Numerical Solution and Global Error Estimation of Peristaltic motion of a Johnson-Segalman Fluid with Heat and Mass Transfer in a Planer channel

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Abstract:

Runge-Kutta-Merson method and Newton iteration in shooting and matching technique were used to obtain the solution of the system of the non-linear ordinary differential equations, which describe the two-dimensional flow of a Johnson-Segalman fluid with heat and mass transfer in a planer channel having walls that are transversely displaced by an infinite, harmonic traveling wave of large wavelength. Accordingly, we obtained the solutions of the momentum, the energy, and the concentration equations. The numerical formula of the stream function, the velocity, the temperature, and the concentration distributions of the problem were illustrated graphically. Effects of some parameters of this problem such as, Weissenberg number W_e, total flux number F, Eckeret number E_c, Prandtle number P_r, Soret number S_r, Schmidt number S_c, Reaction parameter Rc, Radiation parameter R_n, and reaction order m on these formula were discussed. Also we estimate the global error for the numerical values of solution by using Zadunaisky technique.

Keyword and phrases : *Johnson-Segalman fluid, heat transfer, mass transfer, global error, peristaltic.*

সংক্ষিপ্তসার

অ-রৈখিক সাধারণ অবকল সমীকরণগুলির সমাধান নির্ণয় করতে রুঙ্গো-কুটা-মেসন পদ্ধতি এবং সুটিং ও সমবাক্ষ বস্তুর ক্ষেত্রে নিউটন পুনরাবৃত্তিকার কৃৎকৌশল প্রয়োগ করা হয়েছে। প্রাচীর সমূহ সমন্বিত সমতলীয় পথে তাপ ও ভরের স্থানান্তরণ সহ জনসন - সিগালম্যান প্রবাহী পদার্থের প্রবাহকে দ্বি-মাত্রিক প্রবাহরূপে বর্ণনা করা হয়েছে যখন এই প্রাচীর-গুলি দীর্ঘতরঙ্গ দৈর্ঘ্যে অসীম দোলগতি তরঙ্গের পরিক্রমার ফলে তির্যকভাবে স্থানচ্যুত। এইভাবে প্রবাহী পদার্থের ভরবেগ, শক্তি এবং গাঢ়তার সমীকরণগুলির সমাধান করা হয়েছে। উক্ত সমস্যার প্রবাহ-অপেক্ষক, গতিবেগ, তাপমাত্রা এবং গাঢ়তা-বন্টনের সাংখ্যিক সূত্র- গুলিকে লেখচিত্রের সাহায্যে ব্যাখ্যা করা হয়েছে। এই সমস্যার কতিপয় প্রাচলের কার্যকারীতার সূত্রাবলী যথা -- ভাইসেনবার্গ সংখ্যা W_e, সর্বমোট ফ্লাক্স্ সংখ্যা F, একার্টসংখ্যা E_r, প্রাণ্ডল সংখ্যা P_r, সরেট্ সংখ্যা S_r, স্মিড্ সংখ্যা S_c, প্রতিক্রিয়ার প্রাচল R_c, বিকিরণ প্রাচল R_n, প্রতিক্রিয়ার ক্রম m প্রসঙ্গে আলোচনা করা হয়েছে। যাডুনায়েস্কি কৃৎ-কৌশলের সাহায্যে আনুমানিক ভূ - গোলীয় ত্রুটির হিসাব করা হয়েছে।

1. Introduction

The word Peristalsis derives from the Greek word " περισταλτικοσ " which means clasping and compressing. It is used to describe a progressive wave of contraction along a channel or tube whose cross-sectional area consequently varies. Peristalsis is regarded as having considerable relevance in biomechanics and especially as a major mechanism for fluid transport in many biological systems (as it is in the human) [1].

The dynamics of the fluid transport by peristaltic motion of confining walls has received a careful study in the recent literature. The need for peristaltic pumping may arise in circumstances where it is desirable to avoid using any internal moving parts such as pistons to be one of the main mechanisms of fluid transport in a biological system. Specifically, peristaltic mechanism is involved in swallowing food through the esophagus, urine transport from the kidney to the bladder through the ureter, movement of chyme in the gastrointestinal tract. Moreover, in the cervical canal, it is involved in the movement of ovum in the Fallopian tube, transport of lump in the lymphatic vessels, and vasomotion of small blood vessels such as arterioles, venules, and capillaries, as well as in the mechanical and neurological aspects of the peristaltic reflex. In plant physiology, such a mechanism is involved in phloem translocation by driving a sucrose solution along tubules by peristaltic contractions. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems such as roller and finger pumps. The application of peristaltic motion as a mean of transporting fluid has also aroused interest in engineering fields [2 – 4]. In particular, the peristaltic pumping of corrosive fluids and slurries could be useful as it is desirable to prevent their contact with mechanical parts of the pump [5]. The Johnson-Segalman model is a viscoelastic fluid model which was developed to allow for non-affine deformations [6]. This model has attracted a lot of interest because it has been used by a number of researchers [7 – 9] to explain the "spurt" phenomenon. "spurt" is a little understood phenomenon observed in the flow of a number of non-Newtonian fluids in which there is a large increase in the volume throughout for a small increases in the driving pressure gradient, at a critical pressure gradient. Experimentalists usually associate "spurt" with slip at the wall and there have been a number of experiments [10 – 13] to support this hypothesis [14]. Our study is an extension of Hayat, Wang, Siddiqui, and Hutter [5] who studied the peristaltic motion of a Johnson-Segalman fluid in a planer channel, and who obtained some distributions of the stream function and the velocity at different values of Weissenberg number We and Total flux number F. The objective of this work is to investigate the numerical solution by using Runge-Kutta-Merson method and shooting technique [15], [16] for the system of non-linear differential equations which arises from the flow of a Johnson-Segalman fluid with heat and mass transfer in a planer channel having walls that are transversely displaced by an infinite, harmonic traveling wave of large wavelength. We obtained the distributions of the stream function, the velocity, the temperature, and the concentration. Communicated with the stream function and the velocity distribution, we found that a good agreement between our results and the previous results in [5] in spite of the difference between our method of solution. Temperature and concentration distributions are obtained and the effect of the problem parameters on these solutions are discussed and illustrated graphically. Also global error estimation for the error propagation is obtained using Zadunaisky technique [17].

2 Formulation of the problem

Consider a two-dimensional infinite channel of uniform width $2n$

filled with an incompressible non-Newtonian fluid obeying Johnson-Segalman model with heat and mass transfer in the presence of solar radiation with chemical reaction. We choose a rectangular coordinate system for the channel with $\bar{X}$ along the center line and $\bar{Y}$ normal to it (see figure (1)). We assume that an infinite train of sinusoidal waves progresses with velocity c along the walls in the $\bar{X}$−direction.

The geometry of the wall surface is defined as

$$\bar{h}\left(\bar{X},t\right)=n+b\sin\left(\frac{2\pi}{\lambda}\left(\bar{X}-ct\right)\right) \quad , \tag{1}$$

Where b is the amplitude and λ is the wavelength. We also assume that there is no motion of the wall in the longitudinal direction (extensible or elastic wall).

3 Basic equations

The basic equations governing the flow of an incompressible fluid are the following equations

the continuity equation

$$\nabla\cdot V=0 \quad , \tag{2}$$

the momentum equation

$$\nabla\cdot\sigma+\rho f=\rho\frac{dV}{dt} \quad , \tag{3}$$

the temperature equation

$$\rho c_p\left[\frac{\partial T}{\partial t}+\left(V\cdot\nabla\right)T\right]=\kappa\nabla^2T+\Phi+\nabla\cdot q \quad , \tag{4}$$

the concentration equation

$$\left[\frac{\partial C}{\partial t}+\left(V\cdot\nabla\right)C\right]=D\,\nabla^2C+\frac{D\,\kappa_T}{T_m}\nabla^2T-A\left(C-C_2\right)^m \tag{5}$$

Where V is the velocity vector, f is the body force per unit mass, ρ is the density, $\left(\frac{d}{dt}\right)$ is the material time derivative, and σ is the Cauchy stress.

Also T and C are the temperature and concentration of the fluid, F is the dissipation function, κ is the thermal conductivity, c_p is the specific heat capacity at constant pressure, q is the radiative heat flux, D is the coefficient of mass diffusivity, κ_T is the thermal diffusion ratio, T_m is the mean fluid temperature, A is the reaction rate constant, and m is the reaction order.

Johnson and segalman [18] proposed an integral model which can also be written in the rate-type form. With an appropriate choice of kernel function and the time constants, the Cauchy stress S in such a Johnson-segalman fluid is related to the fluid motion through the following relations,

$$\sigma = -PI + \tau \quad , \tag{6}$$

$$\tau = 2\mu L + S \quad , \tag{7}$$

$$S + m_1 \left[\frac{dS}{dt} + S\,(W - a\ L) + (W - a\ L)^T\, S \right] = 2\eta L \quad , \tag{8}$$

Where L is the symmetric part of the velocity gradient and W is the skew-symmetric part of the velocity gradient, that is

$$L = \frac{1}{2}\left[R + R^T\right] \quad ,$$

$$W = \frac{1}{2}\left[R - R^T\right] \quad ,$$

$$R = grad\ V \quad . \tag{9}$$

Also, $-PI$ denotes the indeterminate part of the stress due to the constraint of incompressibility, μ and η are viscosities, m_1 is the relaxation time, and "a" is called the slip parameter. When $a = 1$, the Johnson-Segalman model reduces to the oldroyed–B model [1]; when $a = 1$ and $\mu = 0$, the Johnson-Segalman model reduces to the Maxwell fluid; and when $m_1 = 0$, the model reduces to the classical Navier-Stokes fluid.

Let $\bar{U}$ and $\bar{V}$ be the longitudinal and transverse velocity components of the fluid, respectively.

For unsteady two-dimensional flows, the velocity components can be written as follows

$$V = \left(\bar{U}\left(\bar{X}, \bar{Y}, t\right), \bar{V}\left(\bar{X}, \bar{Y}, t\right), 0\right) \quad . \tag{10}$$

Also, the temperature and the concentration functions can be written as follows,

$$T = T\left(\bar{X}, \bar{Y}, t\right) \quad , \quad C = C\left(\bar{X}, \bar{Y}, t\right) \quad . \tag{11}$$

The equations of motion (2), (3) and the constitutive relations (6), (7), and (8) in the absence of body forces take the following form:-

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \quad , \tag{12}$$

$$\begin{aligned} & \rho \left(\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \bar{U} \\ = & -\frac{\partial \bar{P}\left(\bar{X}, \bar{Y}, t\right)}{\partial \bar{X}} + \mu \left(\frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2} \right) \bar{U} + \frac{\partial \bar{S}_{\bar{X}\bar{X}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} \ , \end{aligned} \tag{13}$$

$$\begin{aligned} & \rho \left(\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \bar{V} \\ = & -\frac{\partial \bar{P}\left(\bar{X}, \bar{Y}, t\right)}{\partial \bar{Y}} + \mu \left(\frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2} \right) \bar{V} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} \ , \end{aligned} \tag{14}$$

$$\begin{aligned} 2\mu \frac{\partial \bar{U}}{\partial \bar{X}} = & \ \bar{S}_{\bar{X}\bar{X}} + m_1 \left[\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right] \bar{S}_{\bar{X}\bar{X}} \\ & -2a \ m_1 \bar{S}_{\bar{X}\bar{X}} \frac{\partial \bar{U}}{\partial \bar{X}} + m_1 \left[(1-a) \frac{\partial \bar{V}}{\partial \bar{X}} - (1+a) \frac{\partial \bar{U}}{\partial \bar{Y}} \right] \bar{S}_{\bar{X}\bar{Y}} \ , \end{aligned} \tag{15}$$

$$\begin{aligned} \eta \left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}} \right) = & \ \bar{S}_{\bar{X}\bar{Y}} + m_1 \left[\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right] \bar{S}_{\bar{X}\bar{Y}} \\ & + \frac{m_1}{2} \left[(1-a) \frac{\partial \bar{U}}{\partial \bar{Y}} - (1+a) \frac{\partial \bar{V}}{\partial \bar{X}} \right] \bar{S}_{\bar{X}\bar{X}} \\ & + \frac{m_1}{2} \left[(1-a) \frac{\partial \bar{V}}{\partial \bar{X}} - (1+a) \frac{\partial \bar{U}}{\partial \bar{Y}} \right] \bar{S}_{\bar{Y}\bar{Y}} \ , \end{aligned} \tag{16}$$

$$\begin{aligned} 2\mu \frac{\partial \bar{V}}{\partial \bar{Y}} = & \ \bar{S}_{\bar{Y}\bar{Y}} + m_1 \left[\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right] \bar{S}_{\bar{Y}\bar{Y}} \\ & -2a \ m_1 \bar{S}_{\bar{Y}\bar{Y}} \frac{\partial \bar{V}}{\partial \bar{Y}} + m_1 \left[(1-a) \frac{\partial \bar{U}}{\partial \bar{Y}} - (1+a) \frac{\partial \bar{V}}{\partial \bar{X}} \right] \bar{S}_{\bar{X}\bar{Y}} \ . \end{aligned} \tag{17}$$

The dissipation function Φ can be written as follows

$$\Phi = \tau_{ij} \frac{\partial V_i}{\partial X_j} \quad , \tag{18}$$

$$\begin{aligned} \Phi = & \ 2\mu \left[\left(\frac{\partial \bar{U}}{\partial \bar{X}} \right)^2 + \left(\frac{\partial \bar{U}}{\partial \bar{Y}} \right)^2 + \left(\frac{\partial \bar{V}}{\partial \bar{X}} \right)^2 + \left(\frac{\partial \bar{V}}{\partial \bar{Y}} \right)^2 \right] + \bar{S}_{\bar{X}\bar{X}} \frac{\partial \bar{U}}{\partial \bar{X}} \\ & + \bar{S}_{\bar{X}\bar{Y}} \left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}} \right) + \bar{S}_{\bar{Y}\bar{Y}} \frac{\partial \bar{V}}{\partial \bar{Y}} \ . \end{aligned} \tag{19}$$

Also, we using Rosselant approximation [19] we have

$$q=-\frac{4\sigma_0}{3\kappa_0}\frac{\partial T^4}{\partial y} \quad , \tag{20}$$

Where, σ_0 is the Stefan Boltizman constant and κ_0 is the mean absorption coefficient. We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_2 , and neglecting higher-order terms [20], one gets,

$$T^4 \approx 4T_2^3 T - 3T_2^4 \quad . \tag{21}$$

Then, equations (4), and (5) can be written as follows:-

$$\begin{aligned}
&\frac{\partial T}{\partial t}+\bar{U}\frac{\partial T}{\partial \bar{X}}+\bar{V}\frac{\partial T}{\partial \bar{Y}} \\
=\;& \frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T}{\partial \bar{X}^2}+\frac{\partial^2 T}{\partial \bar{Y}^2}\right)+\frac{16\sigma_0}{3\rho c_p \kappa_0}T_2^3\frac{\partial^2 T}{\partial \bar{Y}^2} \\
&+\frac{2\mu}{\rho c_p}\left[\left(\frac{\partial \bar{U}}{\partial \bar{X}}\right)^2+\left(\frac{\partial \bar{U}}{\partial \bar{Y}}\right)^2+\left(\frac{\partial \bar{V}}{\partial \bar{X}}\right)^2+\left(\frac{\partial \bar{V}}{\partial \bar{Y}}\right)^2\right] \\
&+\frac{\bar{S}_{\bar{X}\bar{X}}}{\rho c_p}\frac{\partial \bar{U}}{\partial \bar{X}}+\frac{\bar{S}_{\bar{X}\bar{Y}}}{\rho c_p}\left(\frac{\partial \bar{U}}{\partial \bar{Y}}+\frac{\partial \bar{V}}{\partial \bar{X}}\right)+\frac{\bar{S}_{\bar{Y}\bar{Y}}}{\rho c_p}\frac{\partial \bar{V}}{\partial \bar{Y}} \quad ,
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\frac{\partial C}{\partial t}+\bar{U}\frac{\partial C}{\partial \bar{X}}+\bar{V}\frac{\partial C}{\partial \bar{Y}} \\
=\;& D\left(\frac{\partial^2 C}{\partial \bar{X}^2}+\frac{\partial^2 C}{\partial \bar{Y}^2}\right)+\frac{D\kappa_T}{T_m}\left(\frac{\partial^2 T}{\partial \bar{X}^2}+\frac{\partial^2 T}{\partial \bar{Y}^2}\right)-A\left(C-C_2\right)^m \quad .
\end{aligned} \tag{23}$$

In the fixed coordinate system $(\bar{X},\bar{Y})$, the motion is unsteady because of the moving boundary. However, if observed in a coordinate system $(\bar{x},\bar{y})$ moving with the speed c, it can be treated as steady because the boundary shape appears to be stationary. The transformation between the two frames is given by

$$\bar{x}=\bar{X}-ct \quad , \quad \bar{y}=\bar{Y} \quad . \tag{24}$$

The velocities in the fixed and moving frames are related by

$$\bar{u}=\bar{U}-c \quad , \quad \bar{v}=\bar{V} \quad . \tag{25}$$

Where, $(\bar{u},\bar{v})$ are components of the velocity in the moving coordinate system.

Then, the equation's (12)-(17), (22), and (23) can be written in the moving coordinate system as follows:-

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad , \tag{26}$$

$$\begin{aligned} &\rho \left(\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \bar{u} \\ = \quad & -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu \left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} \right) \bar{u} + \frac{\partial \bar{s}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{s}_{\bar{x}\bar{y}}}{\partial \bar{y}} \quad , \end{aligned} \tag{27}$$

$$\begin{aligned} &\rho \left(\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \bar{v} \\ = \quad & -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu \left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} \right) \bar{v} + \frac{\partial \bar{s}_{\bar{x}\bar{y}}}{\partial \bar{x}} + \frac{\partial \bar{s}_{\bar{y}\bar{y}}}{\partial \bar{y}} \quad , \end{aligned} \tag{28}$$

$$\begin{aligned} 2\eta \frac{\partial \bar{u}}{\partial \bar{x}} &= \bar{s}_{\bar{x}\bar{x}} + m_1 \left[\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right] \bar{s}_{\bar{x}\bar{x}} - 2am_1 \bar{s}_{\bar{x}\bar{x}} \frac{\partial \bar{u}}{\partial \bar{x}} \\ &+ m_1 \left[(1-a) \frac{\partial \bar{v}}{\partial \bar{x}} - (1+a) \frac{\partial \bar{u}}{\partial \bar{y}} \right] \bar{s}_{\bar{x}\bar{y}} \quad , \end{aligned} \tag{29}$$

$$\begin{aligned} \eta \left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right) &= \bar{s}_{\bar{x}\bar{y}} + m_1 \left[\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right] \bar{s}_{\bar{x}\bar{y}} \\ &+ \frac{m_1}{2} \left[(1-a) \frac{\partial \bar{u}}{\partial \bar{y}} - (1+a) \frac{\partial \bar{v}}{\partial \bar{x}} \right] \bar{s}_{\bar{x}\bar{x}} \\ &+ \frac{m_1}{2} \left[(1-a) \frac{\partial \bar{v}}{\partial \bar{x}} - (1+a) \frac{\partial \bar{u}}{\partial \bar{y}} \right] \bar{s}_{\bar{y}\bar{y}} \quad , \end{aligned} \tag{30}$$

$$\begin{aligned} 2\eta \frac{\partial \bar{v}}{\partial \bar{y}} &= \bar{s}_{\bar{y}\bar{y}} + m_1 \left[\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right] \bar{s}_{\bar{y}\bar{y}} - 2am_1 \bar{s}_{\bar{y}\bar{y}} \frac{\partial \bar{v}}{\partial \bar{y}} \\ &+ m_1 \left[(1-a) \frac{\partial \bar{u}}{\partial \bar{y}} - (1+a) \frac{\partial \bar{v}}{\partial \bar{x}} \right] \bar{s}_{\bar{x}\bar{y}} \quad , \end{aligned} \tag{31}$$

$$\begin{aligned} \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} &= \frac{\kappa}{\rho c_p} \left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} \right) + \frac{16\sigma_0}{3\rho c_p \kappa_0} T_2^3 \frac{\partial^2 T}{\partial \bar{y}^2} \\ &+ \frac{2\mu}{\rho c_p} \left[\left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{x}} \right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 \right] \\ &+ \frac{\bar{s}_{\bar{x}\bar{x}}}{\rho c_p} \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\bar{s}_{\bar{x}\bar{y}}}{\rho c_p} \left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right) + \frac{\bar{s}_{\bar{y}\bar{y}}}{\rho c_p} \frac{\partial \bar{v}}{\partial \bar{y}} \quad , \end{aligned} \tag{32}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}}+\bar{v}\frac{\partial C}{\partial \bar{y}} = D\left(\frac{\partial^2 C}{\partial \bar{x}^2}+\frac{\partial^2 C}{\partial \bar{y}^2}\right) +\frac{D\kappa_T}{T_m}\left(\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}\right)-A\left(C-C_2\right)^m \tag{33}$$

In order to simplify the governing equations of the motion, we may introduce the following dimensionless transformations:-

$$\begin{aligned} x &= \frac{2\pi}{\lambda}\bar{x} \quad , \quad y=\frac{\bar{y}}{n} \quad , \quad u=\frac{\bar{u}}{c} \quad , \quad v=\frac{\bar{v}}{c} \quad , \\ s &= \frac{n}{\mu c}\bar{s} \quad , \quad p=\frac{2\pi n^2}{\lambda(\mu+\eta)c}\bar{p} \quad , \quad h=\frac{\bar{h}}{n} \quad , \\ \theta &= \frac{T-T_2}{T_1-T_2} \quad , \quad \phi=\frac{C-C_2}{C_1-C_2} \quad . \end{aligned} \tag{34}$$

Substituting (34) into equations (26)-(33) we obtain the following non-dimensional equations:-

$$\delta\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad , \tag{35}$$

$$R_e\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u = -(1+N)\frac{\partial p}{\partial x} +\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u+\delta\frac{\partial s_{xx}}{\partial x}+\frac{\partial s_{xy}}{\partial y} \quad , \tag{36}$$

$$R_e\delta\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v = -(1+N)\frac{\partial p}{\partial y} +\delta\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)v+\delta^2\frac{\partial s_{xy}}{\partial x}+\delta\frac{\partial s_{yy}}{\partial y} \quad , \tag{37}$$

$$2N\delta\frac{\partial u}{\partial x} = s_{xx}+W_e\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)s_{xx}-2a\delta W_e s_{xx}\frac{\partial u}{\partial x} +W_e\left[(1-a)\delta\frac{\partial v}{\partial x}-(1+a)\frac{\partial u}{\partial y}\right]s_{xy} \quad , \tag{38}$$

$$\begin{aligned} N\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right) &= s_{xy}+W_e\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)s_{xy} \\ &+\frac{W_e}{2}\left[(1-a)\frac{\partial u}{\partial y}-(1+a)\delta\frac{\partial v}{\partial x}\right]s_{xx} \\ &+\frac{W_e}{2}\left[(1-a)\delta\frac{\partial v}{\partial x}-(1+a)\frac{\partial u}{\partial y}\right]s_{yy} \quad , \end{aligned} \tag{39}$$

$$\begin{aligned} 2N\frac{\partial v}{\partial y} &= s_{yy} + W_e\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)s_{yy} - 2aW_e s_{yy}\frac{\partial v}{\partial y} \\ &\quad + W_e\left[(1-a)\frac{\partial u}{\partial y} - (1+a)\,\delta\frac{\partial v}{\partial x}\right]s_{xy} \ , \end{aligned} \tag{40}$$

$$\begin{aligned} & R_e\left(\delta u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) \\ = \ & \frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \frac{4}{3R_n}\frac{\partial^2\theta}{\partial y^2} \\ & +2E_c\left[\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \delta^2\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] \\ & +E_c\delta s_{xx}\frac{\partial u}{\partial x} + E_c s_{xy}\left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + E_c s_{yy}\frac{\partial v}{\partial y} \ , \end{aligned} \tag{41}$$

$$\begin{aligned} R_e\left(\delta u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y}\right) &= \frac{1}{S_c}\left(\delta^2\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) \\ & \quad S_r\left(\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - R_c\phi^m \end{aligned} \tag{42}$$

Where, the dimensionless parameters are defined by:-

$\delta = \frac{2\pi n}{\lambda}$ (Wave number),

$R_e = \frac{\rho c n}{\mu}$ (Reynolds number),

$N = \frac{\eta}{\mu}$ (viscosity parameter),

$W_e = \frac{m_1 c}{n}$ (Weissenberg number),

$P_r = \frac{\nu\rho c_p}{\kappa}$ (Prandtle number),

$R_n = \frac{\rho c_p \kappa_0 \nu}{4\sigma_0 T_2^3}$ (Radiation parameter),

$E_c = \frac{c^2}{c_p(T_1 - T_2)}$ (Eckert number),

$S_c = \frac{\nu}{D}$ (Schmidt number),

$S_r = \frac{D\kappa_T(T_1 - T_2)}{T_m \nu (C_1 - C_2)}$ (Soret number),

$R_c = \dfrac{A\,(C_1 - C_2)^{m-1}\, n^2}{\nu}$ (Reaction parameter).

Equation (35) allows the introducing of the dimensionless stream function $\psi\,(x, y)$ in terms of

$$u = \frac{\partial\psi}{\partial y} \quad , \quad v = -\delta\frac{\partial\psi}{\partial x} \quad . \tag{43}$$

In terms of ψ, we fined that (35) is identically satisfied, while the other equations take the forms,

$$\begin{aligned}
&\delta R_e \left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right] \\
= \;& -(1+N)\frac{\partial p}{\partial x} + \left(\delta^2\frac{\partial^3\psi}{\partial x^2\partial y} + \frac{\partial^3\psi}{\partial y^3}\right) + \delta\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} \quad ,
\end{aligned} \tag{44}$$

$$\begin{aligned}
&-\delta^3 R_e \left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial x}\right] \\
= \;& -(1+N)\frac{\partial p}{\partial y} - \delta^2\left(\delta^2\frac{\partial^3\psi}{\partial x^3} + \frac{\partial^3\psi}{\partial y^2\partial x}\right) + \delta^2\frac{\partial s_{xy}}{\partial x} + \delta\frac{\partial s_{yy}}{\partial y} \quad ,
\end{aligned} \tag{45}$$

$$\begin{aligned}
2N\delta\frac{\partial^2\psi}{\partial x\partial y} \;=\;& s_{xx} + W_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)s_{xx} \\
&-2aW_e\delta s_{xx}\frac{\partial^2\psi}{\partial x\partial y} \\
&-W_e\left[(1-a)\,\delta^2\frac{\partial^2\psi}{\partial x^2} + (1+a)\,\frac{\partial^2\psi}{\partial y^2}\right]s_{xy} \quad ,
\end{aligned} \tag{46}$$

$$\begin{aligned}
N\left(\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial^2\psi}{\partial x^2}\right) \;=\;& s_{xy} + W_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)s_{xy} \\
&+\frac{W_e}{2}\left[(1-a)\,\frac{\partial^2\psi}{\partial y^2} - (1+a)\,\delta^2\frac{\partial^2\psi}{\partial x^2}\right]s_{xx} \\
&-\frac{W_e}{2}\left[(1-a)\,\delta^2\frac{\partial^2\psi}{\partial x^2} + (1+a)\,\frac{\partial^2\psi}{\partial y^2}\right]s_{yy} \quad ,
\end{aligned} \tag{47}$$

$$\begin{aligned}
-2N\delta\frac{\partial^2\psi}{\partial x\partial y} \;=\;& s_{yy} + W_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)s_{yy} \\
&+2aW_e\delta s_{yy}\frac{\partial^2\psi}{\partial x\partial y} \\
&+W_e\left[(1-a)\,\frac{\partial^2\psi}{\partial y^2} + (1+a)\,\delta^2\frac{\partial^2\psi}{\partial x^2}\right]s_{xy} \quad ,
\end{aligned} \tag{48}$$

$$\begin{aligned}
&R_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)\\
=\;&\frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{4}{3R_n}\frac{\partial^2\theta}{\partial y^2}\\
&+2E_c\left[\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\delta^4\left(\frac{\partial^2\psi}{\partial x^2}\right)^2+\delta^4\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right]\\
&+E_c\delta s_{xx}\frac{\partial^2\psi}{\partial x^2}+E_c s_{xy}\left[-\delta^2\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right]-E_c\delta s_{yy}\frac{\partial^2\psi}{\partial x\partial y}\ , \qquad (49)
\end{aligned}$$

$$\begin{aligned}
R_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}\right) &= \frac{1}{S_c}\left(\delta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)\\
&+S_r\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-R_c\phi^m\ . \qquad (50)
\end{aligned}$$

Boundary conditions:-

The boundary conditions for the dimensionless stream function in the moving frame are,

$$\left.\begin{aligned}&\psi=0\ , &&\text{(by convection)}\\ &\frac{\partial^2\psi}{\partial y^2}=0\ , &&\text{(by symmetry)}\end{aligned}\right\}\text{on the centerline } y=0\ ,$$

$$\left.\begin{aligned}&\frac{\partial\psi}{\partial y}=-1 &&\text{(no slip condition)}\\ &\psi=F\end{aligned}\right\}\text{at the wall}$$

$$y=h\ . \qquad (51)$$

where, F is the total flux number. We also note that h represents the dimensionless form of the surface of the peristaltic wall.

$$h(x)=1+\chi\sin x$$

Where, $\chi=\dfrac{b}{n}$, (is the amplitude ratio or the occlusion)

The boundary conditions of the temperature and the concentration are

$$\begin{aligned}
T &= T_1\ ,\quad C=C_1 &&\text{at}\quad \bar{y}=0\ ,\\
T &= T_2\ ,\quad C=C_2 &&\text{at}\quad \bar{y}=\bar{h}\ . \qquad (52)
\end{aligned}$$

Where, T_1 and C_1 are the temperature and the concentration of the fluid at a lower wall of the channel, T_2 and C_2 are the temperature and the concentration of the fluid at an upper wall of the channel.

Then, the dimensionless boundary conditions are given by:-

$$\psi = 0 \ , \quad \frac{\partial^2\psi}{\partial y^2} = 0 \ , \quad \theta = 1 \ , \quad \phi = 1 \quad \text{at} \quad y = 0 \ ,$$
$$\psi = F \ , \quad \frac{\partial\psi}{\partial y} = -1 \ , \quad \theta = 0 \ , \quad \phi = 0 \quad \text{at} \quad y = h \ . \tag{53}$$

Equations for large wavelength

A general solution of the dynamic equations (44) – (50) for arbitrary values of all parameters seems to be impossible to find. Even in the case of Newtonian fluids, all analytical solutions obtained so far by Shapiro et al. [21], and by Srivastava [22] are based on assumptions that one or some of the parameters are zero or small. Accordingly, we carry out our investigation on the basis that the dimensionless wave number is small, that is,

$$\delta \ll 1 \ , \tag{54}$$

which corresponds to the long-wavelength approximation [21]. Thus, to lowest order in δ, equations (44) – (50) give

$$(1+N)\frac{\partial p}{\partial x} = \frac{\partial s_{xy}}{\partial y} + \frac{\partial^3\psi}{\partial y^3} \ , \tag{55}$$

$$\frac{\partial p}{\partial y} = 0 \ , \tag{56}$$

$$s_{xx} - W_e\,(1+a)\,s_{xy}\frac{\partial^2\psi}{\partial y^2} = 0 \ , \tag{57}$$

$$N\frac{\partial^2\psi}{\partial y^2} = s_{xy} + \frac{W_e}{2}\,(1-a)\,s_{xx}\frac{\partial^2\psi}{\partial y^2} - \frac{W_e}{2}\,(1+a)\,s_{yy}\frac{\partial^2\psi}{\partial y^2} \ , \tag{58}$$

$$s_{yy} + W_e\,(1-a)\,s_{xy}\frac{\partial^2\psi}{\partial y^2} = 0 \ , \tag{59}$$

$$\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} + \frac{4}{3R_n}\frac{\partial^2\theta}{\partial y^2} + 2E_c\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + E_c s_{xy}\frac{\partial^2\psi}{\partial y^2} = 0 \ , \tag{60}$$

$$\frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2} + S_r\frac{\partial^2\theta}{\partial y^2} - R_c\phi^m = 0 \ . \tag{61}$$

Substituting (57) and (59) into (58) yields,

$$s_{xy} = \frac{N\left(\dfrac{\partial^2\psi}{\partial y^2}\right)}{\left(1 + W_e^2\,(1-a^2)\left(\dfrac{\partial^2\psi}{\partial y^2}\right)\right)} \tag{62}$$

Substituting (62) into (55) and (60), and using (56) we finally have,

$$\frac{d^2}{dy^2}\left[\frac{N\left(\frac{d^2\psi}{dy^2}\right)}{\left(1+W_e^2\left(1-a^2\right)\left(\frac{d^2\psi}{dy^2}\right)\right)}\right]+\frac{d^4\psi}{dy^4}=0 \ , \tag{63-a}$$

$$\left(\frac{1}{P_r}+\frac{4}{3R_n}\right)\frac{d^2\theta}{dy^2}+E_c\left[2+\frac{N}{\left(1+W_e^2\left(1-a^2\right)\left(\frac{d^2\psi}{dy^2}\right)\right)}\right]\left(\frac{d^2\psi}{dy^2}\right)^2=0 \ , \tag{63-b}$$

$$\frac{1}{S_c}\frac{d^2\phi}{dy^2}+S_r\frac{d^2\theta}{dy^2}-R_c\phi^m=0 \ . \tag{63-c}$$

We can see that, the velocity equation $(63-a)$ is the same resulting as the main paper [5], but the temperature equation $(63-b)$, and the concentration equation $(63-c)$are the addition of the main problem.

The system of non-linear ordinary differential equations (63) together with the boundary conditions (53), will be solved numerically by using Runge-Kutta-Merson method and a Newton iteration shooting and matching technique.

4 Numerical solution

The system of non-linear ordinary differential equations (63) can be written as following:-

$$\begin{aligned}\psi^{(4)} &= N\left[\frac{1}{1+\frac{\left(1-W_e^2\left(1-a^2\right)\psi''^2\right)}{\left(1+W_e^2\left(1-a^2\right)\psi''^2\right)^2}}\right] \\ &\quad \left[\frac{2W_e^2\left(1-a^2\right)\psi''\psi'''^2}{\left(1+W_e^2\left(1-a^2\right)\psi''^2\right)^2} + \frac{4W_e^2\left(1-a^2\right)\left(1-W_e^2\left(1-a^2\right)\psi''^2\right)\psi''\psi'''^2}{\left(1+W_e^2\left(1-a^2\right)\psi''^2\right)^3}\right] \\ \theta'' &= \left(\frac{-3P_rE_cR_n}{3R_n+4P_r}\right)\left[2+\frac{N}{\left(1+W_e^2\left(1-a^2\right)\psi''^2\right)}\right]\psi''^2 \ ,\end{aligned}$$

$$\phi'' = S_c S_r \theta'' + S_c R_c \phi^m \quad . \tag{64}$$

With the following boundary conditions:-

$$\begin{aligned} \psi(0) &= 0 \quad , \quad \psi''(0) = 0 \quad , \quad \theta(0) = 1 \quad , \quad \phi(0) = 1 \quad , \\ \psi(h) &= F \quad , \quad \psi'(h) = -1 \quad , \quad \theta(h) = 0 \quad , \quad \phi(h) = 0 \quad . \end{aligned} \tag{65}$$

We take,

$$\begin{aligned} Y_1 &= \psi \ , \quad Y_5 = \theta \quad , \quad Y_7 = \phi \quad , \\ Y_1' &= Y_2 \quad , \\ Y_2' &= Y_3 \quad , \\ Y_3' &= Y_4 \quad , \\ Y_4' &= N \left[\frac{1}{1 + \dfrac{\left(1 - W_e^2 \left(1 - a^2\right) Y_3^2\right)}{\left(1 + W_e^2 \left(1 - a^2\right) Y_3^2\right)^2}} \right] \\ &\quad \left[\frac{2W_e^2 \left(1 - a^2\right) Y_3 Y_4^2}{\left(1 + W_e^2 \left(1 - a^2\right) Y_3^2\right)^2} + \frac{4W_e^2 \left(1 - a^2\right) \left(1 - W_e^2 \left(1 - a^2\right) Y_3^2\right) Y_3 Y_4^2}{\left(1 + W_e^2 \left(1 - a^2\right) Y_3^2\right)^3} \right] , \\ Y_5' &= Y_6 \quad , \\ Y_6' &= \left(\frac{-3P_r E_c R_n}{3R_n + 4P_r} \right) \left[2 + \frac{N}{\left(1 + W_e^2 \left(1 - a^2\right) Y_3^2\right)} \right] Y_3^2 \quad ; \\ Y_7' &= Y_8 \quad , \\ Y_8' &= -S_c S_r Y_6' + S_c R_c Y_7^m \quad . \end{aligned} \tag{66}$$

Subject to the boundary conditions:-

$$Y_1(0) = 0 \quad , \quad Y_3(0) = 0 \quad , \quad Y_5(0) = 1 \quad , \quad Y_7(0) = 1 \quad ,$$

$$Y_1(h) = F \quad , \quad Y_2(h) = -1 \quad , \quad Y_5(h) = 0 \quad , \quad Y_7(h) = 0 \quad . \tag{67}$$

To apply shooting method we use the subroutine D02HAF from the NAG Fortran library, which requires the supply of starting values of missing initial and terminal conditions.

The subroutine uses Runge-Kutta-Merson (as an initial value solver) with variable step size in order to control the local truncation error, then it applies modified Newton-Raphson technique to make successive corrections to the estimated boundary values. The process repeated iteratively until convergence is obtained, i.e. until the absolute value of the difference between every two successive approximations of the missing conditions is less than ε (in our case ε is taken $= 10^{-5}$).

5 Estimation of the global error

We use the Zadunaisky technique [17], [23] for calculating the global error, which can be explained in the following steps:-

1− We fined the interpolating polynomial of Y_i , $(i = 1, 2, 3, ..., 8)$ from the values of Y_i and we named it P_i , $(i = 1, 2, 3, ..., 8)$ and we find the interpolating functions of $\psi^{(4)}$, θ'', ϕ'' , and we named it as

$$\psi^{(4)} = q_1(x) \quad , \quad \theta'' = q_2(x) \quad , \quad \phi'' = q_3(x) \quad .$$

$$\begin{aligned} q_1(x) &= N\left[\frac{1}{1+\dfrac{\left(1-W_e^2\left(1-a^2\right)\left(P_3(x)\right)^2\right)}{\left(1+W_e^2\left(1-a^2\right)\left(P_3(x)\right)^2\right)^2}}\right] \\ &\left[\begin{array}{c} \dfrac{2W_e^2\left(1-a^2\right)P_3(x)\left(P_4(x)\right)^2}{\left(1+W_e^2\left(1-a^2\right)\left(P_3(x)\right)^2\right)^2} \\ +\dfrac{4W_e^2\left(1-a^2\right)\left(1-W_e^2\left(1-a^2\right)\left(P_3(x)\right)^2\right)P_3(x)\left(P_4(x)\right)^2}{\left(1+W_e^2\left(1-a^2\right)\left(P_3(x)\right)^2\right)^3} \end{array}\right] , \end{aligned}$$

$$q_2(x) = \left(\frac{-3P_rE_cR_n}{3R_n+4P_r}\right)\left[2+\frac{N}{\left(1+W_e^2\left(1-a^2\right)\left(P_3(x)\right)^2\right)}\right]\left(P_3(x)\right)^2 \quad ,$$

$$q_3(x) = -S_cS_rq_2(x) + S_cR_c\left(P_7(x)\right)^m \quad . \tag{68}$$

2− We calculate the defect functions $D_i(x), (i = 1, 2, 3, ..., 8)$, which can be written as follows

$$\begin{aligned} D_1(x) &= P_1'(x) - P_2(x) = 0 \quad , \; D_2(x) = P_1''(x) - P_3(x) = 0 \; , \\ D_3(x) &= P_1'''(x) - P_4(x) = 0 \quad , \; D_4(x) = P_1''''(x) - q_1(x) \quad , \\ D_5(x) &= P_5'(x) - P_6(x) = 0 \quad , \; D_6(x) = P_5''(x) - q_2(x) \quad , \\ D_7(x) &= P_7'(x) - P_8(x) = 0 \quad , \; D_8(x) = P_7''(x) - q_3(x) \quad . \end{aligned} \tag{69}$$

3− We add the defect function $D_i(x), (i = 1, 2, 3, ..., 8)$ to the original problem, which can be written as follows

$$
\begin{aligned}
Z_1' &= Z_2 \;, \\
Z_2' &= Z_3 \;, \\
Z_3' &= Z_4 \;, \\
Z_4' &= N\left[\frac{1}{1+\dfrac{\left(1-W_e^2\left(1-a^2\right)Z_3^2\right)}{\left(1+W_e^2\left(1-a^2\right)Z_3^2\right)^2}}\right] \\
&\quad \left[\begin{array}{c} \dfrac{2W_e^2\left(1-a^2\right)Z_3Z_4^2}{\left(1+W_e^2\left(1-a^2\right)Z_3^2\right)^2} \\ +\dfrac{4W_e^2\left(1-a^2\right)\left(1-W_e^2\left(1-a^2\right)Z_3^2\right)Z_3Z_4^2}{\left(1+W_e^2\left(1-a^2\right)Z_3^2\right)^3} \end{array}\right] + D_4(x)\,, \\
Z_5' &= Z_6 \;, \\
Z_6' &= \left(\frac{-3P_rE_cR_n}{3R_n+4P_r}\right)\left[2+\frac{N}{\left(1+W_e^2\left(1-a^2\right)Z_3^2\right)}\right]Z_3^2 + D_6(x) \;, \\
Z_7' &= Z_8 \;, \\
Z_8' &= -S_cS_rZ_6' + S_cR_cZ_7^m + D_8(x) \;.
\end{aligned} \tag{70}
$$

4– We solved the pseudo problem (70) by the same method and we will have the solutions $Z_i, (i = 1, 2, 3, ..., 8)$.

5– We calculate an estimation of the global error from the formula

$$\underset{\sim}{e}_n = \underset{\sim}{Z}_n - \underset{\sim}{Z}(x_n) = \underset{\sim}{Z}_n - \underset{\sim}{P}(x_n) \;, \qquad (n = 1, 2, ..., 6) \tag{71}$$

Where, Z_n is the approximate solution of (70), (the pseudo problem) at the point x_n , and $Z(x_n)$ is the exact solution of the pseudo problem at x_n. obviously the exact solution of (70) is $Z(x) = P(x)$. The values of the global error are shown in table (1). This error is based on using 11 points to find the interpolating polynomials $P_i(x)$ of degree 5.

In order to achieve the above task we used combination of programs in Fortran (using NAG library routine D02HAF) and Mathematica package.

6 Numerical results and discussion

The present work generalized the problem of a Johnson-Segalman fluid with heat and mass transfer in a planer channel. Equations (63) with the boundary conditions (53) are approximated by a system of non-linear ordinary differential equations. This system was solved numerically by using a Runge-Kutta-Merson method and a Newton iteration shooting and matching technique. The functions ψ, u, θ, and ϕ are obtained and illustrated graphically as shown in figures (2) – (17) for different values of , Weissenberg number W_e , total flux number$F$, Eckeret number E_c, Prandtle number P_r , Soret number S_r , Schmidt number S_c , reaction parameter R_c , radiation parameter R_n, and reaction order m. The other parameters are chosen as $h = 1$, $N = 1$, and $a = 0.8$. During the curse of numerical solution, we noticed that the values of ψ and u are the same as obtained by [5]. The effect of Weissenberg number W_e shown in figures $(2)-(3)$. It is observed that, The temperature θ decreases with increasing W_e , while the concentration ϕ increases with increasing W_e . Figures (4) – (7) describe the effects of the total flux number F on the stream function distributions ψ, the velocity distributions u, the temperature distributions θ, and the concentration distributions ϕ. It is observed that as F decreases, ψ, u, and θ increase, while ϕ decreases. Figures (8) and (9) display results for the temperature profiles and the concentration profiles respectively. It is clear that as E_c increases, the temperature increases, while the concentration decreases. Figures (10) and (11) display results for the temperature distributions and the concentration distributions respectively. It is seen that, the temperature increases with increasing the Prandtle number P_r , while the concentration decreases with increasing P_r . Figures (12), (13), and (14) display the variation of the concentration distributions for several values of the Schmidt number S_c , the Soret number S_r , and the Reaction parameter R_c respectively. It is clear that, the concentration decreases with increasing of S_c , S_r , and R_c . Figures (15) and (16) display the variation of the temperature distributions and the concentration distributions for several values of the Radiation parameter R_n. It is noted that, as R_n increases the temperature increases, while the concentration decreases. In figure (17) the concentration distributions are shown. It is observed that, the concentration increases with increasing the reaction order m.

7 Conclusion

In this work, we have studied the motion of a Johnson-Segalman fluid with heat and mass transfer in a planer channel. We concerned our work on obtaining the temperature and the concentration distributions which are illustrated graphically at different values of the parameters of the problem. During the study we have that, the solutions of the stream function and the velocity are the same as the main problem [5] in spite of the different of numerical method which we are used. The technical method which used to solve this problem is Runge-Kutta-Merson method and a Newton-iteration shooting and matching technique. Global error estimation is also obtained using Zadunaisky technique. The errors estimated justify the use of the approximate solution as a suitable approximation to the calculated physical values.

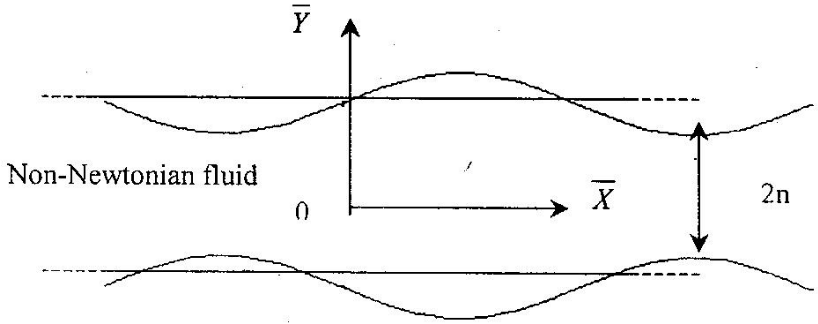
8 Applications

Peristalsis is now well-known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a peristaltic mechanism may be involved in swallowing food through the esophagus, in urine transport from the kidney to the bladder through the urethra, in movement of chyme in the gastro-intestinal tract, in the transport of spermatozoa in the ductus efferents of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotion of small blood vessels such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

Table 1

y	ψ=Y1	error (e1n)	u=Y2	error (e2n)	θ=Y5	error (e5n)	ϕ=Y7	error (e7n)
0	.000D+00	.0000D+00	-.242D+01	.1300D+00	.100D+01	.0000D+00	.100D+01	.0000D+00
0.1	-.242D+00	.1200D-01	-.241D+01	.1000D+00	.927D+00	.0000D+00	.820D+00	.0000D+00
0.2	-.482D+00	.2000D-01	-.239D+01	.6000D-01	.853D+00	.0000D+00	.661D+00	.1000D-02
0.3	-.719D+00	.2400D-01	-.234D+01	.1000D-01	.779D+00	.0000D+00	.520D+00	.1000D-02
0.4	-.949D+00	.2200D-01	-.226D+01	-.4000D-01	.702D+00	.3000D-02	.393D+00	-.1000D-02
0.5	-.117D+01	.2000D-01	-.215D+01	-.7000D-01	.622D+00	.8000D-02	.280D+00	-.6000D-02
0.6	-.138D+01	.1000D-01	-.200D+01	-.8000D-01	.534D+00	.1400D-01	.182D+00	-.1200D-01
0.7	-.157D+01	.0000D+00	-.181D+01	-.5000D-01	.435D+00	.1700D-01	.100D+00	-.1560D-01
0.8	-.174D+01	.0000D+00	-.158D+01	-.1000D-01	.317D+00	.1700D-01	.385D-01	-.1460D-01
0.9	-.188D+01	-.1000D-01	-.131D+01	.1000D-01	.175D+00	.9000D-02	.273D-02	-.7940D-02
1	-.200D+01	.0000D+00	-.100D+01	.0000D+00	.133D-08	-.7693D-06	-.144D-08	.8344D-06

Note: this table contains the values of the dimensionless physical quantities, the streamline function ψ, the velocity u, the temperature θ, and the concentration ϕ at the values of the dimensionless distance y, (y = [0,1]). Also, this table contains the values of the global error e_{1n} of ψ, the global error e_{2n} of u, the global error e_{5n} of θ, and the global error e_{7n} of ϕ.



Figure(1)

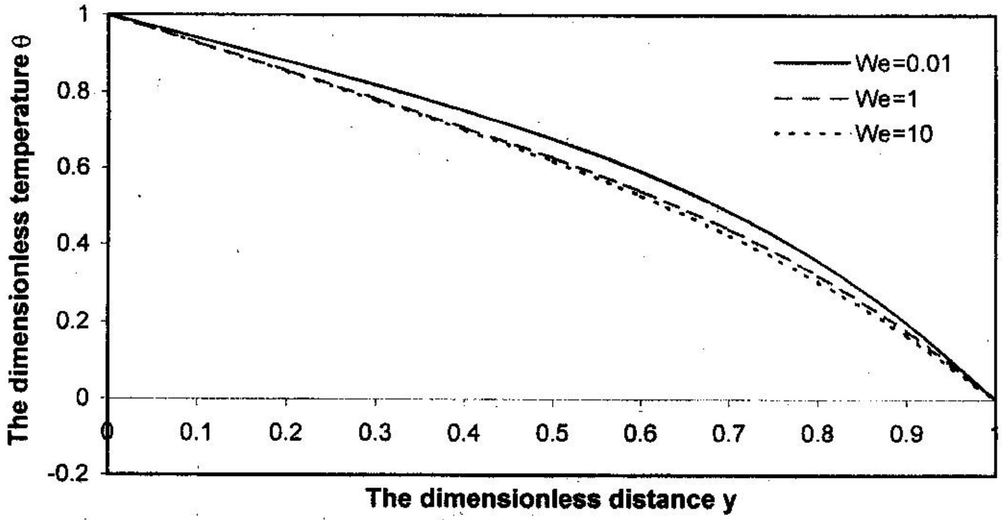


Figure (2). The temperature profiles are plotted versuse y for different values of We for a system have the particulars F=-2, a=0.8, Pr=0.7, Ec=0.5, RN=1, Sc=0.5, Sr=2, Rc=5, m=1

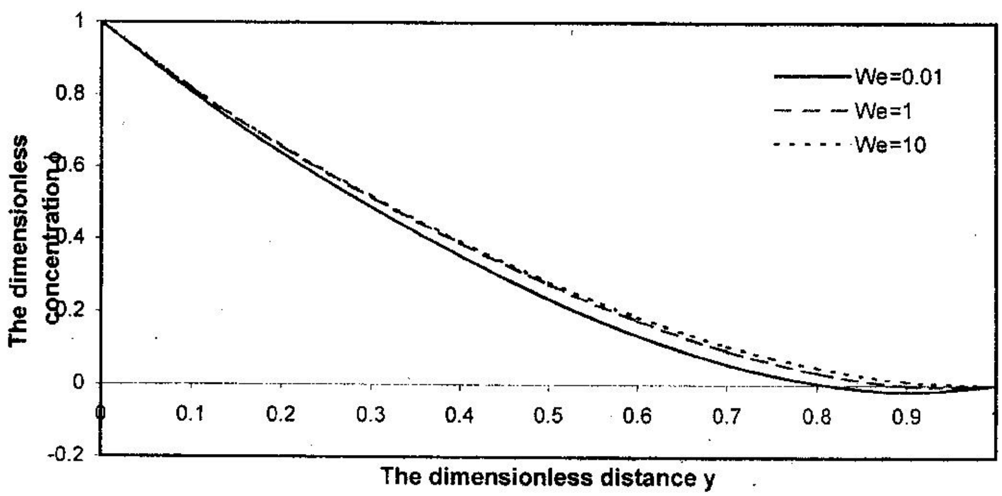


Figure (3). The concentration profiles are plotted versus y for different values of We for a system as in figure (2).

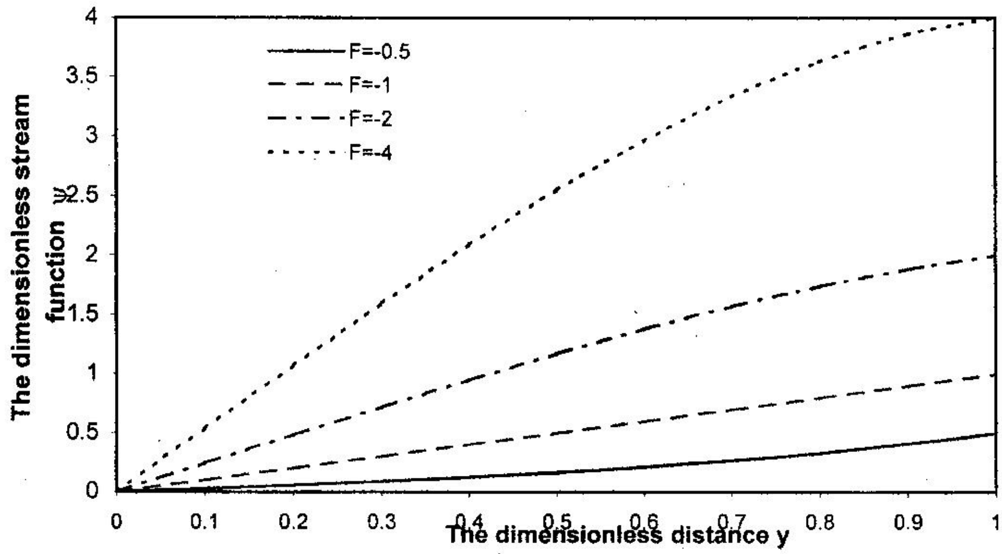


Figure (4). The dimensinless stream function profiles are plotted versus y for different values of F for a system have the particulars We=1.5, a=0.8, Pr=0.7, Ec=0.5, RN=1, Sc=0.5, Sr=2, RC=5, and m=1

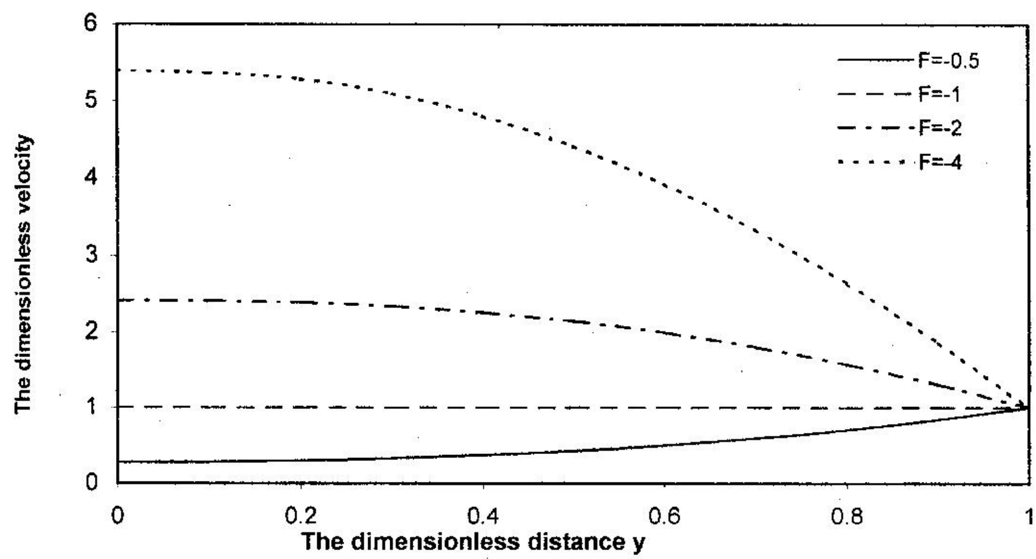


Figure (5). The dimensinless velocity profiles are plotted versus y for different values of F for a system as in figure (4).

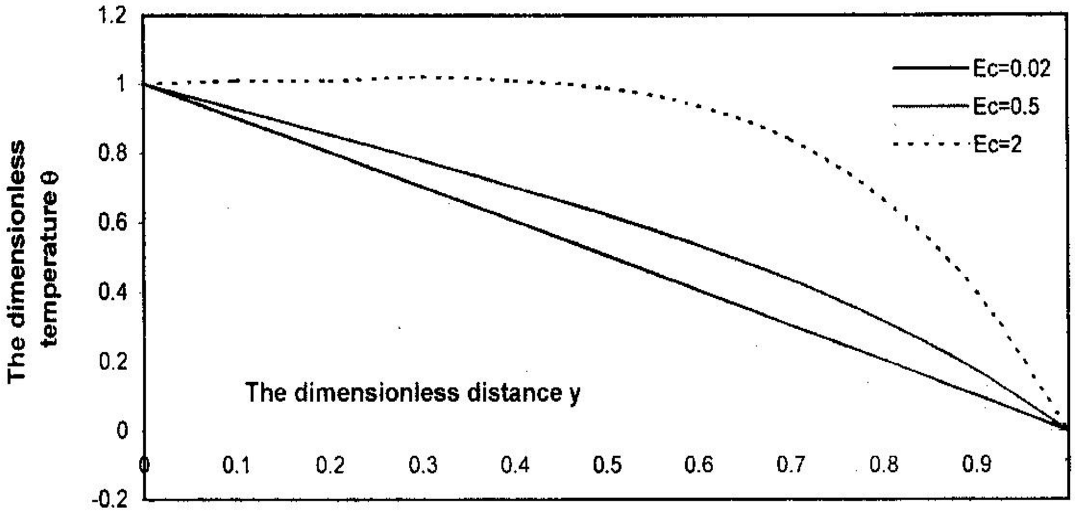


Figure (8). The temperature profiles are plotted versus y for different values of Ec for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, RN=1, Sc=0.5, Sr=2, Rc=5, and m=1

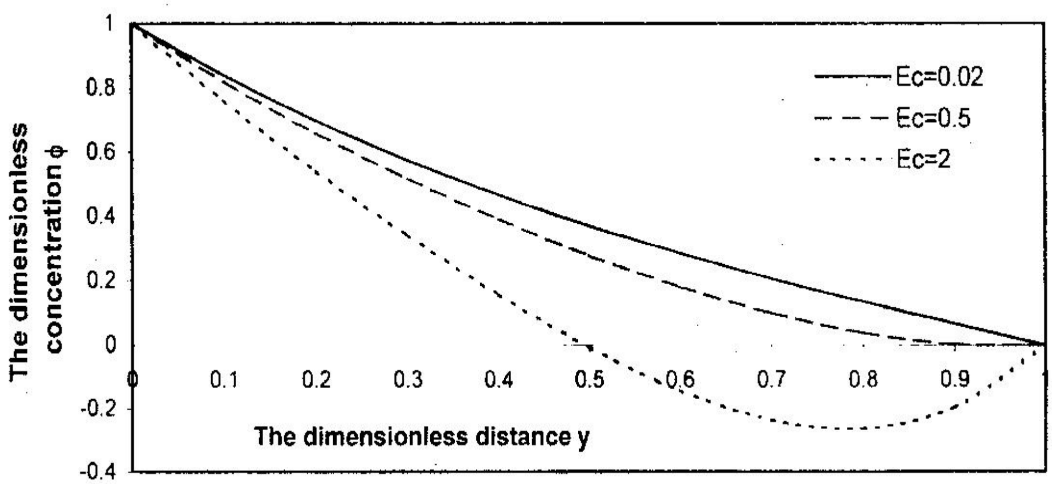


Figure (9). The concentration profiles are plotted versus y for different values of Ec for a system as in figure (8).

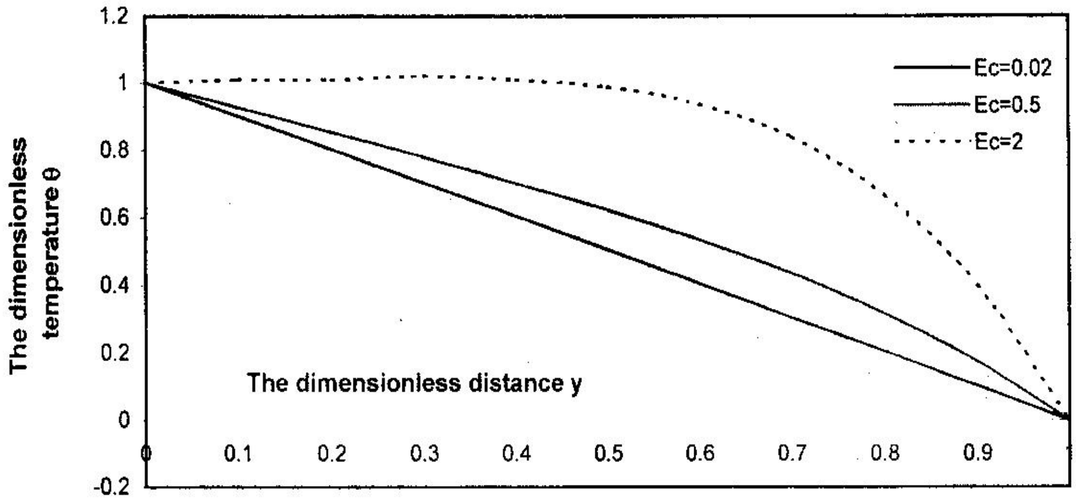


Figure (8). The temperature profiles are plotted versus y for different values of Ec for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, RN=1, Sc=0.5, Sr=2, Rc=5, and m=1

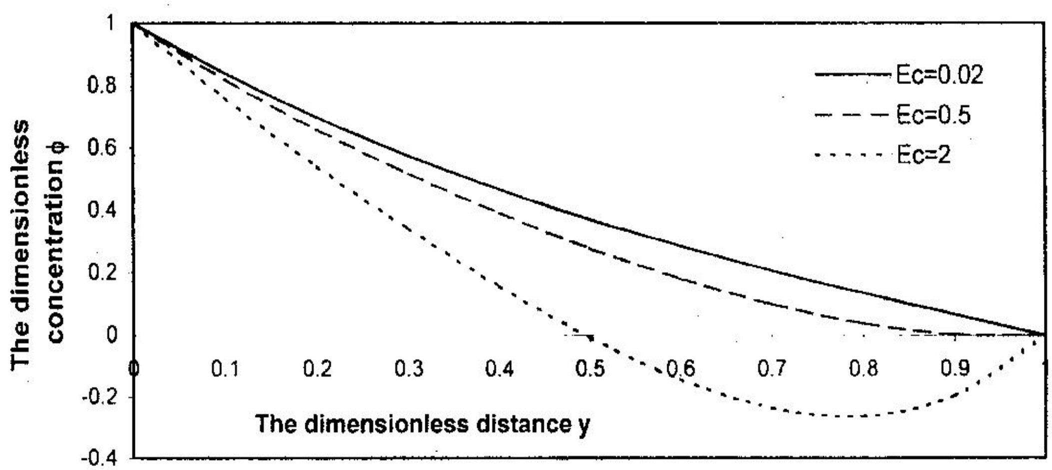


Figure (9). The concentration profiles are plotted versus y for different values of Ec for a system as in figure (8).

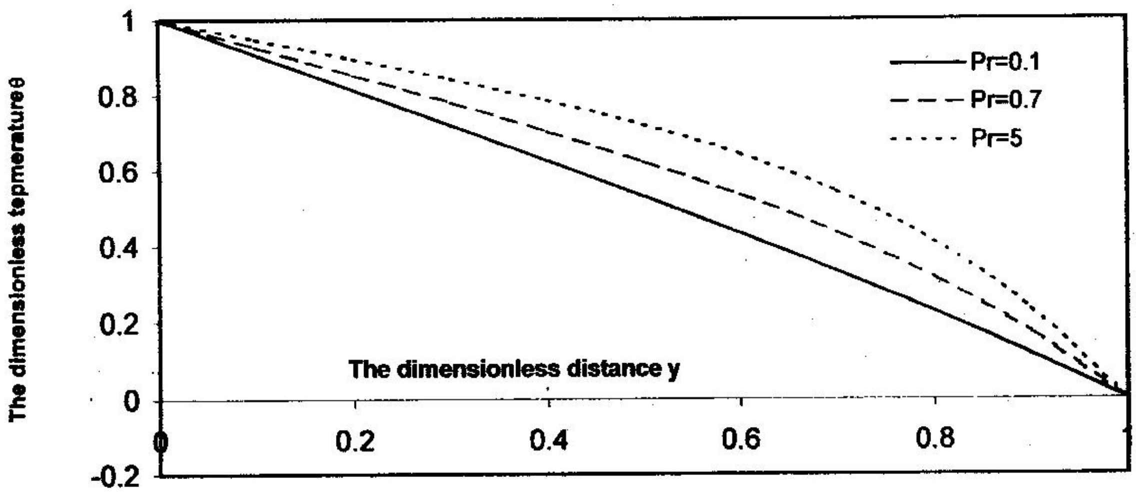


Figure (10). The temperature profiles are plotted versus y for different values of Pr for a system have the particulars F=-2, We=1.5, a=0.8, Ec=0.5, RN=1, Sc=0.5, Sr=2, Rc=5, and m=1

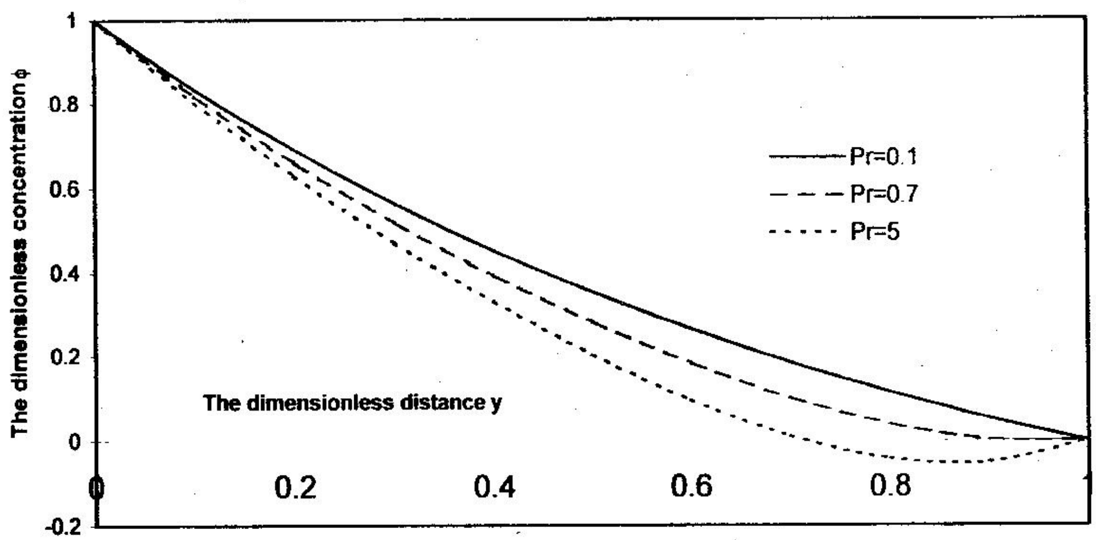


Figure (11). The concentration profiles are plotted versus y for different values of Pr for asystem as in figure (10).

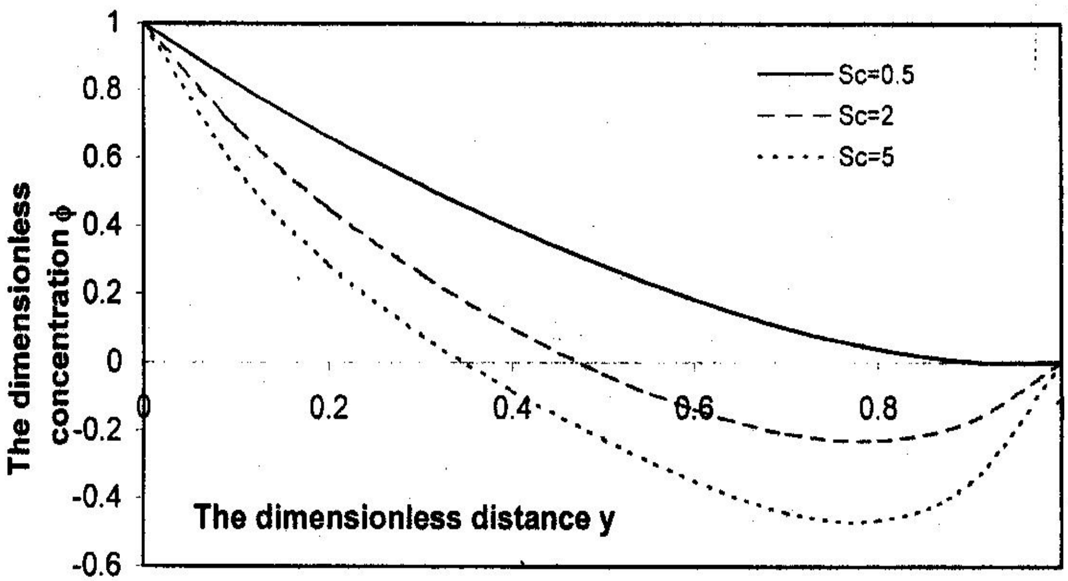


Figure (12). The concentration profiles are plotted versus y for different values of Sc for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, Ec=0.5, RN=1, Sr=2, Rc=5, and m=1

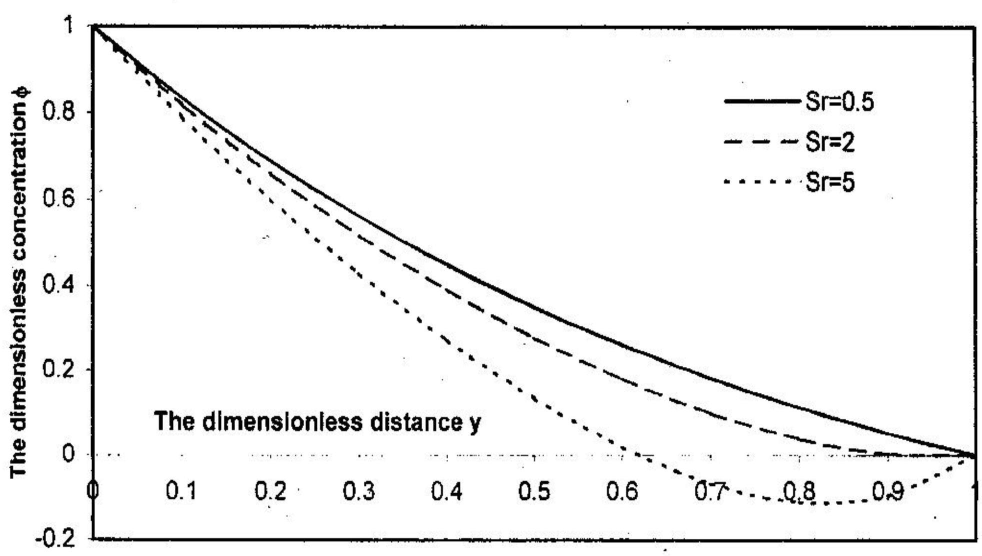


Figure (13). The concentration profiles are plotted versus y for different values of Sr for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, Ec=0.5, RN=1, Sc=0.5, Rc=5, and m=1

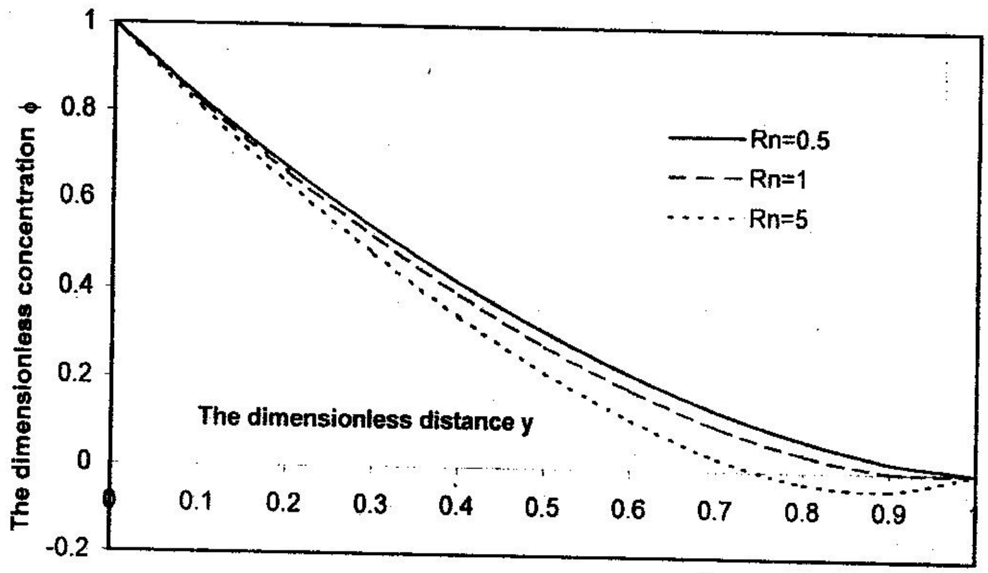


Figure (16). The concentration distributions are plotted versus y for different values of RN for a system as in figure (15).

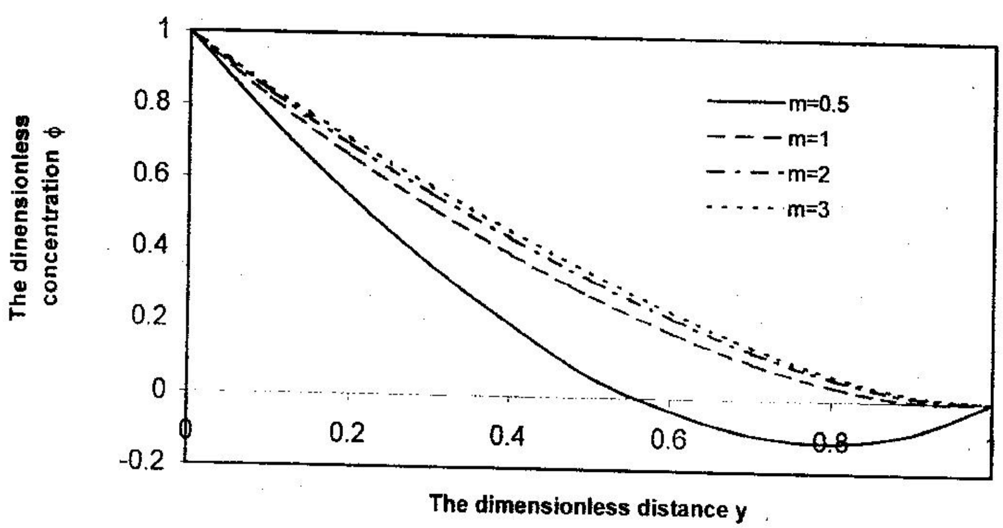


Figure (16). The concentration distributions are plotted versus y for different values of reaction order m for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, Ec=0.5, RN=1, Sc=0.5, Sr=2, and Rc=5

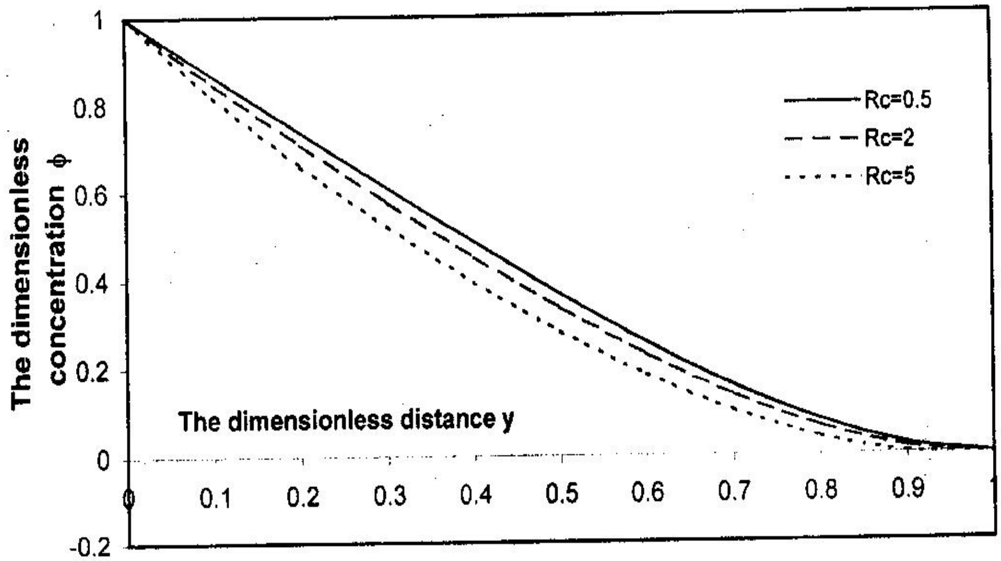


Figure (14). The dimensionless concentration distributions are plotted versuse y for different values of Rc for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, Ec=0.5, RN=1, Sc=0.5, Sr=2, and m=1

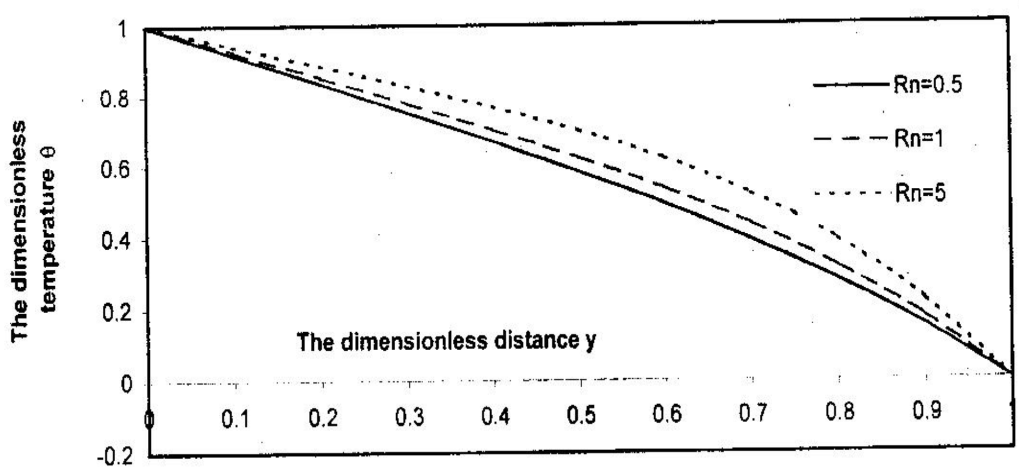


Figure (15). The tempreture profiles are plotted versus y for different values of RN for a system have the particulars F=-2, We=1.5, a=0.8, Pr=0.7, Ec=0.5, Sc=0.5, Sr=2, Rc=5,and m=1

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