ISSN 0973-8975
J.Mech.Cont.& Math. Sci., Vol.-3, No.-2, December (2008) Pages 324-336

ON INTERFACE WAVES IN SECOND ORDER THERMO-VISCOELASTIC SOLID MEDIA UNDER THE INFLUENCE OF GRAVITY

By

[1]D. P. ACHARYA, [2]INDRAJIT ROY and [3]H. S. CHAKRABORTY

109/3, Kailash Roy Chowdhury Road, Barrackpore, Kolkata- 700 120, India.
Email: acharyadp_05@rediffmail.com

Department of Mathematics, University of Kalyani, W B, India.

Abstract

The aim of the present paper is to investigate interface waves (surface waves) of Earthquakes in second order thermo-viscoelastic solid media under the influence of gravity. The displacement components are expressed in terms of displacement potentials. The problem of surface waves, particularly, Rayleigh waves, Love waves and Stoneley waves have been dealt with and the wave velocity equations corresponding to these waves have been determined. All final results and equations are in fair agreement with the corresponding classical results when the effect of temperature, viscosity and gravity are ignored.

Keyword and phrases : *thermo-viscoelastic solid, surface wave, Rayleigh wave, wave velocity, gravity.*

সংক্ষিপ্তসার

এই গবেষণা পত্রের উদ্দেশ্য হচ্ছে অভিকর্ষের প্রভাবাধীন দ্বিতীয় ক্রমের তাপ-সান্দ্র-স্থিতিস্থাপক ঘনবস্তু মাধ্যমে ভূ-কম্পনের আন্তস্থলীয় তরঙ্গ (তল-তরঙ্গ)- এর অনুসন্ধান করা। সরণ উপাংশগুলিকে স্থৈতিক উপাংশে প্রকাশ করা হয়েছে। তল - তরঙ্গের সমস্যাগুলিকে, বিশেষতঃ র‍্যালে তরঙ্গ, লাভ তরঙ্গ এবং স্টোনলী তরঙ্গগুলিকে নিয়ে কাজ করা হয়েছে। তাপ, সান্দ্রতা এবং অভিকর্ষের প্রভাবকে বাদ দিলে সকল চূড়ান্ত ফল এবং সমীকরণগুলি অনুরূপ আদি ফলাফলের সঙ্গে সম্পূর্ণ সঙ্গতিপূর্ণ হয়।

1. INTRODUCTION

The importance of interface waves and their investigations in isotropic elastic solid medium are well recognized in the study of earthquake waves and other phenomena in seismology and geophysics. The theory of surface waves has been developed by several investigators [Love 1911, Stoneley 1924, Ewing *et al* 1957, Bullen 1965, Jeffreys 1970].

The effect of viscosity, curvature, gravity, thermal field, magnetic field and other interacting fields are not considered in the classical problems of waves and vibrations to the desired extent. These effects have already been dealt with to a limited extent by [Ewing *et al* 1957].The influence of gravity on elastic waves and particularly on an elastic globe was first considered by [Bromwich 1898]. The influence of gravity on superficial waves has been investigated by [Love 1911] who has shown that the Rayleigh wave velocity is affected by the gravity field. Love has pointed out also that the effect of gravity is to increase the Rayleigh wave velocity to some significant amount when the wave length is large. Following the same theory and using the dynamical equation of motion for a homogenous isotropic elastic solid [De and Sengupta 1975, 1976, Das and Sengupta 1992, Das *et al* 1995, Ghosh *et al* 2000] have investigated the effect of gravity on elastic waves and vibrations and also on the propagation of waves in an elastic layer. Biot 1965 has studied the influence of gravity on Rayleigh waves assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible.

In this paper, an endeavor has been made to investigate the interface waves in thermo-viscoelastic solid under the influence of gravity where the concept of the second order viscoelastic model [Voigt 1887, Acharya and Mondal 2002] has been assumed.

2. BASIC EQUATIONS AND FORMULATION OF THE PROBLEM

Let M_1 and M_2 be two homogeneous semi-infinite viscoelastic media in welded contact at their common plane surface of separation under the influence of gravity and temperature. We suppose that the two media (M_2 being above M_1) are separated by a horizontal plane boundary of infinite dimensions. As reference system consider a set of orthogonal Cartesian axes $Ox_1x_2x_3$ with the origin O located at any point on the interface $x_3 = 0$ and Ox_3 pointing normally into M_1 $(x_3 = 0)$.

We consider the possibility of a wave moving in the positive x_1-direction and assume that the disturbance is confined to the neighborhood of the boundary thus making it a surface wave. Prior to the appearance of any disturbance both the media are everywhere at absolute temperature T_0.

We further assume that at any instant all the particles in a line parallel to x_2 axis have equal displacement i.e. all partial derivatives with respect to x_2 vanish. Let u_1, u_2, u_3 be the components of displacement at any point(x_1, x_2, x_3) at time t.

The dynamical equations of motion for three dimensional problems under the influence of gravity are [Sengupta and Acharya 1976, Das and Sengupta 1992]

$$\frac{\partial \sigma_{11}}{\partial x_1}+\frac{\partial \sigma_{12}}{\partial x_2}+\frac{\partial \sigma_{13}}{\partial x_3}+\rho g \frac{\partial u_3}{\partial x_1}=\rho \frac{\partial^2 u_1}{\partial t^2} \qquad (1)$$

$$\frac{\partial \sigma_{21}}{\partial x_1}+\frac{\partial \sigma_{22}}{\partial x_2}+\frac{\partial \sigma_{23}}{\partial x_3}+\rho g \frac{\partial u_3}{\partial x_2}=\rho \frac{\partial^2 u_2}{\partial t^2} \qquad (2)$$

$$\frac{\partial \sigma_{31}}{\partial x_1}+\frac{\partial \sigma_{32}}{\partial x_2}+\frac{\partial \sigma_{33}}{\partial x_3}-\rho g (\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2})=\rho \frac{\partial^2 u_3}{\partial t^2} \qquad (3)$$

where ρ is the mass density of the material medium, g is the acceleration due to gravity and σ_{ij} are the stress components.

According to Voigt's definition [Voigt 1887, Acharya and Mondal 2002] the stress strain relations for thermo-viscoelastic medium with second order viscosity may be presented as

$$\sigma_{ij}=2D_\mu e_{ij}+\{D_\lambda \Delta - D_\beta T\}\delta_{ij} \qquad (4)$$

where

$$D_\mu=\sum_{r=0}^{2}\mu^r \frac{\partial^r}{\partial t^r}\ ;\ D_\lambda=\sum_{r=0}^{2}\lambda^r \frac{\partial^r}{\partial t^r}\ ;\ D_\beta=\sum_{r=0}^{2}\beta^r \frac{\partial^r}{\partial t^r}\ ;$$

in which λ^0, μ^0 are the elastic parameters and λ^r, $\mu^r\ (r=1,2)$ are the parameters representing the effect of viscosity, $\beta^r\ (r=0,1,2)$ are the thermal parameters, T is the absolute temperature over the initial temperature T_0.

To suit the actual situation of the present problem the dynamical equations of motion with the accommodation of thermal field and gravity are [Sengupta and Acharya 1976]

$$\left(D_\lambda + D_\mu\right)\frac{\partial \Delta}{\partial x_1} + D_\mu \nabla^2 u_1 - D_\beta \frac{\partial T}{\partial x_1} + \rho g \frac{\partial u_3}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{5}$$

$$D_\mu \nabla^2 u_2 = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{6}$$

$$\left(D_\lambda + D_\mu\right)\frac{\partial \Delta}{\partial x_3} + D_\mu \nabla^2 u_3 - D_\beta \frac{\partial T}{\partial x_3} - \rho g \frac{\partial u_1}{\partial x_1} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{7}$$

Two displacement potentials $\varphi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ associated with the displacement components u_1 and u_3 may be defined in such a manner that

$$u_1 = \frac{\partial \varphi}{\partial x_1} - \frac{\partial \psi}{\partial x_3} \;;\; u_3 = \frac{\partial \varphi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \tag{8}$$

so that

$$\nabla^2 \varphi = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \equiv \Delta\,,\quad \nabla^2 \psi = \frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}.$$

Introduction of the functions φ and ψ into the above equations leads one to obtain two set of equations in compact form, valid for the media M_1 $(j=1)$ and M_2 $(j=2)$

$$D_\alpha \nabla^2 \varphi_j + g \frac{\partial \psi_j}{\partial x_1} - D_U T_j = \frac{\partial^2 \varphi_j}{\partial t^2}, \tag{9}$$

$$D_S \nabla^2 \psi_j - g \frac{\partial \varphi_j}{\partial x_1} = \frac{\partial^2 \psi_j}{\partial t^2}, \tag{10}$$

$$D_S \nabla^2 (u_2)_j = \frac{\partial^2 (u_2)_j}{\partial t^2}\;; \tag{11}$$

where suffixes $j=1$ and $j=2$ have been used to designate quantities for the media M_1 and M_2 respectively

$$D_\alpha = \sum_{r=0}^{2} \alpha_j^{r2} \frac{\partial^r}{\partial t^r},\; D_S = \sum_{r=0}^{2} S_j^{r2} \frac{\partial^r}{\partial t^r},\quad D_U = \sum_{r=0}^{2} U_j^{r2} \frac{\partial^r}{\partial t^r}.$$

in which

$$\alpha_j^{r2}=\sum_{r=0}^{2}\frac{\lambda_j^r+2\mu_j^r}{\rho_j},\ U_j^{r2}=\sum_{r=0}^{2}\frac{\beta_j^r}{\rho_j},\ S_j^{r2}=\sum_{r=0}^{2}\frac{\mu_j^r}{\rho_j},\ \beta_j^r=(3\lambda_j^r+2\mu_j^r)\alpha_t,\ r=0,1,2;$$

α_t is the coefficient of linear expansion of solid.

The generalized law of heat conduction in absence of heat source is [Rakshit and Sengupta 1998]

$$K_j\nabla^2T_j=\rho_jC_{vj}\frac{\partial T_j}{\partial t}+T_0D_\beta\frac{\partial}{\partial t}(\nabla^2\varphi_j), \tag{12}$$

where $K_j\,(j=1,2)$ are the coefficients of heat conduction and $c_{vj}\,(j=1,2)$ are the specific heats of the media at constant volume $\rho_j\,(j=1,2)$ are the densities of M_1 and M_2 respectively

The solutions of (9)-(12) for the medium M_1 may be taken in the forms

$$[\varphi_1,\psi_1,(u_2)_1,T_1]=[\hat{\varphi}(x_3),\hat{\psi}(x_3),\hat{u}_2(x_3),\hat{T}(x_3)]\exp\{i\omega(x_1-ct)\}, \tag{13}$$

Using (13) into (9)-(12) one obtains for the medium M_1

$$\left[\frac{d^2}{dx_3^2}+A_3^2\right]\hat{\varphi}+\frac{ig\omega\hat{\psi}}{\sum_{r=0}^{2}(-i\omega c)^r(\alpha_1^r)^2}=\frac{\sum_{r=0}^{2}(-i\omega c)^r(U_1^r)^2}{\sum_{r=0}^{2}(-i\omega c)^r(\alpha_1^r)^2}\hat{T}, \tag{14}$$

$$\left[\frac{d^2}{dx_3^2}+B_3^2\right]\hat{\psi}=\frac{ig\omega\hat{\varphi}}{\sum_{r=0}^{2}(-i\omega c)^r(S_1^r)^2}, \tag{15}$$

$$\left[\frac{d^2}{dx_3^2}-C_3^2\right]\hat{T}=-\frac{i\omega c\rho T_0}{K}\sum_{r=0}^{2}(-i\omega c)^r(U_1^r)^2\left[\frac{d^2}{dx_3^2}-\omega^2\right]\hat{\varphi}, \tag{16}$$

$$\frac{d^2\hat{u}_2}{dx_3^2}+B_3^2\hat{u}_2=0, \tag{17}$$

in which

$$A_3^2 = \omega^2\left\{\frac{c^2}{\sum_{r=0}^{2}(-i\omega c)^r(\alpha_1^r)^2}-1\right\};B_3^2 = \omega^2\left\{\frac{c^2}{\sum_{r=0}^{2}(-i\omega c)^r(S_1^r)^2}-1\right\};C_3^2 = \omega^2\left\{1-\frac{ic\rho C_v}{\omega K}\right\}.$$

Obviously the equations (14)-(17) have exponential solutions and in order that φ_1, ψ_1, T_1 and $(u_2)_1$ shall describe surface waves they must become vanishingly small as $x_3 \to \infty$.

Imposing this condition for the medium M_1, one may get

$$\left.\begin{aligned}
&\varphi_1 = \{A\exp(i\omega\alpha_1 x_3) + B\exp(i\omega\alpha_2 x_3) + C\exp(i\omega\alpha_3 x_3)\}\exp\{i\omega(x_1 - ct)\},\\
&T_1 = \{A_1\exp(i\omega\alpha_1 x_3) + B_1\exp(i\omega\alpha_2 x_3) + C_1\exp(i\omega\alpha_3 x_3)\}\exp\{i\omega(x_1 - ct)\},\\
&\psi_1 = \{A_2\exp(i\omega\alpha_1 x_3) + B_2\exp(i\omega\alpha_2 x_3) + C_2\exp(i\omega\alpha_3 x_3)\}\exp\{i\omega(x_1 - ct)\},\\
&(u_2)_1 = D\exp\{i\omega(H_1 x_3 + x_1 - ct)\},
\end{aligned}\right\} \quad (18)$$

where $H_1^2 = \left\{\frac{\rho c^2}{\sum_{r=0}^{2}(-i\omega c)^r \mu_1^r}-1\right\}$ of which the imaginary part is positive.

Using (18) in (14)-(17) it is seen that $\alpha_j^2\ (j=1,2,3)$ are the roots of the equation

$$\omega^6\alpha^6 + N_1\omega^4\alpha^4 + N_2\omega^2\alpha^2 + N_3 = 0, \quad (19)$$

where $N_1 = \nu_4^2 - B_3^2, N_2 = \nu_3^2 - \nu_4^2 B_3^2 - \nu_5^2, N_3 = B_3^2\nu_5^2 + \nu_3^2 C_3^2$, in which

$$\nu_1^2 = \frac{\left\{\sum_{r=0}^{2}(-i\omega c)^r(U_1^r)^2\right\}^2}{\sum_{r=0}^{2}(-i\omega c)^r(\alpha_1^r)^2}, \nu_2^2 = \frac{i\omega c\rho T_0}{B_3}, \nu_3^2 = \frac{(ig\omega)^2}{\left\{\sum_{r=0}^{2}(-i\omega c)^r(\alpha_1^r)^2\right\}\left\{\sum_{r=0}^{2}(-i\omega c)^r(S_1^r)^2\right\}},$$

$\nu_4^2 = C_3^2 - A_3^2 - \nu_1^2\nu_4^2$ and $\nu_5^2 = A_3^2 C_3^2 + \nu_1^2\nu_2^2\omega^2$.

The constants A_1, B_1, C_1, A_2, B_2 and C_2 are related with the constants A, B and C by means of the following expressions

$$A_2 = n_1 A, B_2 = m_1 B, C_2 = l_1 C, A_1 = n_1' A, B_1 = m_1' B, C_1 = l_1' C, \tag{20}$$

where

$$n_1 = \frac{ig\omega}{(B_3^2 - \omega^2\alpha_1^2)\left\{\sum_{r=0}^{2}(-i\omega c)^r (S_1^r)^2\right\}}, m_1 = \frac{ig\omega}{(B_3^2 - \omega^2\alpha_2^2)\left\{\sum_{r=0}^{2}(-i\omega c)^r (S_1^r)^2\right\}},$$

$$l_1 = \frac{ig\omega}{(B_3^2 - \omega^2\alpha_3^2)\left\{\sum_{r=0}^{2}(-i\omega c)^r (S_1^r)^2\right\}}, n_1' = -\frac{i\omega c\rho_1 T_0\left\{\sum_{r=0}^{2}(-i\omega c)^r (U_1^r)^2\right\}\left(\omega^2\alpha_1 + \omega^2\right)}{B_3\left(\omega^2\alpha_1^2 + C_3^2\right)},$$

$$m_1' = -\frac{i\omega c\rho_1 T_0\left\{\sum_{r=0}^{2}(-i\omega c)^r (U_1^r)^2\right\}\left(\omega^2\alpha_2 + \omega^2\right)}{B_3\left(\omega^2\alpha_2^2 + C_3^2\right)}, l_1' = -\frac{i\omega c\rho_1 T_0\left\{\sum_{r=0}^{2}(-i\omega c)^r (U_1^r)^2\right\}\left(\omega^2\alpha_3 + \omega^2\right)}{B_3\left(\omega^2\alpha_3^2 + C_3^2\right)}.$$

Corresponding equations and expressions valid for the medium M_2 may be determined similarly using different notations for physical parameters and field functions.

3. BOUNDARY CONDITIONS

Let us now formulate two boundary conditions which must be satisfied for the present problem.

I. The components of displacement, temperature and normal flux at the boundary surface between the two media M_1 and M_2 must be continuous and the continuity is independent of time and position on the boundary surface $x_3 = 0$.

II. The stress components σ_{31}, σ_{32} and σ_{33} where

$$(\sigma_{31})_j = D_\mu\left(2\frac{\partial^2\varphi_j}{\partial x_1 \partial x_3} + \frac{\partial^2\psi_j}{\partial x_1^2} - \frac{\partial^2\psi_j}{\partial x_3^2}\right),$$

$$(\sigma_{32})_j = D_\mu \frac{\partial(u_2)_j}{\partial x_3},$$

$$(\sigma_{33})_j = D_\lambda \nabla^2 \varphi_j + 2D_\mu \left(\frac{\partial^2 \varphi_j}{\partial x_3^2} + \frac{\partial^2 \psi_j}{\partial x_1 \partial x_3} \right) - D_\beta T_j,$$

where as before $j = 1,2$ for the media M_1 and M_2 and across the boundary surface between M_1 and M_2 must be continuous at all times and places.

Using A', B', C', D', n_2, m_2, l_2, α_1', α_2', α_3', n_2', m_2', l_2', H_2 as the corresponding quantities for the medium and imposing the boundary conditions one obtains the following

$$A(1-n_1\alpha_1)+B(1-m_1\alpha_2)+C(1-l_1\alpha_3) = A'(1+n_2\alpha_1')+B'(1+m_2\alpha_2')+C'(1+l_2\alpha_3') \tag{21}$$

$$D = D' \tag{22}$$

$$A(\alpha_1+n_1)+B(\alpha_2+m_1)+C(\alpha_3+l_1) = A'(n_2-\alpha_1')+B'(m_2-\alpha_2')+C'(l_2-\alpha_3') \tag{23}$$

$$n_1'A + m_1'B + l_1'C = n_2'A' + m_2'B' + l_2'C' \tag{24}$$

$$B_3(\alpha_1 n_1' A + \alpha_2 m_1' B + \alpha_3 l_1' C) = -B_3'(\alpha_1' n_2' A' + \alpha_2' m_2' B' + \alpha_3' l_2' C') \tag{25}$$

$$\begin{aligned} &\mu_1^*[A(n_1\alpha_1^2 - 2\alpha_1 - n_1) + B(m_1\alpha_2^2 - 2\alpha_2 - m_1) + C(l_1\alpha_3^2 - 2\alpha_3 - l_1)] \\ &= \mu_2^*[A'(n_2\alpha_1'^2 - 2\alpha_1' - n_2) + B'(m_2\alpha_2'^2 - 2\alpha_2' - m_2) + C'(l_2\alpha_3'^2 - 2\alpha_3' - l_2)] \end{aligned} \tag{26}$$

$$H_1\mu_1^* D = -H_2\mu_2^* D' \tag{27}$$

$$\begin{aligned} &A[\lambda_1^*(1+\alpha_1^2) + 2\mu_1^*\alpha_1(\alpha_1+n_1) + n_1'\beta_1^*] + B[\lambda_1^*(1+\alpha_2^2) + 2\mu_1^*\alpha_2(\alpha_2+m_1) + m_1'\beta_1^*] \\ &+ C[\lambda_1^*(1+\alpha_3^2) + 2\mu_1^*\alpha_3(\alpha_3+l_1) + l_1'\beta_1^*] = A'[\lambda_2^*(1+\alpha_1'^2) + 2\mu_2^*\alpha_1'(\alpha_1'-n_2) - n_2'\beta_2^*] \\ &+ B'[\lambda_2^*(1+\alpha_2'^2) + 2\mu_2^*\alpha_2'(\alpha_2'-m_2) - m_2'\beta_2^*] + C'[\lambda_2^*(1+\alpha_3'^2) + 2\mu_2^*\alpha_3'(\alpha_3'-l_2) - l_2'\beta_2^*] \end{aligned} \tag{28}$$

where the asterisk indicates the complex quantities

$$\mu_j^* = \sum_{r=0}^{2}(-i\omega c)^r \mu_j^r,\ \lambda_j^* = \sum_{r=0}^{2}(-i\omega c)^r \lambda_j^r,\ \beta_j^* = \sum_{r=0}^{2}(-i\omega c)^r \beta_j^r,\ j = 1,2.$$

It follows from the equations (22) and (27) that $D = D' = 0$. Hence there is no displacement in the x_2 direction i.e. there is no transverse component of displacement. Thus no SH waves occur in this case.

Eliminating the constants A, B, C, A', B' and C' from the equations (21), (23)-(26) and (28) one finally obtains the following wave velocity equation in determinantal form as

$$\begin{vmatrix} (1-n_1\alpha_1) & (1-m_1\alpha_2) & (1-l_1\alpha_3) & -(1+n_2\alpha_1') & -(1+m_2\alpha_2') & -(1+l_2\alpha_3') \\ (\alpha_1+n_1) & (\alpha_2+m_1) & (\alpha_3+l_1) & (\alpha_1'-n_2) & (\alpha_2'-m_2) & (\alpha_3'-l_2) \\ n_1' & m_1' & l_1' & -n_2' & -m_2' & -l_2' \\ K_1\alpha_1 n_1' & K_1\alpha_2 m_1' & K_1\alpha_3 l_1' & K_2\alpha_1' n_2' & K_2\alpha_2' m_2' & K_2\alpha_3' l_2' \\ F_1(\alpha_1,n_1) & F_1(\alpha_2,m_1) & F_1(\alpha_3,l_1) & F_2(\alpha_1',n_2) & F_2(\alpha_2',m_2) & F_2(\alpha_3',l_2) \\ G_1(\alpha_1,n_1,n_1') & G_1(\alpha_2,m_1,m_1') & G_1(\alpha_3,l_1,l_1') & G_2(\alpha_1',n_2,n_2') & G_2(\alpha_2',m_2,m_2') & G_2(\alpha_3',l_2,l_2') \end{vmatrix} = 0 \tag{29}$$

where $F_1(x, y) = \mu_1^*(x^2 y - 2x - y)$, $F_2(x, y) = \mu_2^*(y - yx^2 - 2x)$,

$G_1(x, y, z) = -\lambda_1^*(1 + x^2) - 2\mu_1^* x(x + y) - z\beta_1^*$ and $G_2(x, y, z) = \lambda_2^*(1 + x^2) - 2\mu_2^* x(y - x) - z\beta_2^*$.

The above equation (29) represents the frequency equation for the general surface waves propagated near the interface of two different thermo-viscoelastic solid semi-infinite media of Voigt type with second order viscosity under the influence of gravity.

4. EXAMPLES OF SURFACE WAVES

Rayleigh waves: For the case of viscoelastic Rayleigh waves of second order under the thermal field and gravity, medium M_2 is replaced by vacuum, so that the plane interface now becomes a free surface. Moreover, in this case, there comes a thermal boundary condition [Chadwick 1960, Mukherjee and Sengupta 1991]

$$\frac{\partial T_1}{\partial x_3} + hT_1 = 0 \quad \text{on} \quad x_3 = 0 \tag{30}$$

where h is Planck's constant.

Since the temperature difference across the free surface is always small this linearized form of radiative condition is valid on the boundary $x_3 = 0$.

As found earlier, we note here again that there can be no SH waves. From the equations (26) and (28) putting $A' = B' = C' = 0$ the relevant equations take the modified forms

$$A(n_1\alpha_1^2 - 2\alpha_1 - n_1) + B(m_1\alpha_2^2 - 2\alpha_2 - m_1) + C(l_1\alpha_3^2 - 2\alpha_3 - l_1) = 0, \tag{31}$$

$$\begin{aligned} &A[\lambda_1^*(1+\alpha_1^2) + 2\mu_1^*\alpha_1(\alpha_1 + n_1) + n_1'\beta_1^*] + B[\lambda_1^*(1+\alpha_2^2) + 2\mu_1^*\alpha_2(\alpha_2 + m_1) + m_1'\beta_1^*] \\ &\qquad + C[\lambda_1^*(1+\alpha_3^2) + 2\mu_1^*\alpha_3(\alpha_3 + l_1) + l_1'\beta_1^*] = 0 \end{aligned} \tag{32}$$

From (30) we also have

$$(i\omega\alpha_1 + h)n_1'A + (i\omega\alpha_2 + h)m_1'B + (i\omega\alpha_3 + h)l_1'C = 0 \tag{33}$$

Eliminating the constants A, B and C from the equations (31)-(33) we get

$$\begin{aligned} &(n_1\alpha_1^2 - 2\alpha_1 - n_1)[\{\lambda_1^*(1+\alpha_2^2) + 2\mu_1^*\alpha_2(\alpha_2 + m_1) + m_1'\beta_1^*\}(\alpha_3 + h)l_1' \\ &-\{\lambda_1^*(1+\alpha_3^2) + 2\mu_1^*\alpha_3(\alpha_3 + l_1) + l_1'\beta_1^*\}(\alpha_2 + h)m_1'] - (m_1\alpha_2^2 - 2\alpha_2 - m_1) \\ &[\{\lambda_1^*(1+\alpha_1^2) + 2\mu_1^*\alpha_1(\alpha_1 + n_1) + n_1'\beta_1^*\}(\alpha_3 + h)l_1' - \{\lambda_1^*(1+\alpha_3^2) + 2\mu_1^*\alpha_3(\alpha_3 + l_1) + \\ &l_1'\beta_1^*\}(\alpha_1 + h)n_1'] + (l_1\alpha_3^2 - 2\alpha_3 - l_1)[\{\lambda_1^*(1+\alpha_1^2) + 2\mu_1^*\alpha_1(\alpha_1 + n_1) + n_1'\beta_1^*\}(\alpha_2 + h)m_1' - \\ &\{\lambda_1^*(1+\alpha_2^2) + 2\mu_1^*\alpha_2(\alpha_2 + m_1) + m_1'\beta_1^*\}(\alpha_1 + h)n_1'] = 0 \end{aligned} \tag{34}$$

This is the required wave velocity equation of Rayleigh waves in a thermo-viscoelastic solid semi-infinite medium with second order viscosity under the influence of gravity. When the effects of viscosity, temperature and gravity are neglected, the equation (34) is in well agreement with the corresponding classical results.

Love waves: For Love type of interface waves to exit we assume a layered semi infinite medium in which M_2 is bounded by two horizontal plane surfaces at a finite distance H apart, while M_1 is a semi-infinite medium as before. In the case of Love waves it is known that only the displacement component u_2 play the role.

For the medium M_1 we proceed exactly as in the general case and thus $(u_2)_1$ is given by the last equation of (18) with the imaginary part of H_1 positive. However, for the medium M_2, we must preserve the full solution, since the displacement no longer diminishes with the increasing distance from the interface of the two media. Hence

$$(u_2)_2 = A_4 \exp\{i\omega(H_2x_3 + x_1 - ct)\} + B_4 \exp\{i\omega(-H_2x_3 + x_1 - ct)\} \tag{35}$$

Since the displacement component and the stress component must be continuous across the plane of contact, we have

$$(u_2)_1 = (u_2)_2, \quad (\sigma_{32})_1 = (\sigma_{32})_2 \quad \text{at } x_3 = 0 \tag{36}$$

From the last equation of (18) and (35) with these conditions we get

$$D = A_4 + B_4 \text{ and } \mu_1^* H_1 D = \mu_2^* H_2 (A_4 - B_4) \tag{37}$$

Eliminating D we get

$$A_4(\mu_2^* H_2 - \mu_1^* H_1) = B_4(\mu_2^* H_2 + \mu_1^* H_1) \tag{38}$$

We now introduce the boundary condition that there is no stress across the free surface $x_3 = -H$

$$(\sigma_{32})_2 = 0 \quad \text{at} \quad x_3 = -H \tag{39}$$

Using (39) one obtains the following

$$A_4 \exp(-i\omega H_2 H) = B_4 \exp(i\omega H_2 H) \tag{40}$$

Eliminating the constants A_4 and B_4 from equations (38) and (40) one gets after a little simplification the following consistency equation

$$\mu_2^* H_2 \tan(H_2 \omega H) + i\mu_1^* H_1 = 0. \tag{41}$$

Equation (41) represents the required frequency equation for Love waves propagated near the interface of thermo-viscoelastic layered semi-infinite media with second order viscosity under the influence of gravity. It is observed from this equation that Love waves are not affected by the presence of a temperature field and gravitational field. Putting the parameters due to viscosity of the medium as zero in (41) one obtains the classical frequency equation for Love waves.

Stoneley waves: The generalized form of Rayleigh waves is the Stoneley waves in which we assume that the waves are propagated along the common boundary of the semi-infinite media M_1 and M_2. Thus the equation (29) will represent the wave velocity equation for Stoneley waves propagated near the interface of two different thermo-viscoelastic semi-infinite media with second order viscosity under the influence of gravity. This equation (29) reduces to the classical Stoneley wave velocity equation when the additional effects are not considered.

5. Conclusions

On conclusion it is noted that the general interface waves (Stoneley waves) and Rayleigh waves are greatly influenced by the viscous nature of the material media, the presence of gravity and thermal field. Such general interface waves and Rayleigh waves are dispersive in nature which is a contrast of the classical situation. It is remarkable to note that Love waves are not affected by the presence of temperature and gravitational fields. Further dispersion of Love waves occurs due to the presence of viscosity.

References

1) Love A E H (1911) *Some Problems of Geodynamics*. Dover, New York.
2) Stoneley R (1924) Elastic waves at the surface of separation of two solids. *Proc.Roy Soc. London* A-106, pp. 416-428.
3) Bullen KE (1965) *An introduction to the theory of seismology*. Cambridge University Press, London. pp. 85-99.
3) Jeffreys Sir H (1970) *The Earth,* Cambridge Univ. Press.
4) Bromwich TJIA (1898) On the influence of gravity on elastic waves and in particular on the vibrations of an elastic globe. *Proc. London Math. Soc.*30, pp. 98-120.
5) De SN and Sengupta PR (1975) Thermo-elastic Rayleigh waves under the influence of gravity. *Gerlands Betir. Geophysik.* 84(6), pp. 509-514.
6) De SN and Sengupta PR (1976) Surface waves under the influence of gravity. *Gerlands Betir. Geophysik.* 85(4), pp. 311-318.

7) Das TK and Sengupta PR (1992) Effect of gravity on visco-elastic surface waves in solids involving time rate of strain and stress of first order. *Sādhanā*. 17(2), pp. 315-323.

8) Das TK, Sengupta PR, and Debnath L (1995). Effect of gravity on visco-elastic surface waves in solids involving time rate of strain and stress of higher order. *Int. J. Math. Math. Sci.* 18(1), pp. 71-76.

9) Ghosh NC, Roy I and Biswas PK (2000) Effect of gravity and couple-stress on thermo-visco-elastic Rayleigh waves involving stress rate and strain rate. *J. Bih. Math. Soc.* 20, pp. 89-99.

10) Biot MA (1965) *Mechanics of incremental deformations*, Theory of elasticity and viscoelasticity of initially stressed solids and fluids, including thermodynamic foundations and applications to finite strain, John Wiley & Sons, New York. 44-45, 273-281.

11) Voigt W (1887) Theoretische Studien uber die Elasticitats Verhaltnisse der Krystalle. *Abh.Ges. Wiss. Gottingen* 34.

12) Acharya DP and Mondal A (2002) Plane waves in a rotating higher order viscoelastic solid with special reference to first and second order. *Acta Ciencia Indica*, 28(3), pp. 303-310.

13) Sengupta PR and Acharya DP (1976) Thermo elastic surface waves in the influence of gravity. *Acta Ciencia Indica* 2(4), pp. 406.

14) Rakshit AK and Sengupta PR (1998) Magneto-thermo-elastic waves in initially stressed conducting layer. *Sādhanā* 23(3), pp. 233-246.

15) Chadwick P (1960) *Progress in Solid Mechanics* Vol. 1 (Ed. I. N. Sneddon and R. Hill) North Holland Pub. Co., Amsterdam. 263.

16) Mukherjee A and Sengupta PR (1991) Surface waves in thermo visco-elastic media of higher order. *Ind. J. Pure Appl. Math.* 22 (2), pp. 159-167.