ISSN 0973-8975

J.Mech.Cont.& Math. Sci., Vol.-4, No.-2, January (2010) Pages 523-529

STEADY FLOW OF MICROPOLAR FLUID UNDER UNIFORM SUCTION

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Abstract

This paper is concerned with the steady flow of a micropolar fluid past an infinite flat plate subjected to uniform suction.

Keyword and phrases : *micro polarfluid, steady flow, flate plate, uniform suction .*

সংক্ষিপ্তসার

এই পত্রটি সম - শোষণ শর্ত সাপেক্ষে একটি অসীম চেপটা প্লেটকে অপরিবর্ত্তী প্রবাহসহ মাইক্রোপোলার প্রবাহী পদার্থের অতিক্রমন সম্পর্কিত একটি বিষয়।

1. Introduction

Theory of micropolar fluids was introduced by Eringen[1] in order to explain the properties of polymeric fluids, fluids with certain additives and animal bloods, milk etc. for which the classical Navier-Stokes' theory seems to be inadequate. In addition to their usual motion, fluid particles of these type of fluids possess the ability to rotate about the centroid of the volume element in an average sense described by the skew-symmetric gyration tensor $\vec{\sigma}$.

Hoyl and Fabula [2] investigated the flow of fluids containing extremely small amount of polymeric additives and found that the skin-friction near a rigid body in such fluids are lower than the same without the additive. Several basic flows such as plane shear flow, flow between two rotating cylinders and surface waves in a micropolar liquid was studied by Willson [3]. He [4] further studied the stability of a layer of micropolar liquid flowing down an inclined plane. Again, boundary layer flows in micropolar liquids was discussed by the same author [5] with special reference to flow near a stagnation point. Sengupta and Ghosh [6] investigated asymptotic suction problem in unsteady flow of micropolar liquids. Gupta and Gupta [7] discussed the problem of steady flow of micropolar liquids. In the present paper an attempt has been made to study the problem of steady flow of micripolar fluid past an infinite flat plate subjected to uniform suction. In solving the problem the physical quantities have been made non-dimensional. Solutions for velocity field and micro-rotation field are found out both analytically and numerically. Numerical results are presented by means of graphs.

2. Field Equations of a Micropolar Fluid

In the absence of external body forces and body couples the equations for steady motion of a micropolar fluid can be written as

$$(\lambda+2\mu+\chi)\nabla\nabla\cdot\vec{V}-(\mu+\chi)\nabla\times\nabla\times\vec{V}+\chi(\nabla\times\vec{\sigma})-\nabla p=\rho\left[\frac{1}{2}\nabla(V^2)-\vec{V}\times(\nabla\times\vec{V})\right] \quad (1)$$

$$(\alpha+\beta+\gamma)\nabla\nabla.\vec{\sigma}-\gamma\nabla\times\nabla\times\vec{\sigma}+\chi(\nabla\times\vec{V})-2\chi\vec{\sigma}=\rho j(\vec{V}.\nabla)\vec{\sigma} \quad (2)$$

where $\vec{V},\vec{\sigma}$ respectively are velocity and micro-rotation vectors and p is the scalar pressure. The constant j is the gyration parameter, ρ is the density while α, β, γ, μ, χ, λ are material constants.

Assuming that the fluid is incompressible and homogeneous with constant

$$\nabla\cdot\vec{V}=0 \quad (3)$$

Considering the velocity field as

$$\vec{V}=[u(x,y),v(x,y),0], \quad \vec{\sigma}=[0,0,\sigma(x,y)]$$

and introducing from [5]

$$\mu+\chi=\gamma\rho, \qquad p=\rho P, \qquad \chi=k\rho, \qquad \gamma=G\chi$$

the equations of motion (1) and (2) and the equation of continuity (3) become

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+k\frac{\partial \sigma}{\partial y} \tag{4}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\gamma\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-k\frac{\partial \sigma}{\partial x} \tag{5}$$

$$j\left(u\frac{\partial \sigma}{\partial x}+v\frac{\partial \sigma}{\partial y}\right)=k\left[G\left(\frac{\partial^2 \sigma}{\partial x^2}+\frac{\partial^2 \sigma}{\partial y^2}\right)-2\sigma+\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right] \tag{6}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

3. Formulation of the Problem

Let us consider that a micropolar fluid is flowing, with a uniform velocity u_0 parallel to x-axis, along an infinite flat plate at zero incidence. We assume that the plate coincides with $y=0$ where y-axis is taken perpendicular to the plate.

For a fluid flowing with uniform velocity u_0 parallel to x-axis we have $u=u(y)$ and therefore equation of continuity (7) gives v = constant = $-V_o$(say) where $V_o(>0)$ is the uniform suction velocity.

Then equations (4) and (6) give

$$-V_0\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}+k\frac{\partial \sigma}{\partial y} \tag{8}$$

$$-jV_0\frac{\partial \sigma}{\partial y}=k\left(G\frac{\partial^2 \sigma}{\partial y^2}-2\sigma-\frac{\partial u}{\partial y}\right) \tag{9}$$

whereas equation (5) shows that $P=$ constant everywhere in the flow.

Introducing non-dimensional variables

$$\eta=\frac{V_0 y}{\gamma},\qquad U(\eta)=\frac{u(y)}{u_0},\qquad \Omega(\eta)=\frac{\sigma(y)\gamma}{u_0^{\,2}}$$

we get from (8) and (9)

$$\frac{\partial^2 U}{\partial \eta^2}+\frac{\partial U}{\partial \eta}+s\frac{\partial \Omega}{\partial \eta}=0 \tag{10}$$

and

$$R_1\frac{\partial\Omega}{\partial\eta}+R_2\frac{\partial^2\Omega}{\partial\eta^2}-2sR_3\Omega-s\frac{\partial U}{\partial\eta}=0 \tag{11}$$

where

$$s=\frac{ku_0}{\gamma V_0}\,,\qquad R_1=\frac{u_0^{\ 2}}{\gamma^2}\cdot j\,,\qquad R_2=\frac{kGu_0^{\ 2}}{\gamma}\qquad \text{and}\qquad R_3=\frac{u_0}{V_0}.$$

The boundary conditions are

$U=0,\ \Omega=0$ when $\eta=0$ (12)

$U=1,\ \Omega=0$ when $\eta\to\propto$ (13)

4. Solution of the Problem

Integrating (10) with respect to η and using the boundary condition (13) we get

$$\Omega=-\frac{1}{s}\frac{\partial U}{\partial\eta}-\frac{U}{s}+\frac{1}{s} \tag{14}$$

With the help of (14) equation (11) becomes

$$\frac{R_2}{s}\frac{\partial^3U}{\partial\eta^3}+\frac{1}{s}(R_1+R_2)\frac{\partial^2U}{\partial\eta^2}+\left(\frac{R_1}{s}-2R_3+s\right)\frac{\partial U}{\partial\eta}-2R_3U+2R_3=0 \tag{15}$$

Assuming a solution $U(\eta)$ proportional to $e^{m\eta}$ of the homogeneous part of (15) we observe that

m has to satisfy the cubic equation

$$\frac{R_2}{s}m^3+\frac{1}{s}(R_1+R_2)m^2+\left(\frac{R_1}{s}-2R_3+s\right)m-2R_3=0 \tag{16}$$

By Decartes' rule of sign it follows that (16) has atmost one positive root, since R_1, R_2, R_3, s

are all real and positive (considering $\frac{R_1}{s}-2R_3+s>0$). Again, since all the coefficients of (16) are real, it has at least one real root. Now if m_1, m_2, m_3 be the roots of (16), then $m_1\, m_2\, m_3=\frac{2R_3s}{R_2}>0$. Therefore, taking $m_3>0$ there arises two possibilities : either (i) $m_1<0$, $m_2<0$ or (ii) m_1, m_2 are complex.

But the second possibility leads to the conclusion that the velocity $U(\eta)$ is periodic which contradicts the assumption of this problem. Therefore we reject the second possibility. On the other hand the first possibility is reasonable from the physical point of view and in this case (i.e., when $m_1 < 0$, $m_2 < 0$, $m_3 > 0$) the solution of (15) becomes

$$U(\eta) = C_1 e^{m_1\eta} + C_2 e^{m_2\eta} \tag{17}$$

Here we have rejected the root m_3 since it leads to infinite value of velocity when $\eta \to \infty$.

Now, using the boundary condition (12) (i.e., $U(0) = 0$) we get

$C_2 = - C_1 - 1$.

Then equation (17) reduces to

$$U(\eta) = C_1(e^{m_1\eta} - e^{m_2\eta}) - e^{m_2\eta} + 1 \tag{18}$$

Substituting (18) in (14) we obtain

$$\Omega(\eta) = \frac{1}{s}\left[(1+m_2)e^{m_2\eta} - C_1(m_1 e^{m_1\eta} + e^{m_1\eta} - m_2 e^{m_2\eta} - e^{m_2\eta})\right] \tag{19}$$

Now, using the boundary condition (12) (i.e., $\Omega(0) = 0$) we calculate C_1 from (19) as

$$C_1 = \frac{1+m_2}{m_1 - m_2} .$$

Therefore from (18) and (19) the velocity $U(\eta)$ and the micro-rotation $\Omega(\eta)$ are respectively obtained as

$$U(\eta) = \frac{1+m_2}{m_1 - m_2}\left(e^{m_1\eta} + e^{m_2\eta}\right) - e^{m_2\eta} + 1 \tag{20}$$

and

$$\Omega(\eta) = \frac{(1+m_1)(1+m_2)}{s(m_1 - m_2)}\left(e^{m_2\eta} - e^{m_1\eta}\right) \tag{21}$$

The solutions (20) and (21) are valid when all the roots of (16) are real, which is possible only when $\Delta < 0$ where Δ is the discriminent of the cubic equation (16).

We now calculate $U(\eta)$ and $\Omega(\eta)$ numerically for different values of η when the parameters R_1, R_2, R_3 and s satisfy the condition $\Delta < 0$.

5. Numerical Results

The numerical results of U and Ω are exhibited in the following table :

$s = 2.00$	$m_1 = -1.55$	$m_2 = -2.55$
η	U	Ω
0.4	0.36451	-0.7559×10^{-1}
0.8	0.62297	-0.6792×10^{-1}
1.2	0.78449	-0.4637×10^{-1}
1.6	0.87950	-0.2849×10^{-1}
2.0	0.93353	-0.1660×10^{-1}
2.4	0.96365	-0.0939×10^{-1}
2.8	0.98023	-0.0522×10^{-1}
3.2	0.98929	-0.0287×10^{-1}
3.6	0.99421	-0.0154×10^{-1}
4.0	0.99687	-0.0085×10^{-1}
4.4	0.99831	-0.0046×10^{-1}
4.8	0.99909	-0.0025×10^{-1}
5.2	0.99951	-0.0013×10^{-1}

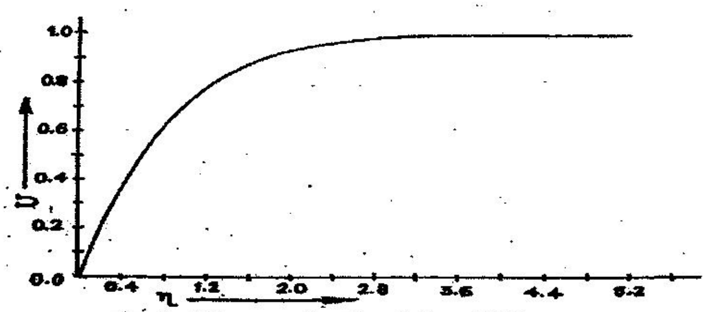

Fig.1. Graph of equation (20).

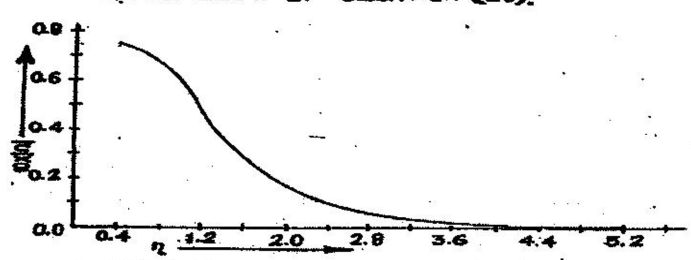

Fig.2. Graph of equation (21).

6. Discussion

It is clear from the table and the graphs that $U(\eta)$ increases as η increases and tends to unity and the micro-rotation vector $\Omega(\eta)$ are negative everywhere which implies that the sense of rotation of $\Omega(\eta)$ is opposite to that of the vorticity of the basic flow. It is also found from the figure 2 that $|\Omega(\eta)|$ decreases as η increases. Therefore we may conclude that the micro-structure of the fluid exerts a significant influence on the steady flow.

Acknowledgement

We acknowledge our deep gratitude to Prof.(Dr.) P. R. Sengupta, Ph.D., D.Sc., F.A.Sc.T., F.I.M.A.(U.K.), F.N.A.Sc. for his help in preparing this paper.

References

1) Eringen, A. C. (1964) – Simple Microfluids; Int. Jour. Engng. Sci., 2, 205.
2) Hoyl, J. W. and Fabula, A. C. (1964) – The effect of additives on fluid
3) Willson, A. J. (1969) - Basic flows of a micropolar liquid; Appl. Sci. Res., 20, 338.
4) Willson, A. J. (1968) – The Flow of a Micropolar liquid layer down an inclined plane; Proc. Camb. Phil. Soc., 64, 513.
5) Willson, A. J. (1970) – Boundary layer in Micropolar liquid; Proc. Camb. Phil. Soc., 67, 469.
6) Sengupta, P. R. and Ghosh, P. C. (1982) – Asymptotic suction problem in the unsteady flow of Micropolar liquids; Journal of Technology, XXVII, 1, 21.
7) Gupta, P. S. and Gupta, A. S. (1972) – Steady flow of Micropolar liquids; Acta Mechanica, 15, 141.