

DECAY OF FIRST ORDER REACTANT IN INCOMPRESSIBLE MHD TURBULENT FLOW BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN A ROTATING SYSTEM.

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Abstract

Following Deissler's approach the decay for the concentration fluctuation of a dilute contaminant undergoing a first order chemical reaction in MHD turbulent flow at times before the final period in a rotating system for the case of multi-point and multi-time correlation equations is studied. Two-point, two-time and three-point, three-time correlation equations have been obtained and to make the set of equations determinate, the terms containing quadruple correlations are neglected in comparison with second and third order correlation terms. The solution obtained gives the decay law for the concentration fluctuations before the final period in a rotating system.

Keyword and phrases : MHD turbulent flow, rotating system, concentration fluctuation.

সংক্ষিপ্তসার

আবর্তন তন্ত্রে প্রথম ক্রমের রাসায়নিক বিক্রিয়ার অধীন লঘু সংক্রমণের গাঢ়তার হ্রাস - বৃদ্ধির জন্য শেখ পিরিয়ডের পূর্ব সময়ে (MHD) বিক্ষোভের অবক্ষয়কে বহু - বিন্দু এবং বহু - সময়ের জন্য সহ - পরিবর্তনের সমীকরণকে অনুসন্ধান করা হয়েছে ডিস্লে (Dessler) অভিজ্ঞতাকে অনুসরণ করে। দ্বি বিন্দু, দ্বি - সময় এবং ত্রি - বিন্দু, ত্রি - সময় সহ - পরিবর্তন সমীকরণগুলিকে নির্ণয় করা হয়েছে এবং সমীকরণগুলিকে গঠন করার জন্য দ্বিতীয় ও তৃতীয় ক্রমের সহ - পরিবর্তনের পদগুলিকে বাদ দেওয়া হয়েছে। নির্ণীত সমাধানটি আবর্তন তন্ত্রে শেখ পিরিয়ডের পূর্বের গাঢ়তার হ্রাস - বৃদ্ধির জন্য অবক্ষয় সূত্রটিকে প্রকাশ করেছে।

1. Introduction

Loeffler and Deissler⁽⁷⁾ used the theory, developed by Deissler^(3,4) to study the temperature fluctuations in homogeneous turbulence before the final period.

In their approach it is considered the two- and three-point correlation equations and solution were obtained of these equations after neglecting the fourth and higher order correlation terms. Using Deissler's theory ,Kumar and patel⁽⁵⁾ studied the first order reactant in homogeneous turbulence before the final period for the case of multi-point and multi-time consideration.

In our present study, the same approach of Deissler is applied to the study of magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulent flow before the final period in a rotating system.

In this problem, we considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally, we obtained the decay law for magnetic energy fluctuation of concentration before the final period in a rotating system.

2. Fundamental Equations

The equations of motion for viscous, incompressible MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k} (u_i u_k - h_i h_k) = -\frac{\partial W}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\varepsilon_{mki} \Omega_m u_i \quad (2.1)$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k} (h_i u_k - u_j h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad (2.2)$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0, \quad (2.3)$$

where

$u_i(\hat{x}, t)$ = component of turbulent velocity,

$h_i(\hat{x}, t)$ = component of magnetic field fluctuation ,

$W(\hat{x}, t) = \frac{p}{\rho} + \frac{1}{2} \langle h \rangle^2$ = total MHD pressure,

$P(\hat{x}, t)$ = hydrodynamic pressure,

ρ = fluid density,

ν = kinematic viscosity,

$\lambda = \frac{\nu}{P_M}$ = magnetic diffusivity,

P_M = magnetic prandtle number,

x_k = space coordinate; the subscripts can take on the values 1,2 or 3 ,

Ω_m = constant angular velocity component,

ε_{mkl} = alternating tensor.

3. Two-Point, Two-Time Correlation and Spectral Equations

If the turbulence and concentration magnetic field are homogeneous , chemical reaction and the local mass transfer have no effect on the velocity field , the reaction rate and the magnetic diffusivity are constant, then induction equation of a magnetic field governing the concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p'

separated by the vector \hat{r} could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - R h_i \quad (3.1)$$

and

$$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \lambda \frac{\partial^2 h'_j}{\partial x'_k \partial x'_k} - R h'_j \quad (3.2)$$

where R is the constant reaction rate.

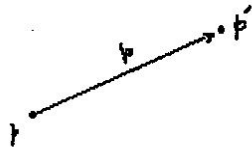


Fig.1. Vector configuration for two point correlation equations.

Multiplying equation (3.1) by h'_j and equation (3.2) by h_i and taking ensemble average, we get,

$$\frac{\partial \langle h_j h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k} [\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = \lambda \frac{\partial^2}{\partial x_k \partial x_k} - R \langle h_i h'_j \rangle \quad (3.3)$$

and

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x'_k} [\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle] = \lambda \frac{\partial^2}{\partial x'_k \partial x'_k} - R \langle h_i h'_j \rangle \quad (3.4)$$

Angular $\langle \dots \rangle$ which is used to denote an ensemble average.

Using the transformations

$$\begin{aligned} \frac{\partial}{\partial x_k} &= -\frac{\partial}{\partial r_k}, \quad \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k} \\ \left(\frac{\partial}{\partial t} \right)_{t'} &= \left(\frac{\partial}{\partial t} \right)_{\Delta t} - \frac{\partial}{\partial \Delta t}, \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \end{aligned} \quad (3.5)$$

into equations (3.3) and (3.4), one obtains

$$\begin{aligned} \frac{\partial \langle h_j h'_j \rangle}{\partial t} + \frac{\partial}{\partial r_k} [\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle](\hat{r}, \Delta t, t) \\ - \frac{\partial}{\partial r_k} [\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle](\hat{r}, \Delta t, t) = 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - 2R \langle h_i h'_j \rangle \end{aligned} \quad (3.6)$$

and

$$\frac{\partial \langle h_i h'_j \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k} [\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle](\hat{r}, \Delta t, t) = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - R \langle h_i h'_j \rangle \quad (3.7).$$

Using the relations (cf. Chandrasekhar⁽¹⁾)

$$\langle u_k h_i h'_j \rangle = -\langle u'_k h_i h'_j \rangle, \quad \langle u'_j h_i h'_k \rangle = -\langle u_i h_k h'_j \rangle$$

Equations (3.6) and (3.7) become

$$\frac{\partial \langle h_j h'_j \rangle}{\partial t} + 2 \frac{\partial}{\partial r_k} [\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - 2R \langle h_i h'_j \rangle \quad (3.8)$$

and

$$\frac{\partial \langle h_i h'_j \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k} [\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - R \langle h_i h'_j \rangle \quad (3.9)$$

Now, we write equations (3.8) and (3.9) in spectral form by use of the three dimensional Fourier transforms

$$\langle h_i h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \varphi_i \varphi'_j \rangle(\hat{K}, \Delta t, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (3.10)$$

and

$$\langle u_i h_k h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \varphi_k \varphi'_j \rangle(\hat{K}, \Delta t, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (3.11)$$

Interchanging the subscripts i and j and then interchanging the points p and p' gives

$$\begin{aligned} \langle u'_i h_j h'_j \rangle(\hat{r}, \Delta t, t) &= \langle u_k h_i h'_j \rangle(-\hat{r}, -\Delta t, t + \Delta t) \\ &= \int_{-\infty}^{\infty} \langle \alpha_i \varphi_i \varphi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \end{aligned} \quad (3.12)$$

where \hat{K} is known as a wave- number vector and $d\hat{K} = dK_1 dK_2 dK_3$. The magnitude of \hat{K} has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substituting equations (3.10) to (3.12) into equations (3.8) and (3.9), one obtains

$$\begin{aligned} \frac{\partial \langle \varphi_i \varphi'_j \rangle}{\partial t} + 2[\lambda k^2 + R] \langle \varphi_i \varphi'_j \rangle &= 2ik_k [\langle \alpha_i \varphi_k \varphi'_j \rangle(\hat{K}, \Delta t, t) \\ &\quad - \langle \alpha_k \varphi_i \varphi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t)] \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} \frac{\partial \langle \varphi_i \varphi'_j \rangle}{\partial \Delta t} + [\lambda k^2 + R] \langle \varphi_i \varphi'_j \rangle &= ik_k [\langle \alpha_i \varphi_k \varphi'_j \rangle(\hat{K}, \Delta t, t) \\ &\quad - \langle \alpha_k \varphi_i \varphi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t)] \end{aligned} \quad (3.14)$$

The tensors equations (3.13) and (3.14) becomes a scalar equation by contraction of the indices i and j

$$\begin{aligned} \frac{\partial \langle \varphi_i \varphi'_i \rangle}{\partial t} + 2[\lambda k^2 + R] \langle \varphi_i \varphi'_i \rangle &= 2ik_k [\langle \alpha_i \varphi_k \varphi'_i \rangle(\hat{K}, \Delta t, t) \\ &\quad - \langle \alpha_k \varphi_i \varphi'_i \rangle(-\hat{K}, -\Delta t, t + \Delta t)] \end{aligned} \quad (3.15)$$

and

$$\frac{\partial \langle \varphi_i \varphi_i' \rangle}{\partial \Delta t} + [\lambda k^2 + R] \langle \varphi_i \varphi_i' \rangle = ik_k [\langle \alpha_i \varphi_k \varphi_i' \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \varphi_i \varphi_i' \rangle (-\hat{K}, -\Delta t, t + \Delta t)] . \quad (3.16)$$

4. Three-point, Three-time correlation equations and solution for times before the final period

In the present investigation, under the same assumption as before, we take the momentum equation of MHD turbulence at the point p and inductions of magnetic field fluctuation of concentration at p' and p'' separated by the vector \hat{r} and \hat{r}' as

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} - h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial W}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\varepsilon_{mkl} \Omega_m u_l \quad (4.1)$$

$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \lambda \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} - R h_i' \quad (4.2)$$

$$\frac{\partial h_i''}{\partial t''} + u_k'' \frac{\partial h_i''}{\partial x_k''} - h_k'' \frac{\partial u_i''}{\partial x_k''} = \lambda \frac{\partial^2 h_i''}{\partial x_k'' \partial x_k''} - R h_i'' \quad (4.3)$$

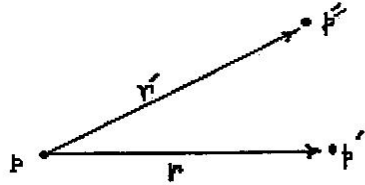


Fig.2. Vector configuration for three point correlation equations.

Multiplying equations (4.1) by $h_i h_j''$, (4.2) by $u_i h_j''$ and (4.3) by $u_i h_i'$ and taking ensemble average, one obtains

$$\frac{\partial \langle u_i h_i' h_j'' \rangle}{\partial t} + \frac{\partial}{\partial x_k} [\langle u_k u_i h_i' h_j'' \rangle - \langle h_k h_i h_i' h_j'' \rangle] = -\frac{\partial \langle W h_i' h_j'' \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i h_i' h_j'' \rangle}{\partial x_k \partial x_k} - 2\varepsilon_{mkl} \Omega_m u_l \langle u_i h_i' h_j'' \rangle, \quad (4.4)$$

$$\frac{\partial \langle u_i h_i' h_j'' \rangle}{\partial t'} + \frac{\partial}{\partial x_k'} [\langle u_i u_k' h_i' h_j'' \rangle - \langle u_i u_i' h_k' h_j'' \rangle] = \lambda \frac{\partial^2 \langle u_i h_i' h_j'' \rangle}{\partial x_k' \partial x_k'} - R \langle u_i h_i' h_j'' \rangle \quad (4.5)$$

$$\frac{\partial \langle \varphi_i \varphi_i' \rangle}{\partial \Delta t} + [\lambda k^2 + R] \langle \varphi_i \varphi_i' \rangle = ik_k [\langle \alpha_i \varphi_k \varphi_i' \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \varphi_i \varphi_i' \rangle (-\hat{K}, -\Delta t, t + \Delta t)] . \quad (3.16)$$

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$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} - h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial W}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\varepsilon_{mkl} \Omega_m u_l \quad (4.1)$$

$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \lambda \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} - R h_i' \quad (4.2)$$

$$\frac{\partial h_i''}{\partial t''} + u_k'' \frac{\partial h_i''}{\partial x_k''} - h_k'' \frac{\partial u_i''}{\partial x_k''} = \lambda \frac{\partial^2 h_i''}{\partial x_k'' \partial x_k''} - R h_i'' \quad (4.3)$$

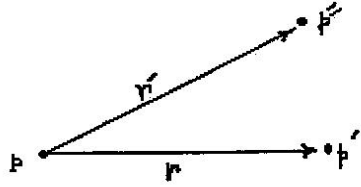


Fig.2. Vector configuration for three point correlation equations.

Multiplying equations (4.1) by $h_i h_j''$, (4.2) by $u_i h_j''$ and (4.3) by $u_i h_j'$ and taking ensemble average, one obtains

$$\frac{\partial \langle u_i h_i' h_j'' \rangle}{\partial t} + \frac{\partial}{\partial x_k} [\langle u_k u_i h_i' h_j'' \rangle - \langle h_k h_i h_i' h_j'' \rangle] = -\frac{\partial \langle W h_i' h_j'' \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i h_i' h_j'' \rangle}{\partial x_k \partial x_k} - 2\varepsilon_{mkl} \Omega_m u_l \langle u_i h_i' h_j'' \rangle, \quad (4.4)$$

$$\frac{\partial \langle u_i h_i' h_j'' \rangle}{\partial t'} + \frac{\partial}{\partial x_k'} [\langle u_i u_k' h_i' h_j'' \rangle - \langle u_i u_i' h_k' h_j'' \rangle] = \lambda \frac{\partial^2 \langle u_i h_i' h_j'' \rangle}{\partial x_k' \partial x_k'} - R \langle u_i h_i' h_j'' \rangle \quad (4.5)$$

$$\frac{\partial \langle u_i h'_i h''_j \rangle}{\partial t''} + \frac{\partial}{\partial x''_k} [\langle u_i u''_k h'_i h''_j \rangle - \langle u_i u'_i h'_i h''_k \rangle] = \lambda \frac{\partial^2 \langle u_i h'_i h''_j \rangle}{\partial x''_k \partial x''_k} - R \langle u_i h'_i h''_j \rangle. \quad (4.6)$$

Using the transformations

$$\frac{\partial}{\partial x_k} = - \left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r'_k} \right), \frac{\partial}{\partial x''_k} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r'_k}$$

$$\left(\frac{\partial}{\partial t} \right)_{t', t''} = \left(\frac{\partial}{\partial t} \right)_{\Delta t', \Delta t''} - \frac{\partial}{\partial \Delta t} - \frac{\partial}{\partial \Delta t'},$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial t''} = \frac{\partial}{\partial \Delta t'}$$

and six dimensional Fourier transforms of the type

$$\begin{aligned} \langle u_i h'_i h''_j \rangle(\hat{r}, \hat{r}', \Delta t, \hat{\Delta t}', t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \\ &\cdot \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \end{aligned} \quad (4.7)$$

$$\begin{aligned} \langle u_i u'_k h'_i h''_j \rangle(\hat{r}, \hat{r}', \Delta t, \hat{\Delta t}', t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_i \phi'_k \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \\ &\cdot \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \end{aligned} \quad (4.8)$$

and

$$\begin{aligned} \langle W h'_i h''_j \rangle(\hat{r}, \hat{r}', \Delta t, \hat{\Delta t}', t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \gamma \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \\ &\cdot \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \end{aligned} \quad (4.9)$$

into equations (4.4) to (4.6), we have

$$\frac{\partial}{\partial t} \langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [(1 + p_M)(k^2 + k'^2) + 2p_M k k' + \frac{2}{\lambda}(R + \varepsilon_{mkl} \Omega_m)]$$

$$\langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad (4.10)$$

$$\frac{\partial}{\partial \Delta t} \langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [k^2 + \frac{R}{\lambda}] \langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad (4.11)$$

and

$$\frac{\partial}{\partial \Delta t'} \langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [k'^2 + \frac{R}{\lambda}] \langle \phi_i \beta'_i \beta''_j \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \quad (4.12)$$

The tensor equations (4.10) to (4.12) can be converted to scalar equations by contraction of the indices i and j and inner multiplication by k_i

$$\frac{\partial}{\partial t} k_i \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [(1 + p_M)(k^2 + k'^2) + 2p_M k k' + \frac{2}{\lambda} (R + \varepsilon_{mkl} \Omega_m)] \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad (4.10a)$$

$$\frac{\partial}{\partial \Delta t} k_i \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [k^2 + \frac{R}{\lambda} \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)] = 0 \quad (4.11a)$$

$$\frac{\partial}{\partial \Delta t'} k_i \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [k'^2 + \frac{R}{\lambda} \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)] = 0 \quad (4.12a)$$

Integrating equations (4.10a) to (4.12a) between t_0 and t , we obtain

$$k_i \langle \phi_i \beta'_i \beta''_i \rangle = f_i \exp \left\{ -\lambda [(1 + p_M)(k^2 + k'^2) + 2p_M k k' \cos \theta + \frac{2}{\lambda} (R + \varepsilon_{mkl} \Omega_m)] (t - t_0) \right\},$$

$$k_i \langle \phi_i \beta'_i \beta''_i \rangle = g_i \exp \left[-\lambda \left(k^2 + \frac{R}{\lambda} \right) \Delta t \right]$$

and

$$k_i \langle \phi_i \beta'_i \beta''_i \rangle = q_i \exp \left[-\lambda \left(k'^2 + \frac{R}{\lambda} \right) \Delta t' \right].$$

For these relations to be consistent, we have

$$k_i \langle \phi_i \beta'_i \beta''_i \rangle = k_i \langle \phi_i \beta'_i \beta''_i \rangle_0 \exp \left\{ -\lambda [(1 + p_M)(k^2 + k'^2)(t - t_0) + k'^2 \Delta t + k^2 \Delta t' + 2p_M k k' \cos \theta (t - t_0) + \frac{2}{\lambda} (R + \varepsilon_{mkl} \Omega_m)(t - t_0 + \frac{\Delta t + \Delta t'}{2})] \right\}, \quad (4.13)$$

where θ is the angle between \hat{k} and \hat{k}' and $\langle \phi_i \beta'_i \beta''_i \rangle_0$ is the value of $\langle \phi_i \beta'_i \beta''_i \rangle$ at

$$t = t_0, \Delta t = \Delta t' = 0, \lambda = \frac{\nu}{P_M}.$$

By letting $\hat{r}' = 0, \Delta t' = 0$ in the equation (4.10) and comparing with equation (3.11) and (3.12), we get

$$\langle \alpha_i \varphi_i \varphi'_i \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_i \beta'_i \beta''_i \rangle (\hat{K}, \hat{K}', \Delta t, 0, t) d\hat{k}' \quad (4.14)$$

and

$$\langle \alpha_k \varphi_k \varphi'_k \rangle (-\hat{K}, -\Delta t, t + \Delta t) = \int_{-\infty}^{\infty} \langle \phi_i \beta'_i \beta''_i \rangle (-\hat{K}, -\hat{K}', -\Delta t, 0, t) d\hat{k}'. \quad (4.15)$$

Substituting equations (4.13) to (4.15) into equation (3.15), one obtains

$$\begin{aligned} \frac{\partial}{\partial t} \langle \phi_i \phi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda [k^2 + \frac{R}{\lambda}] \langle \phi_i \phi_i' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} 2ik_i [\langle \phi_i \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, 0, t) \\ - \langle \phi_i \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}', \Delta t, 0, t)]_0 \exp[-\lambda \{ (1 + p_M)(k^2 + k'^2)(t - t_0) + k^2 \Delta t \\ + 2p_M(t - t_0)kk' \cos \theta + \frac{2}{\lambda} (R + \varepsilon_{mkl} \Omega_m)(t - t_0) + \Delta t \}] d\hat{K}'. \end{aligned} \quad (4.16)$$

Now, $d\hat{K}'$ can be expressed in terms of k' and θ as

$$-2\pi k' d(\cos \theta) dk' \text{ [cf. Deissler}^{(4)}]$$

$$\text{Hence, } d\hat{K}' = -2\pi k' d(\cos \theta) dk'. \quad (4.16a)$$

Substituting (4.16a) in (4.16), we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \langle \phi_i \phi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda [k^2 + \frac{R}{\lambda}] \langle \phi_i \phi_i' \rangle (\hat{K}, \Delta t, t) = 2 \int_0^{\infty} 2\pi k_i [\langle \phi_i \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') \\ - \langle \phi_i \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0 k'^2 \left[\int_{-1}^1 \exp[-\lambda \{ (1 + p_M)(k^2 + k'^2)(t - t_0) + k^2 \Delta t \right. \\ \left. + 2p_M(t - t_0)kk' \cos \theta + \frac{2}{\lambda} (R + \varepsilon_{mkl} \Omega_m)(t - t_0) + \frac{\Delta t}{2} \}] d(\cos \theta) \right] dk' \end{aligned} \quad (4.17)$$

The quantity $[\langle \phi_i \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_i \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0$ depends on the initial condition of the turbulence. In order to make further calculation it is necessary to assume a relation which gives

$ik_i [\langle \phi_i \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_i \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0$ as a function of k and k' (cf.

Loeffler & Deissler⁽⁷⁾).

The relation assumed is

$$(2\pi)^2 ik_i [\langle \phi_i \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_i \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0 = \delta_0 [k^2 k'^4 - k^4 k'^2], \quad (4.18)$$

where δ_0 is a constant depending on the initial conditions.

Substituting equation (4.18) in equation (4.17) and multiplying both sides by k^2 , we get

$$\frac{\partial E}{\partial t} + 2\lambda k^2 E = G, \quad (4.19)$$

where $E = 2\pi k^2 \langle \phi_i \phi_j' \rangle$ is the magnetic energy spectrum function and G is the magnetic energy transfer term which is given below

$$G = -2\delta_0 \int_0^\infty [k^2 k'^4 - k^4 k'^2] \left[\int_{-1}^1 \exp[-\lambda \left\{ (1+p_M)(k^2 + k'^2)(t-t_0) + k^2 \Delta t \right. \right. \\ \left. \left. + 2p_M(t-t_0)kk' \cos \theta + \frac{2}{\lambda} (R + \varepsilon_{mkl} \Omega_m)(t-t_0 + \frac{\Delta t}{2}) \right\} d(\cos \theta)] dk' \right] \quad (4.20)$$

Integrating equation (4.20) with respect to $\cos \theta$ and k' , we obtain

$$G = -\frac{\delta_0 p_M \sqrt{\pi}}{4\lambda^{3/2} (t-t_0)^{3/2} (1+p_M)} \exp[-k^2 \lambda \left(\frac{1+2p_M}{1+p_M} \right) \left(t-t_0 + \frac{1+p_M}{1+2p_M} \Delta t \right)] \\ - 2(R + \varepsilon_{mkl} \Omega_m)(t-t_0 + \frac{\Delta t}{2}) \left[\frac{15k^4}{4p_M^2 (t-t_0)^2 \lambda^2} \left(\frac{p_M}{1+p_M} \right) + \left\{ 5 \left(\frac{p_M}{1+p_M} \right)^2 - \frac{3}{2} \right\} \frac{k^6}{p_M \lambda (t-t_0)} \right] \\ + \left\{ \left(\frac{p_M}{1+p_M} \right)^3 - \frac{p_M}{1+p_M} \right\} k^8 - \frac{\delta_0 p_M \sqrt{\pi}}{4\lambda^{3/2} (t-t_0 + \Delta t)^{3/2} (1+p_M)^{5/2}} \exp[-k^2 \lambda \left(\frac{1+2p_M}{1+p_M} \right) \\ \left(t-t_0 + \frac{p_M}{1+p_M} \Delta t \right)] - 2(R + \varepsilon_{mkl} \Omega_m)(t-t_0 + \frac{\Delta t}{2}) \left[\frac{15k^4}{4\lambda^2 p_M^2 (t-t_0)^2} \left(\frac{p_M}{1+p_M} \right) \right. \\ \left. + \left\{ 5 \left(\frac{p_M}{1+p_M} \right)^2 - \frac{3}{2} \right\} \frac{k^6}{p_M \lambda (t-t_0 + \Delta t)} + \left\{ \left(\frac{p_M}{1+p_M} \right)^3 - \frac{p_M}{1+p_M} \right\} k^8 \right] \quad (4.21)$$

The series of equation (4.18) contains only even power of k and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (4.21) for $\Delta t = 0$ over all wave number, we find that

$$\int_0^\infty G d\hat{k} = 0, \quad (4.22)$$

which indicates that the expression for G satisfies the condition of continuity and homogeneity. Physically it was it was to be expected as G is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

For obtaining the magnetic energy spectrum function G , equation (4.19) can be written in integral form as

$$E = \exp[-2\lambda k^2 (t-t_0 + \frac{\Delta t}{2})] \int G \cdot \exp[2\lambda (k^2 + \frac{R + \varepsilon_{mkl} \Omega_m}{\lambda} (t-t_0 + \frac{\Delta t}{2}))] dt \\ + J(k) \exp[-2\lambda (k^2 + \frac{R + \varepsilon_{mkl} \Omega_m}{\lambda} (t-t_0 + \frac{\Delta t}{2}))], \quad (4.23)$$

where $J(k) = \frac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained by

Corrsin⁽²⁾.

Substituting the values of G as given by equation (4.21) into equation (4.23), gives the equation

$$E = \frac{N_0 k^2}{\pi} \exp\left[-2\lambda\left(k^2 + \frac{R + \varepsilon_{mkl}\Omega_m}{\lambda}\right)\left(t - t_0 + \frac{\Delta t}{2}\right)\right] + \frac{\delta_0 p_M \sqrt{\pi}}{4\lambda^{3/2}(1+p_M)^{3/2}} \exp\left[-k^2 \lambda \left(\frac{1+2p_M}{1+p_M}\right)\right. \\ \left.\left(t - t_0 + \frac{1+p_M}{1+2p_M} \Delta t\right) - 2(R + \varepsilon_{mkl}\Omega_m)\left(t - t_0 + \frac{\Delta t}{2}\right)\right] + \left[\frac{3k^4}{2p_M(t-t_0)^{3/2}\lambda^2} + k^9 F(\omega)\right. \\ \left. + \frac{\delta_0 \sqrt{\pi} p_M}{4\lambda^{3/2}(1+p_M)}\right] \cdot \exp\left[-k^2 \lambda \left(\frac{1+2p_M}{1+p_M}\right)\left(t - t_0 + \frac{p_M}{1+p_M} \Delta t\right) - 2(R + \varepsilon_{mkl}\Omega_m)\left(t - t_0 + \frac{\Delta t}{2}\right)\right] \\ \frac{3k^4}{2\lambda^2 p_M(t-t_0 + \Delta t)^{5/2}} + \frac{(7p_M - 6)k^6}{3\lambda(1+p_M)(t-t_0 + \Delta t)^{3/2}} + \frac{4(3p_M^2 - 2p_M + 3)}{3(1+p_M)^2(t-t_0 + \Delta t)^{3/2}} \\ \frac{8\lambda^{1/2}(3p_M^2 - 2p_M + 3)}{(1+p_M)^{5/2}} k^9 F(\omega)], \quad (4.24)$$

where $F(\omega) = e^{-\omega^2} \int_0^\infty e^{-x^2} dx$, $\omega = k \sqrt{\frac{\lambda(t-t_0)}{1+p_M}}$ or $k \sqrt{\frac{\lambda(t-t_0 + \Delta t)}{1+p_M}}$

By setting $\hat{r} = 0$, $j = i$, $d\hat{K} = -2\pi k^2 d(\cos\theta) d\hat{k}$ and $E = 2\pi k^2 \langle \phi_i \phi_j' \rangle$ in equation (3.10), we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h_i' \rangle}{2} = \int_0^\infty E dk \quad (4.25)$$

Substituting equation (4.24) into equation (4.25) and after integration, we obtain

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\lambda^{3/2} \sqrt{2\pi} \left(T + \frac{\Delta T}{2}\right)^{3/2}} \exp\left[-2(R + \varepsilon_{mkl}\Omega_m)\left(T + \frac{\Delta T}{2}\right)\right] + \frac{\pi\delta_0}{4\lambda^2(1+p_M)(1+2p_M)^{3/2}} \\ + \frac{35p_M(3p_M^2 - 2p_M + 3)}{8(1+2p_M)T^{1/2}\left(T + \frac{1+p_M}{1+2p_M}\Delta T\right)^{3/2}} + \frac{35p_M(3p_M^2 - 2p_M + 3)}{8(1+2p_M)(T + \Delta T)^{1/2}\left(T + \frac{p_M}{1+2p_M}\Delta T\right)^{3/2}} \\ \cdot \exp\left[-2(R + \varepsilon_{mkl}\Omega_m)\left(T + \frac{\Delta T}{2}\right)\right] \cdot \left[\frac{9}{16T^{3/2}\left(T + \frac{1+p_M}{1+2p_M}\Delta T\right)^{3/2}} + \frac{9}{16(T + \Delta T)^{3/2}\left(T + \frac{p_M}{1+2p_M}\Delta T\right)^{3/2}} \right] \\ + \frac{8p_M(3p_M^2 - 2p_M + 3)(1+2p_M)^{3/2}}{3.2^{23/2}(1+p_M)^{11/2}} \sum \frac{1.3.5.....(2n+9)}{n(2n+1)2^{2n}(1+p_M)^n}$$

$$\left\{ \frac{T^{(2n+1)/2}}{(T + \Delta T)^{(2n+1)/2}} + \frac{(T + \Delta T)^{(2n+1)/2}}{(T + \Delta T/2)^{(2n+1)/2}} \right\}, \quad (4.26)$$

where $T = t - t_0$.

Thus the decay law for magnetic energy fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system may be written as

$$\begin{aligned} \frac{\langle h_i h_i' \rangle}{2} = & \exp[-2(R + \varepsilon_{mM} \Omega_m) T_m] \left[\frac{N_0}{8\lambda^{3/2} \sqrt{2\pi} T_m^{3/2}} + \frac{\pi \delta_0}{4\lambda^6 (1 + p_M)(1 + 2p_M)^{5/2}} \right. \\ & \left. \left\{ \frac{9}{16(T - \Delta T/2)^{5/2} \left(T_m + \frac{\Delta T}{1 + 2p_M} \right)^{5/2}} + \frac{9}{16(T_m + \Delta T/2)^{5/2} \left(T_m - \frac{\Delta T}{2(1 + 2p_M)} \right)^{5/2}} \right. \right. \\ & + \frac{5p_M(7p_M - 6)}{16(1 + 2p_M)(T_m - \Delta T/2)^{3/2} \left(T_m + \frac{\Delta T}{2(1 + 2p_M)} \right)^{7/2}} \\ & \left. \left. + \frac{5p_M(7p_M - 6)}{16(1 + 2p_M)(T_m + \Delta T/2)^{3/2} \left(T_m - \frac{\Delta T}{2(1 + 2p_M)} \right)^{7/2}} + \dots \right\} \right], \quad (4.27) \end{aligned}$$

where $T_m = T + \Delta T/2$, which is analogous to the equation (43) of kumar and patel [5&6].

For non- rotating system $\Omega_m = 0$, we can easily find out that

$$\begin{aligned} \frac{\langle h_i h_i' \rangle}{2} = & \exp[-2RT_m] \left[\frac{N_0}{8\lambda^{3/2} \sqrt{2\pi} T_m^{3/2}} + \frac{\pi \delta_0}{4\lambda^6 (1 + p_M)(1 + 2p_M)^{5/2}} \right. \\ & \left\{ \frac{9}{16(T_m - \Delta T/2)^{5/2} \left(T_m + \frac{\Delta T}{1 + 2p_M} \right)^{5/2}} + \frac{9}{16(T_m + \Delta T/2)^{5/2} \left(T_m - \frac{\Delta T}{2(1 + 2p_M)} \right)^{5/2}} \right. \\ & + \frac{5p_M(7p_M - 6)}{16(1 + 2p_M)(T_m - \Delta T/2)^{3/2} \left(T_m + \frac{\Delta T}{2(1 + 2p_M)} \right)^{7/2}} \\ & \left. \left. + \frac{5p_M(7p_M - 6)}{16(1 + 2p_M)(T_m + \Delta T/2)^{3/2} \left(T_m - \frac{\Delta T}{2(1 + 2p_M)} \right)^{7/2}} + \dots \right\} \right] \end{aligned}$$

$$+ \frac{5p_M(7p_M - 6)}{16(1 + 2p_M)(T_m + \Delta T/2)^{3/2} \left(T_m - \frac{\Delta T}{2(1 + 2p_M)} \right)^{7/2} + \dots \dots \dots \}}] \quad (4.28)$$

Which is obtained by Sarker and Islam [10].

If we put $\Delta T = 0, R = 0$, equation (5.1) becomes

$$\begin{aligned} \frac{\langle h^2 \rangle}{2} &= \frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}} T^{-3/2} + \frac{\pi\delta_0}{2\lambda^6(1+p_M)(1+2p_M)^{3/2}} T^{-5} \left\{ \frac{9}{16} + \frac{5p_M(7p_M-6)}{16(1+2p_M)} + \dots \right\} \\ &= XT^{-3/2} + YT^{-5} \\ &= X(t-t_0)^{-3/2} + Y(t-t_0)^{-5} \end{aligned} \quad (4.29)$$

where

$$X = \frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}} \text{ and } Y = \frac{\pi\delta_0}{2\lambda^6(1+p_M)(1+2p_M)^{3/2}} \left\{ \frac{9}{16} + \frac{5p_M(7p_M-6)}{16(1+2p_M)} + \dots \right\},$$

which is obtained by Sarker and kishore [9].

5. Concluding Remarks

This study shows that the turbulent energy in the magnetic field decays more rapidly due to the effect of rotation than the energy for non-rotating fluid. From the assumption we conclude that the higher order correlation terms may be neglected in comparison with lower order correlation terms. By neglecting the quadruple correlations terms in three- point, three-time correlation equation the result (4.27) applicable to the first order reactant in MHD turbulence in a rotating system before the final period of decay were obtained. If higher order correlation equations are considered in the analysis, i.e. if the quadruple correlations were not neglected, it appears that more terms of higher power of $(t-t_0)$ would be added to the equation (4.27). For large times the last term in the equation becomes negligible, leaving the $-3/2$ power decay law for the final period.

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