, but the fluid absorbs radiation emitted by the boundaries.

Using the relation [11]

$$\frac{\partial q_R}{\partial x} = 4(T^{\cdot} - T_\omega)\int_0^\infty k_{\lambda\omega}\left(\frac{de_{b\lambda}}{dT}\right)_\omega d\lambda \tag{7}$$

for optically thin limit and for non-grey gas near equilibrium, we have from (5)

$$Nv = \alpha \frac{d^2\Theta^*}{dx^2} - C\Theta^2 \tag{8}$$

where

$$C = \frac{4}{\rho C_P}\int_0^\infty k_{\lambda 0}\left(\frac{de_{b\lambda}}{dT}\right)_0 d\lambda \tag{9}$$

In the above, $k_\lambda$ is the absorption coefficient, $e_{b\lambda}$ is the Plank function, $T_\infty$ is the reference temperature and the subscript zero indicates that all the quantities have been evaluated at the reference temperature.

Integrating (2) w.r.t.x, we get

$$p = \frac{B^2}{2\mu} + f(z) \tag{10}$$

Using (10) we have from (3)

$$\nu \frac{d^2 v}{dx^2} + \frac{B_0}{\mu}. \tag{11}$$

The L.H.S. is a function of x alone and the R.H.S. is a function of z only. So each side must be equal to the same constant $C_1$ (say). Therefore we must have

$$\nu \frac{d^2 v}{dx^2} + \frac{B_0}{\mu 0}\frac{db}{dx} + g\beta\Theta^* = C_1. \tag{12}$$

The constant $C_1$ is to be determined either from the conditions of pressure to which the channel is subjected or from the mass flow through the channel.

Introducing the following dimensionless quantities for convenience

$$\eta = \frac{x}{L}, \quad u = \frac{Lv}{\alpha}, \quad t = \frac{\Theta^*}{NL}, \quad b = \frac{B}{B_0} \tag{13}$$

$M = B_0 L\left[\dfrac{\sigma}{\rho\upsilon}\right]^{\frac{1}{2}}$ = Hartmann number,

$R_a = g\beta\dfrac{NL^4}{\upsilon\alpha}$ = Rayleigh number,

$P_m = \alpha\sigma\mu$ = Magnetic Prandtl number, the equations (12), (6) and (8) reduce respectively to

$$\frac{d^2u}{d\eta^2}+\frac{M^2}{P_m}\frac{db}{d\eta}-R_a t=C_2, \tag{14}$$

$$\frac{1}{P_m}\frac{d^2b}{d\eta^2}+\frac{du}{d\eta}=0 \tag{15}$$

$$\frac{d^2t}{d\eta^2}-Ft=-u \tag{16}$$

where

$$F=\frac{L^2C}{\alpha},C_2=\frac{C_1L^3}{\alpha v} \tag{17}$$

Integrating (15) w.r.t.$\eta$ we get

$$\frac{1}{P_m}\frac{db}{d\eta}+u=\text{Constant}=C_3 \tag{18}$$

Eliminating u and b from (14), (16) and (18), we have,

$$\frac{d^4t}{d\eta^4}-(F+M^2)\frac{d^2t}{d\eta^2}(M^2F+R_a)t=C_4 \tag{19}$$

where

$$C_4=M^2C_3-C_2.$$

**3. Solutions**

We assume that the fluid flows through the vertical channel with zero heat flux on the non-conducting boundaries so that

$$\frac{dt}{dn}0 \qquad \text{at} \qquad \eta=\pm 1, \tag{20}$$

$$b=0 \qquad \text{at} \qquad \eta=\pm 1. \tag{21}$$

The no-slip boundary conditions are

$$u = 0 \qquad \text{at} \qquad \eta = \pm 1 . \tag{22}$$

The solutions for the temperature t ($\eta$), velocity u ($\eta$) and the induced magnetic field b ($\eta$) are obtained from (19), (14) and (18) satisfying the boundary conditions (20),(22) and (21) as

$$t(\eta) = A_1 \cosh k_1\eta + A_2 \cosh k_2\eta + \frac{C_4}{M^2F+R_a} \tag{23}$$

$$u(\eta) = (F - K_1{}^2)A_1 \cosh K_1\eta + (F - K_2{}^2)\, A_2 \cosh K_2\eta + \frac{FC_4}{M^2F+R_a} \tag{24}$$

$$\frac{b(\eta)}{P_m} = (F-k_1^2)\frac{A_1}{K_1}[\eta \sinh K_1 - \sinh K_1\eta] + (F-K_2^2)\frac{A_2}{K_2}[\eta \sinh K_2 - \sinh K_2\eta] \tag{25}$$

where

$$K_1, K_2 = \left[\frac{1}{2}(F+M^2) \pm \frac{1}{2}\{(F-M^2)^2 - 4R_a\}^{1/2}\right]^{1/2}$$

$$A_1 = \frac{-FC_4K_2 \sinh K_2}{[(M^2F+R_a)\{(F-K_1^2)K_2 Cosh K_1 \sinh K_2 - (F-K_2^2)K_1 \sinh K_1 \cosh K_2\}]}$$

$$A_1 = \frac{FC_4K_1 \sinh K_1}{[(M^2F+R_a)\{(F-K_1^2)K_2 Cosh K_1 \sinh K_2 - (F-K_2^2)K_1 \sinh K_1 \cosh K_2\}]}$$

The non dimensional flow rate $\tilde{\omega}$ due to thermal conduction are

$$\tilde{\omega} = \int_4^1 u d\eta = 2\left[\frac{A_1}{K_1}(F-K_1^2)\sinh K_1 + \frac{A_2}{K_2}(F-K_2^2)\sinh K_2 + \frac{FC_4}{M^2F+R_a}\right] \tag{26}$$

**4. Numerical results and disscussions**

Taking $R_a = 1$, $M^2 = 10$, $C_4 = 1$, the effect of radiation has been shown in figures 1 and 2, where we find that the temperature $t(\eta)$ decreases with increasing F but the velocity $u(\eta)$ increases with F. The behaviour of $b(\eta)$ with F is shown in fig. 3, which is symmetric but reversed in the regions $-1 \leq \eta \leq 0$ and $0 \leq \eta \leq 1$

The variation of flow rate $\tilde{\omega}$ with F for $M^2 = 10$ is shown in table 1.

Table 1
Flow rate $\tilde{\omega}$

$M^2 = 10$

| F | | |
|---|---|---|
| 1 | 2 | 3 |
| 0.11 | 0.12 | 0.16 |

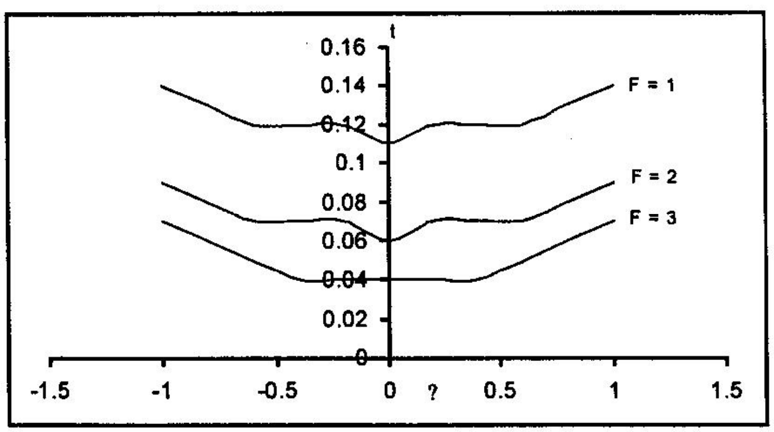


**Fig. 1 Variation of temperature with F.**

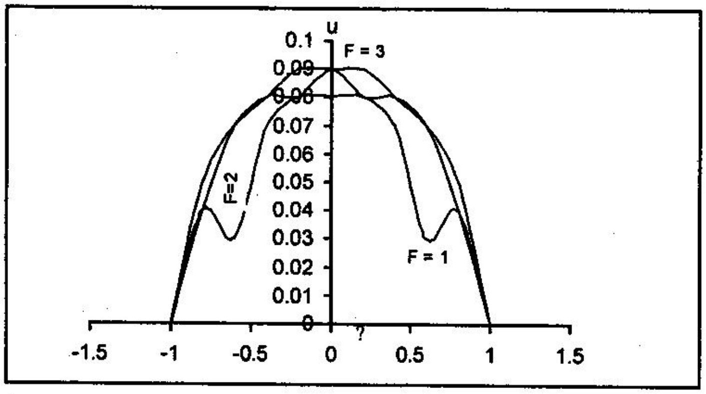


**Fig. 2 Variation of velocity with F.**

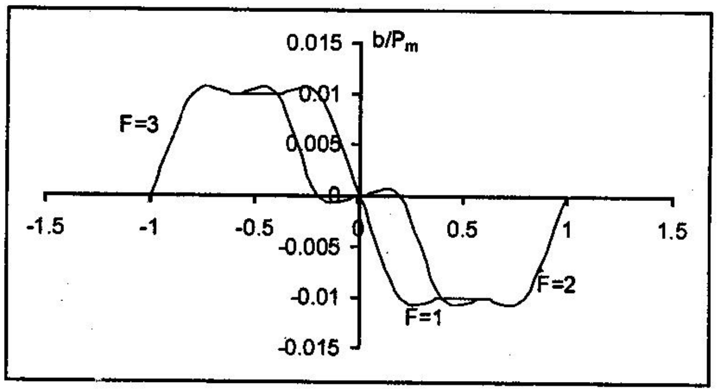


**Fig. 3 Behaviour of b(η) with F.**

## REFERENCES


[1] M.F. Romig : **Advances of Heat Transfer** (Ed.T.F. Invine, Jr., J.P. Hartnett), Vol.1. Academic Press, Newyork, 1964.

[2] R Siegel : *J.Appl. Mech.* **25** (1958) 415.

[3] M.Perlmutter , R Siegel .: *Report NASA TN D* 875, August 1965.

[4] R.A. Alpher : *Int. J. Heat & Mass Transfer* **3** (1961)108.

[5] G.Z. Gershuni , E.M. Zhukhovitsky : *Sov. Phys* JETP **34** (1958) 461.

[6] C.P.Yu : AIAA.J.3 (1965) 1184.

[7] R.Greif, I.S. Habib, J.C. Lin : *J. Fluid Mech.* **46** (1971) 513.

[8] R.Viskanta : *Z.Angev. Math & Phys.* **14** (1965) 353.

[9] P.S.Gupta , A.S. Gupta : *Int. J. Heat &Mass Transfer* **17** (1974) 1437.

[10] D.C. Sanyal, and S.K.Samanta, *Czeck. J.Phys* **B 39**, P 384 (1989).

[11] A.C.Cogley , W.C.Vincenti , S.E. Gilles : *AIAA.* **J.6** (1968) 551.