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AN ASYMPTOTIC METHOD FOR TIME DEPENDENT NONLINEAR SYSTEMS WITH VARYING COEFFICIENTS

By

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Abstract

An asymptotic method has been found to obtain approximate solution of a second order nonlinear differential system based on the extension of Krylov-Bogoliubov-Mitropolskii method, whose coefficients change slowly and periodically with time. Moreover, a non-autonomous case also investigated in which an external periodic force acts in the system. The solutions for different initial conditions show a good agreement with those obtained by numerical method. The method is illustrated by examples.

Keyword and phrases : *non-linear system, varying coefficient, periodic force, asymptotic method.*

সংক্ষিপ্তসার

ক্রাইলভ-বোগোলিউভব-মিট্রোপোলস্কি পদ্ধতির সম্প্রসারণের উপর ভিত্তি করে দ্বিতীয় ক্রমের অরৈখিক অবকল তন্ত্র যার সহগগুলি সময়ের সঙ্গে ধীরে এবং পর্যাবৃত্তাকারে পরিবর্তিত হয় তার আসন্ন সমাধান নির্ণয়ের জন্য স্পর্শপ্রবন পদ্ধতিটিকে পাওয়া গেছে। উপরন্তু একটি অ-স্বশাসিত ক্ষেত্রকেও অনুসন্ধান করা হয়েছে যখন একটি বহিঃ পর্যাবৃত্ত বল এই তন্ত্রের উপর *ক্রিয়াশীল। বিভিন্ন প্রারম্ভিক শর্তের জন্য যে সকল সমাধান পাওয়া যায় তা সাংখ্যমান পদ্ধতির সাহায্যে নির্ণিত সমাধানের সঙ্গে সংগতিপূর্ণ। এই পদ্ধতিটিকে উদাহরণের সাহায্যে ব্যাখ্যা করা হয়েছে।*

1. Introduction

Most of the well-known perturbation methods (*e.g.*, Poincare method [1], WKB method [2-4], Multi time-scale method [5-6] or Krylov-Bogoliubov-Mitropolskii (KBM) method [7-9]) were developed to find periodic solution of nonlinear differential system with constant and slowly varying coefficients. Among the above methods KBM method is convenient and widely used. Krylov and Bogoliubov [7] originally developed a perturbation method to obtain approximate solution of a second order nonlinear differential system described by

$$\ddot{x} + \omega_0^2 x = -\varepsilon f(x, \dot{x}), \tag{1}$$

where the over dots denotes the differentiation with respect to t, ω_0 is a positive constant and ε is a small parameter. Then the method was amplified and justified by Bogoliubov and Mitropolskii [8]. Mitropolskii [9] has extended the method to nonlinear system with slowly varying coefficients as

$$\ddot{x} + \omega_0^2(\tau).x = -\varepsilon f(x, \dot{x}, \tau), \qquad \tau = \varepsilon t \tag{2}$$

Following the extended Krylov-Bogoliubov-Mitropolskii (KBM) method [7-9]), Bojadziev and Edwards [10] studied some damped oscillatory and purely non-oscillatory systems with slowly varying coefficients modeled by

$$\ddot{x} + c(\tau).\dot{x} + \omega^2(\tau).x = -\varepsilon f(x, \dot{x}, \tau), \tag{3}$$

were $c(\tau)$ and $\omega(\tau)$ are positive. Murty [11] has presented a unified KBM method for both under-damped and over-damped system with constant coefficients. Shamsul [12] has presented a unified formula to obtain a general solution of an n-th order ordinary differential equation with constant and slowly varying coefficients. Hung and Wu [13] obtained an exact solution of a differential system in terms of Bessel's functions where the coefficients varying with time in an exponential order. Recently Roy and Shamsul [14] found an asymptotic solution of a differential system in which the coefficient changes in an exponential order of slowly varying time. The aim of this article is to extend the work of paper [14] to similar nonlinear problems in which the coefficients change slowly and periodically with time. Such problems arise in different branches of engineering, *e.g.*, rotor with slowly and periodically changing mass.

2. Method: Let us consider the nonlinear differential system

$$\ddot{x} + (k_1^2 + k_2 \sin\tau)x = -\varepsilon f(x, \tau), \qquad \tau = \varepsilon t \tag{4}$$

where the over-dots denote differentiation with respect to t, ε is a small parameter, k_1, k_2 are constants, $k_2 = O(\varepsilon)$, f is a given nonlinear function. Setting $\omega^2(\tau) = (k_1^2 + k_2 \sin\tau)$, $\omega(\tau)$ is known as frequency.

For $\varepsilon = 0$ and $\tau = \tau_0 =$ constant, $\lambda_1(\tau_0) = i\omega(\tau_0)$, $\lambda_2(\tau_0) = -i\omega(\tau_0)$ are two eigen values of the unperturbed equation of (4) and has the solution

$$x(t,0) = a_{1,0}e^{\lambda_1(\tau_0)t} + a_{2,0}e^{\lambda_2(\tau_0)t}. \tag{5}$$

When $\varepsilon \neq 0$ we seek a solution in accordance with KBM method, of the form

$$x(t,\varepsilon) = a_1(t,\tau) + a_2(t,\tau) + \varepsilon u_1(a_1,a_2,\tau) + \varepsilon^2 \ldots, \tag{6}$$

where a_1 and a_2 satisfy the equations

$$\begin{aligned} \dot{a}_1 &= \lambda_1(\tau)a_1 + \varepsilon A_1(a_1,a_2,\tau) + \varepsilon^2 \ldots, \\ \dot{a}_2 &= \lambda_2(\tau)a_2 + \varepsilon A_2(a_1,a_2,\tau) + \varepsilon^2 \ldots, \end{aligned} \tag{7}$$

Confining attention to the first few term 1, 2...m in the series expansion of (6) and (7), we evaluate functions $u_1, \ldots, A_1, A_2 \ldots$, such that a_1 and a_2 appearing in (6) and (7) satisfy (4) with an accuracy of ε^{m+1}. In order to determine these unknown functions it was early assumed by Murty [11], Shamsul [12] that the functions $u_1, \ldots$ exclude all fundamental terms, since these are included in the series expansion (6) at order ε^0.

Now differentiating (6) twice with respect to t, substituting for the derivatives $\ddot{x}$ and x in (1), utilizing relation (7) and comparing the coefficients of ε, we obtain

$$\begin{aligned} &\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right)A_1 + \lambda_1' a_1 + \left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right)A_2 + \lambda_2' a_2 \\ &+ \left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right)\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right)u_1 = -f^{(0)}(a_1,a_2,\tau), \end{aligned} \tag{8}$$

where $\lambda_1' = \frac{d\lambda_1}{d\tau}$, $\lambda_2' = \frac{d\lambda_2}{d\tau}$, $f^{(0)} = f(x_0, \tau)$ and $x_0 = a_1 + a_2$.

It is assumed that both $f^{(0)}$ can be expanded in Taylor's series [11-12]

$$f^{(0)} = \sum_{r_1, r_2 = 0}^{\infty} F_{r_1, r_2}(\tau) a_1^{r_1} a_2^{r_2}, \tag{9}$$

We have assumed that u_1 does not contain fundamental terms and for the reason the solution will be free from secular terms, namely $t\cos t$, $t\sin t$ and te^{-t} (see [12]). To obtain this solution (4), it has been proposed in [12] $u_1 \ldots$, excluded the terms $a_1^{r_1} a_2^{r_2}$ of $f^{(0)}$ where $r_1 - r_2 = \pm 1$. This restriction guarantees that the solution always excludes *secular*-type terms or first harmonics term and the solution becomes uniformly valid [7-9]. Moreover, we assume that A_1 and A_2 respectively contain terms a_1 and a_2. We have already mentioned that equation (4) is not a standard form of KBM method. We shall be able to transform (6) to the exact formal KBM [7-9] solution by substituting $a_1 = \frac{a}{2} e^{i\varphi}$ and $a_2 = \frac{a}{2} e^{-i\varphi}$. Herein a and φ are respectively amplitude and phase variable. (see [19,20]).

3. Example:

3.1. A nonlinear problem in absence of external force.

We consider a second order nonlinear system with constant and slowly varying coefficient

$$\ddot{x} + (k_1^2 + k_2 \sin \tau) x = -\varepsilon x^3, \tag{10}$$

Here over dots denote differentiation with respect to t, k_1, k_2 are constants, $k_2 = O(\varepsilon)$, $x_0 = a_1 + a_2$ and the function $f^{(0)}$ becomes,

$$f^{(0)} = -(a_1^3 + 3a_1^2 a_2 + 3a_1 a_2^2 + a_2^3). \tag{11}$$

Following the assumption (discussed in section 2) u_1 excludes the terms $3a_1^2 a_2$, $3a_1 a_2^2$.

We substitute in (8) and separate it into two parts as

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right) A_1 + \lambda_1' a_1 + \left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right) A_2 + \lambda_2' a_2, \\ = -(3a_1^2 a_2 + 3a_1 a_2^2) \tag{12}$$

and

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right)\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right) u_1 = -(a_1^3 + a_2^3) \tag{13}$$

The particular solution of (13) is

$$u_1 = -\frac{a_1^3}{2\lambda_1(3\lambda_1 - \lambda_2)} - \frac{a_2^3}{2\lambda_2(3\lambda_2 - \lambda_1)} \tag{14}$$

Now we have to solve (12) for two functions A_1 and A_2. According with unified KBM method A_1 contains the term $3a_1^2 a_2$ and A_2 contains the term $3a_1 a_2^2$ (Shamsul [12]) obtain the following equations

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right) A_1 + \lambda_1' a_1 = -3a_1^2 a_2 \tag{15}$$

and

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right) A_2 + \lambda_2' a_2 = -3a_1 a_2^2 \tag{16}$$

The particular solutions of (15)-(16) are

$$\begin{aligned} A_1 &= -\frac{\lambda_1' a_1}{\lambda_1 - \lambda_2} - \frac{3a_1^2 a_2}{2\lambda_1} \\ A_2 &= \frac{\lambda_2' a_2}{\lambda_1 - \lambda_2} - \frac{3a_1 a_2^2}{2\lambda_2} \end{aligned} \tag{17}$$

Substituting the functional values of A_1 and A_2 (17) into (7) and rearranging, we obtain

$$\dot{a}_1 = \lambda_1 a_1 + \varepsilon\left(-\frac{\lambda_1' a_1}{\lambda_1 - \lambda_2} - \frac{3a_1^2 a_2}{2\lambda_1}\right)$$
$$\dot{a}_2 = \lambda_2 a_2 + \varepsilon\left(\frac{\lambda_2' a_2}{\lambda_1 - \lambda_2} - \frac{3a_1 a_2^2}{2\lambda_2}\right) \tag{18}$$

In the case of un-damped, we have unable to find the exact solutions of a_1, a_2 and u_1.

Therefore, the first order solution of (10) is

$$x(t,\varepsilon) = a_1 + a_2 + \varepsilon u_1 \tag{19}$$

where a_1, a_2 are solution of (18) and u_1 is given by (14). Under the transformations, $a_1 = \frac{a}{2}e^{i\varphi}$ and $a_2 = \frac{a}{2}e^{-i\varphi}$ together with $\lambda_1 = i\omega$, $\lambda_2 = -i\omega$ and if we replace $A_1 e^{-i\varphi} + A_2 e^{i\varphi} = \widetilde{A}_1$ and $-i(A_1 e^{-i\varphi} - A_2 e^{i\varphi}) = a\widetilde{B}_1$ (where $\widetilde{A}_1$ and $\widetilde{B}_1$ are usual notation) equations reduce to

$$\dot{a} = \varepsilon \widetilde{A}_1(a) + \varepsilon^2 \ldots \text{and} \qquad \dot{\varphi} = \omega + \varepsilon \widetilde{B}_1(a) + \varepsilon^2 \ldots \tag{20}$$

We shall obtain the variational equations of a and φ in the real form (a and φ are know as amplitude and phase) which transform (18) to

$$\dot{a} = -\frac{\varepsilon a \omega'}{2\omega}, \quad \dot{\varphi} = \omega + \frac{3\varepsilon a^2}{8\omega}. \tag{21}$$

where $\omega = \sqrt{k_1^2 + k_2 \sin\tau}$, $\quad \omega' = \dfrac{k_2 \cos\tau}{2\sqrt{k_1^2 + k_2 \sin\tau}}$.

The variational equation (21) is a form of KBM solution. The variational equations for amplitude and phase are usually appeared in a set of first order differential equation and solved by numerical technique (see [12]).

3.2. Let us consider another form of the nonlinear differential problem (10)

$$\ddot{x} + k_1^2 x = -k_2 \sin\tau\, x - \varepsilon x^3 = -\varepsilon\, k \sin\tau\, x - \varepsilon x^3 , \tag{22}$$

where $k_2 = \varepsilon\, k$ and $k_1^2 = \omega^2$. Here,

$$f^{(0)} = -(\alpha_1^3 + 3\alpha_1^2\alpha_2 + 3\alpha_1\alpha_2^2 + \alpha_2^3) - k\sin\tau(\alpha_1 + \alpha_2) \tag{23}$$

In our assumption u_1 excludes the terms $3\alpha_1^2\alpha_2, 3\alpha_1\alpha_2^2$ and in our assumption u_1 excludes $k\sin\tau(\alpha_1 + \alpha_2)$. The equations of u_1, A_1 and A_2 become (discussed in Section 2)

$$\left(\lambda_1\alpha_1\frac{\partial}{\partial\alpha_1} + \lambda_2\alpha_2\frac{\partial}{\partial\alpha_2} - \lambda_1\right)\left(\lambda_1\alpha_1\frac{\partial}{\partial\alpha_1} + \lambda_2\alpha_2\frac{\partial}{\partial\alpha_2} - \lambda_2\right)u_1 = -(\alpha_1^3 + \alpha_2^3) \tag{24}$$

and

$$\begin{aligned}\left(\lambda_1\alpha_1\frac{\partial}{\partial\alpha_1} + \lambda_2\alpha_2\frac{\partial}{\partial\alpha_2} - \lambda_2\right)A_1 &= -3\alpha_1^2\alpha_2 - k\alpha_1\sin\tau,\\ \left(\lambda_1\alpha_1\frac{\partial}{\partial\alpha_1} + \lambda_2\alpha_2\frac{\partial}{\partial\alpha_2} - \lambda_1\right)A_2 &= -3\alpha_1\alpha_2^2 - k\alpha_2\sin\tau\end{aligned} \tag{25}$$

Solution of Eqs. (24)-(25) are

$$u_1 = -\frac{\alpha_1^3}{2\lambda_1(3\lambda_1 - \lambda_2)} - \frac{\alpha_2^3}{2\lambda_2(3\lambda_2 - \lambda_1)} \tag{26}$$

$$A_1 = -\frac{3\alpha_1^2\alpha_2}{2\lambda_1} - \frac{k\alpha_1 \sin\tau}{\lambda_1 - \lambda_2},$$
$$A_2 = -\frac{3\alpha_1\alpha_2^2}{2\lambda_2} + \frac{k\alpha_2 \sin\tau}{\lambda_1 - \lambda_2} \tag{27}$$

Substituting the functional values of A_1 and A_2 (27) into (7) and rearranging, we obtain

$$\dot{\alpha}_1 = \lambda_1 a_1 + \varepsilon\left(-\frac{3\alpha_1^2\alpha_2}{2\lambda_1} - \frac{k\alpha_1 \sin\tau}{\lambda_1 - \lambda_2}\right),$$
$$\dot{\alpha}_2 = \lambda_2 a_2 + \varepsilon\left(-\frac{3\alpha_1\alpha_2^2}{2\lambda_2} + \frac{k\alpha_2 \sin\tau}{\lambda_1 - \lambda_2}\right) \tag{28}$$

Therefore, the first order solution of (22) is

$$x(t,\tau) = \alpha_1 + \alpha_2 + \varepsilon u_1 \tag{29}$$

where α_1, α_2 are solutions of (28) and u_1 is given by(26).Under the transformations $\alpha_1 = \frac{\alpha}{2}e^{i\varphi}$ and $\alpha_2 = \frac{\alpha}{2}e^{-i\varphi}$, and substitution $\lambda_1 = i\omega$, $\lambda_2 = -i\omega$, we shall obtain the variational equation of α and φ in the real form (α and φ are know as amplitude and phase) which transform (28) to

$$\dot{\alpha} = 0, \quad \dot{\varphi} = \omega + \frac{3\varepsilon\alpha^2}{8\omega} + \frac{\varepsilon k \sin\tau}{2\omega}, \tag{30}$$

where $k_1^2 = \omega^2$.

4. Non-linear system with external force.

The method is used to similar nonlinear differential system with an external force $E\sin\nu t$,

$$\ddot{x} + (k_1^2 + k_2\sin\tau)x = -\varepsilon f(x,\tau) + \varepsilon E\sin\nu t, \qquad \tau = \varepsilon t \tag{31}$$

where ν is the frequency of the external force.

4.1. Let us consider a second order nonlinear differential system with an external force

$$\ddot{x} + (k_1^2 + k_2 \sin\tau)x = -\varepsilon x^3 + \varepsilon E \sin \nu t, \tag{32}$$

Here over dots denote differentiation with respect to t; k_1, k_2 are constants, $k_2 = O(\varepsilon)$, $x_0 = a_1 + a_2$ and the function

$$f^{(0)} = -(a_1^3 + 3a_1^2 a_2 + 3a_1 a_2^2 + a_2^3) + \frac{\varepsilon E}{2i}\left(e^{i\nu t} - e^{-i\nu t}\right). \tag{33}$$

Under the restriction (discussed in Section 2) u_1 excludes the terms $3a_1^2 a_2$, $3a_1 a_2^2$. Moreover in our assumption u_1 excludes $\varepsilon E\left(e^{i\nu t} - e^{-i\nu t}\right)/(2i)$. We substitute in (8) and separate it into two parts as

$$\begin{aligned}&\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right)A_1 + \lambda_1' a_1 + \left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right)A_2 + \lambda_2' a_2 \\ &= -(3a_1^2 a_2 + 3a_1 a_2^2) + \frac{\varepsilon E}{2i}(e^{i\nu t} - e^{-i\nu t}),\end{aligned} \tag{34}$$

and

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right)\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right)u_1 = -(a_1^3 + a_2^3). \tag{35}$$

The particular solution of (35) is

$$u_1 = -\frac{a_1^3}{2\lambda_1(3\lambda_1 - \lambda_2)} - \frac{a_2^3}{2\lambda_2(3\lambda_2 - \lambda_1)} \tag{36}$$

Now we have to solve (34) for two functions A_1 and A_2. According with unified KBM method A_1 contains the term $3a_1^2 a_2$, $Ee^{i\nu t}/(2i)$ and A_2 contains the term $3a_1 a_2^2$, $Ee^{-i\nu}/(2i)$ (see [12]) obtain the following equation

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_2\right) A_1 + \lambda_1' a_1 = -3a_1^2 a_2 + \frac{E}{2i} e^{i\nu} \tag{37}$$

and

$$\left(\lambda_1 a_1 \frac{\partial}{\partial a_1} + \lambda_2 a_2 \frac{\partial}{\partial a_2} - \lambda_1\right) A_2 + \lambda_2' a_2 = -3a_1 a_2^2 - \frac{E}{2i} e^{-i\nu} \tag{38}$$

The particular solutions of (37)-(38) are

$$\begin{aligned} A_1 &= -\frac{\lambda_1' a_1}{\lambda_1 - \lambda_2} - \frac{3a_1^2 a_2}{2\lambda_1} - \frac{Ee^{i\nu}}{2(\nu + \omega)} \\ A_2 &= \frac{\lambda_2' a_2}{\lambda_1 - \lambda_2} - \frac{3a_1 a_2^2}{2\lambda_2} - \frac{Ee^{-i\nu}}{2(\nu + \omega)} \end{aligned} \tag{39}$$

Substituting the functional values of A_1 and A_2 (39) into (7) and rearranging, we obtain (see Sub-section 3.1)

$$\begin{aligned} \dot{a}_1 &= \lambda_1 a_1 + \varepsilon\left(-\frac{\lambda_1' a_1}{\lambda_1 - \lambda_2} - \frac{3a_1^2 a_2}{2\lambda_1} - \frac{Ee^{i\nu}}{2(\nu + \omega)}\right), \\ \dot{a}_2 &= \lambda_2 a_2 + \varepsilon\left(\frac{\lambda_2' a_2}{\lambda_1 - \lambda_2} - \frac{3a_1 a_2^2}{2\lambda_2} - \frac{Ee^{-i\nu}}{2(\nu + \omega)}\right). \end{aligned} \tag{40}$$

Therefore, the first order solution of (32) is

$$x(t, \varepsilon) = a_1 + a_2 + \varepsilon u_1, \tag{41}$$

where a_1, a_2 are solutions of (40) and u_1 is given by (36). The variational equation of a and φ in the real form (a and φ are know as amplitude and phase), which transform (34) to

$$\dot{a} = -\frac{\varepsilon a \omega'}{2\omega} - \frac{\varepsilon E \cos\psi}{\nu + \omega}, \quad \dot{\varphi} = \omega - \nu + \frac{3\varepsilon a^2}{8\omega} + \frac{\varepsilon E \sin\psi}{a(\nu + \omega)}. \tag{42}$$

where $\omega = \sqrt{k_1^2 + k_2 \sin\tau}$, $\omega' = \dfrac{k_2 \cos\tau}{2\sqrt{k_1^2 + k_2 \sin\tau}}$.

Equation (42) is similar to that obtained by KBM method (see [19-20]).

5. Result and Discussions.

An approximate solution of second-order time dependent nonlinear system with constant and varying coefficient has been obtained based on the KBM [7-9] method. Theoretically, the solution can be obtained up to the accuracy of any order of approximation. However owing to the rapidly growing algebraic complexity for the derivation of the function, the solution is in general confined to a low order, usually the first. In order to test the accuracy of an approximate solution obtained by a certain perturbation method, one can easily compare the approximate solution to the numerical solution (considered to be exact). Due to such a comparison concerning the presented KBM method of this paper, we refer to the works of Murty [11], and Shamsul [12, 14-16] have been compared to the corresponding numerical solution. In this paper we have also compared the perturbation solutions (19), (29) and (41) of *Duffing's* equation (10) and (32) to those obtained by Range-kutta (Forth-order) procedure.

First of all, $x(t,\varepsilon)$ has been computed by perturbation solution (19) with initial condition $[x(0)=1, \dot{x}(0)=0]$ or $a=1.00000, b=-.001434$ for $\varepsilon=.05$. The corresponding numerical solutions has been also computed by forth order Runge-Kutta method. All the results are shown in Fig.1. From Fig.1 it is clear that the

asymptotic solution (19) shows a good agreement with the numerical solution of equation (10).

We have find the approximate solution of the same problem utilize the classical KBM method [7-8] (see Sub-section 3.2) and presented in Fig.2. Seeing the graph it is clear that the perturbation solution (29) does not agree with the numerical solution after a short time interval. Thus the extended KBM method is important.

In Section 4.1, a perturbation solution (41) has been derived when an external force acts and the solution has been presented in Fig.3 for $\varepsilon = .05$ $\nu = 1.1$, $E = .5$ with initial condition $[x(0) = 1, \dot{x}(0) = 0]$, or, $a = 1.00534$, $b = .103118$. This solution also shows a good coincidence with the numerical solution.

6. Conclusion. An approximate solution of a second order nonlinear deferential system with slowly varying coefficients has been found. This method is a generalization of KBM method. The solution for different initial condition shows good coincidence with corresponding numerical solution.

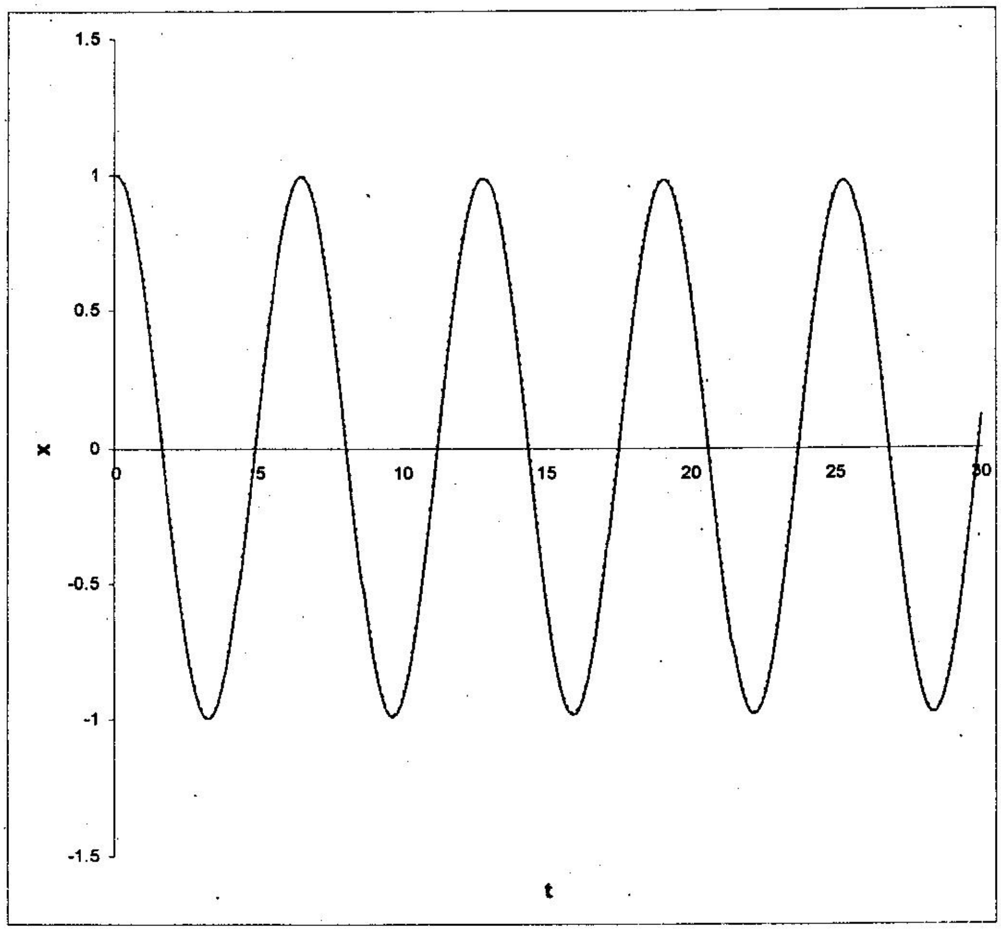

Fig 1: perturbation solution (dotted line) and numerical solution (solid line). In this $[x(0)=1, \dot{x}(0)=0]$ or $a=1.0$, $b=-.001434$ and $\varepsilon=.05$.

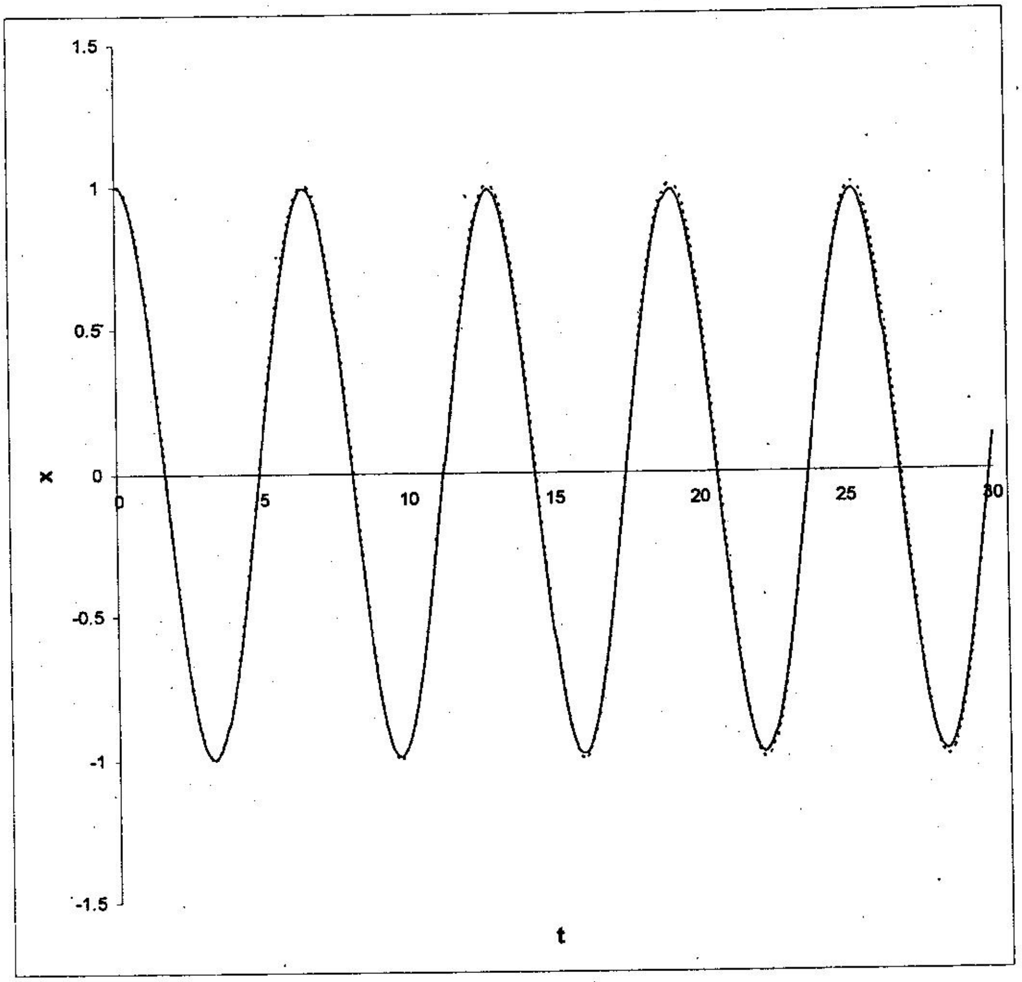

Fig.2: Perturbation solution (dotted line)and numerical solutions (solid line).In this $[x(0)=1,\ \dot{x}(0)=0]$ or $a=1.$, $b=0$ and $\varepsilon=.05$.

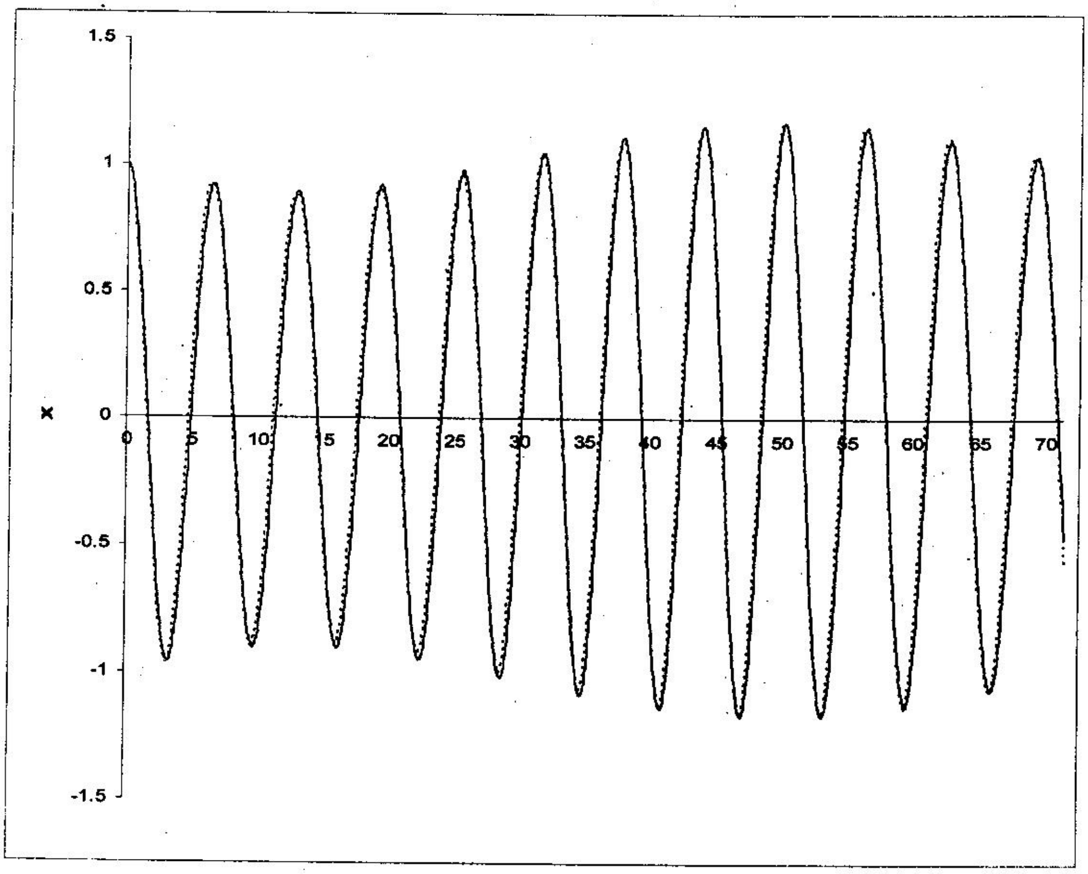

Fig.3: Perturbation solution (dotted line) and numerical solutions (solid line).In this $[x(0)=1,\ \dot{x}(0)=0]$ or $a=1.005340$ $b=.103118$ and $\varepsilon=.05$, $\nu=1.1$, $E=.5$.

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