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MHD FREE CONVECTION FLOW OF FLUID FROM A VERTICAL FLAT PLATE

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Abstract

A two-dimensional natural convection flow of a viscous incompressible and electrically conducting fluid past a vertical impermeable flat plate is considered in presence of a uniform transverse magnetic field. The governing equations are reduced to non-similar boundary layer equations by introducing coordinate transformations appropriate to the cases (i) near the leading edge (ii) in the region far away from the leading edge and (iii) for the entire regime from leading edge to down stream. The governing equations for the flow in the up stream regime are investigated by perturbation method for smaller values of ξ, the stream wise distributed magnetic field parameter. The equations governing the flow for large ξ and for all ξ, have been investigated by employing the implicit finite difference method with Keller box scheme. The effect of Prandtl number Pr and the magnetic field parameter ξ, on the skin friction as well as on the rate of heat transfer for the fluid of low Prandtl number will be shown in tabular form. The effect of Pr and different level of velocity, in the boundary layer region, will also be shown graphically.

Keyword and phrases : *viscous incompressible fluid, convection flows, skin friction, heat transfer.*

সংক্ষিপ্তসার

একটি সম-তির্যক চৌম্বক ক্ষেত্রের উপস্থিতে একটি উলম্ব অভেদ্য চেপ্টা প্লেটের ক্ষেত্রে সান্দ্র অসংনম্য এবং তড়িৎ পরিবাহী পদার্থের অতিক্রমনের একটি দ্বি-মাত্রিক স্বাভাবিক পরিচালন প্রবাহকে বিবেচনা করা হয়েছে। নিয়ন্ত্রক সমীকরণ-গুলিকে অসদৃশ প্রান্তিক স্তরের সমীকরণে সমানীত করা হয়েছে স্থানাঙ্ক রূপান্তরণকে যথোপযুক্ত ক্ষেত্রে প্রয়োগ করে — i) মুখ্য কিনারার কাছে ii) মুখ্য কিনারা থেকে কম দূরবর্তী অঞ্চলে ii) মুখ্য কিনারা থেকে ভাটির অভিমুখে সমগ্র অঞ্চলের জন্য। স্রোতের টান অনুযায়ী বন্টিত চৌম্বক ক্ষেত্রের প্রাচল, ξ এর মান ক্ষুদ্র ধরে বিচলন পদ্ধতির সাহায্যে জোয়ার প্রবাহিত অঞ্চলের জন্য নিয়ন্ত্রক সমীকরণগুলিকে অনুসন্ধান করা হয়েছে। কীলার বক্স (Keller bo x) প্রকল্পসহ অপ্রতক্ষ সসীম - অন্তর পদ্ধতি প্রয়োগ করে ξ এর বৃহত্তম মান এবং ξ এর সকল মানের জন্য প্রবাহের নিয়ন্ত্রক সমীকরণগুলিকে অনুসন্ধান করা হয়েছে। স্কিন ঘর্ষণে (Skin friction) প্রাণভল্ সংখ্যা P_r এবং চৌম্বক ক্ষেত্রের প্রাচল এবং নিম্ন প্রাণভল্ সংখ্যার প্রবাহী পদার্থের জন্য তাপ-স্থানান্তরণের হারকে তালিকা আকারে দেখানো হয়েছে। P_r এর প্রভাব এবং প্রান্তিক স্তরের অঞ্চলে গতিবেগের বিভিন্ন তলকে লেখচিত্রের সাহায্যে প্রকাশ করা হয়েছে।

several authors, such as Sparrow and Cess (19671), Reley (1964) and Kuiken(1970). Simultaneous occurrence of buoyancy and magnetic field forces in the flow of an electrically conducting fluid up a hot vertical flat plate in the presence of a strong cross magnetic field was studied by Sing and Cowling (1963) who had shown that regardless of strength of applied magnetic field there will always be a region in the neighborhood of the leading edge of the plate where electromagnetic forces are unimportant. Creamer and Pai (1974) presented a similarity solution for the above problem with uniform heat flux by formulating it in terms of both a regular and inverse series expansions of characterizing coordinate that provided a link between the similarity states closed to and far from the leading edge. Hossain and Ahmed (1984) studied the combined effect of the free and forced convection with uniform heat flux in the presence of strong magnetic field. Hossain et al (1996) also investigated the MHD free convection flow along a vertical porous flat plate with a power law surface temperature in the presence of a variable transverse magnetic field employing two different methods namely (i) perturbation methods for small and large values of the scaled stream-wise transpiration velocity variable ξ_s (=V_0 $\sqrt{(2x/\nu U_\infty}$, where V_0 is the transpiration velocity) and (ii) the finite difference together with the Keller box method (1978). Wilks (1976) recognized a parameter ξ, defined by $\xi = (\sigma H_0^2/\rho_\infty)^2 x/g\beta(T_0-T_\infty)$ to investigate the MHD free convection flow about a semi-infinite vertical plate in a strong cross magnetic field. The work of that follows reformulates the problem in terms of coordinates expansions with respect to a non-dimensional characteristic length which is fundamental to the problem in its reflection to the relative magnetudes of buoyancy and magnetic forces at varying locations along the plate. A step by step numerical solution has been obtained to supplement the series solutions for small and large ξ.

In the above analysis, the solutions for the problem, Wilks (1976) used only series solutions method. But we use three different methods namely (i) perturbation method for small ξ, (ii) asymptotic solution for large ξ and (iii) finite difference method for all ξ. We have found the results from the three methods and compared our results that with the Wilks and heve shown that there is an excellent agreement with them..

2. The governing equations:

The basic equations steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscosity depending on temperature and also thermal conductivity depending on temperature past a semi-infinite vertical impermeable flat plate in the presence

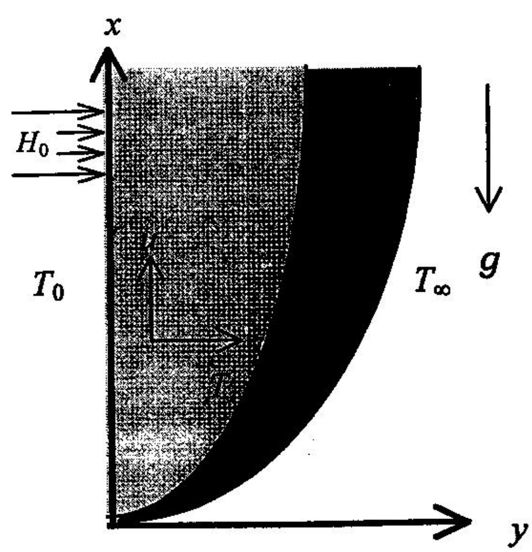

Figure 1: The flow configuration and coordinates system

of a uniformly distributed transverse magnetic field of strength H_0 are as given below

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma H_0^{\,2}u}{\rho_\infty}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho_\infty c_p}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

with the boundary conditions

$$\begin{aligned} &u=v=0, \quad T=T_0 \quad \text{at } y=0 \\ &u\to 0, \quad T\to T_\infty \quad \text{as } y\to\infty \end{aligned}, \tag{4}$$

Here u, v is the velocity components associated with the direction of increase of coordinates x and y measured along and normal to the vertical plate. T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, β is the coefficient of thermal expansion, κ is the thermal conductivity, ρ_∞ is the density of the fluid, c_p is the specific heat at constant pressure and T_∞ is the temperature of the ambient fluid and v the kinematics viscosity of the fluid.

3. Solution for entire regime for all ξ:

Now we introduce the following transformations to the equation (2) and (3)

$$\begin{aligned} &\psi=c\nu x^{3/4}(1+\xi)^{-1/4}f(\xi,\eta), \quad \eta=\frac{cy(1+\xi)^{-1/4}}{x^{1/4}} \\ &T-T_\infty=(T_0-T_\infty)\theta(\xi,\eta), \quad c=\left[\frac{g\beta(T_0-T_\infty)}{\nu^2}\right]^{1/4}, \xi=\frac{(\sigma H_0^2/\rho_\infty)^2 x}{g\beta(T_0-T_\infty)} \end{aligned}, \tag{5}$$

and we get the following equations

$$f''' + \frac{3+2\xi}{4(1+\xi)} ff'' - \frac{1}{2(1+\xi)} f'^2 + (1+\xi)\theta - \xi^{1/2}(1+\xi)^{1/2} f' = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right), \tag{6}$$

$$\theta'' + \Pr\frac{3+2\xi}{4(1+\xi)} f\theta' = \Pr\xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right), \tag{7}$$

with the boundary conditions

$$\begin{aligned} &f = f' = 0; \quad \theta = 1 \quad \text{at}\, \eta = 0 \\ &f' \to 0; \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty, \\ &f = \theta = 0; \quad \text{at} \quad \xi = 0, \eta \rangle 0 \end{aligned} \tag{8}$$

Here the coefficient of skin-friction, τ, and the coefficient of the rate of heat transfer, Q are defined as follows.

$$\tau = \frac{H_0}{g\beta(T_0 - T_\infty)}\left(\frac{\sigma\upsilon}{\rho_\infty}\right)^{1/2}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \xi^{1/4}(1+\xi)^{-3/4} f''(\xi,0) \tag{9}$$

$$Q = \frac{1}{(T_0 - T_\infty)}\left(\frac{\rho_\infty\upsilon}{\sigma H_0^2}\right)^{1/2}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \xi^{-1/4}(1+\xi)^{-1/4}\theta'(\xi,0) \tag{10}$$

4. Solution near the leading edge for small ξ:

For small ξ from the equation (6) and (7) we can approximate the following equations (11) and (12) respectively

$$f''' + \frac{3}{4} ff'' - \frac{1}{2} f'^2 + \theta - \xi^{1/2} f' = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \tag{11}$$

$$\theta'' + \frac{3}{4}\Pr f\theta' = \Pr\xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \tag{12}$$

where the boundary conditions (8) remains the same

To get the solutions of the above equations (11) and (12) we use the 2nd and 3rd order series solution and the finite difference method.

Series solution methods

For series solution we consider the following series

$$f(\xi,\eta) = f_0 + \xi^{1/2} f_1 + \xi f_2 + \cdots\cdots \tag{13}$$

$$\theta(\xi,\eta) = \theta_0 + \xi^{1/2}\theta_1 + \xi\theta_2 + \cdots\cdots \tag{14}$$

where f_0, θ_0 are the well known free convection similarity solutions for flow around a constant temperature semi-infinite vertical plate and where f_1, θ_1 are effectively the first order correction and f_2, θ_2 are effectively second order correction to the flow due to the presence of magnetic field.

Using (13) and (14) in (11) and (12) we get the following equations

For the coefficient of ξ^0

$$f''' \frac{3}{4} f_0 f_0'' - \frac{1}{2} f_0'^2 + \theta_0 = 0 \tag{15}$$

$$\theta_0'' + \frac{3}{4} \Pr f_0 \theta_0' = 0 \tag{16}$$

with the boundary conditions

$$\begin{aligned} & f_0 = f_0' = 0; \quad \theta_0 = 1 \quad \text{at} \, \eta = 0 \\ & f_0' \to 0; \quad \theta_0 \to 0 \quad \text{as} \, \eta \to \infty \\ & f_0 = \theta_0 = 0; \quad \text{at} \quad \xi = 0, \eta \rangle 0 \end{aligned} \tag{17}$$

For the coefficients of $\xi^{1/2}$

$$f_1''' + \frac{3}{4} f_0 f_1'' + \frac{5}{4} f_0'' f_1 - \frac{3}{2} f_0' f_1' + \theta_1 - f_0' = 0 \tag{18}$$

$$\theta_1'' + \Pr\left\{\frac{3}{4} f_0 \theta_1' + \frac{5}{4} f_1 \theta_0' - \frac{1}{2} f_0' \theta_1\right\} = 0 \tag{19}$$

with the boundary conditions

$$\begin{aligned}
&f_1 = f_1' = 0 \quad ;\theta_1 = 1 \quad \text{at}\,\eta = 0 \\
&f_1' \to 0 \quad ;\theta_1 \to 0 \quad \text{as}\,\eta \to \infty \\
&f_1 = \theta_1 = 0 \quad ;\text{at} \quad \xi = 0, \eta \rangle 0
\end{aligned} \tag{20}$$

For the coefficients of ξ

$$f_2''' + \frac{3}{4} f_0 f_2'' + \frac{5}{4} f_1 f_1'' + \frac{7}{4} . f_0'' f_2 + \theta_2 - f_1' - f_1'^2 - 2 . f_0' f_2' = 0 \tag{21}$$

$$\theta_2'' + \Pr(\frac{3}{4} f_0 \theta_2' + \frac{5}{4} f_1 \theta_1' - f_0' \theta_2 + \frac{7}{4} f_2 \theta_0' - \frac{1}{2} f_1' \theta_1) = 0 \tag{22}$$

with the boundary conditions

$$\begin{aligned}
&f_2 = f_2' = 0; \quad \theta_2 = 1 \quad \text{at}\,\eta = 0 \\
&f_2' \to 0; \quad \theta_2 \to 0 \quad \text{as}\,\eta \to \infty \\
&f_2 = \theta_2 = 0; \quad \text{at} \quad \xi = 0, \eta \rangle 0
\end{aligned} \tag{23}$$

Here we also calculate the skin friction and the rate of heat transfer.

5. For large ξ solution

For large ξ from the equations (6) and (7) we get

$$f''' + \frac{1}{2} f f'' + \xi(\theta - f') = \xi\left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \tag{24}$$

$$\theta'' + \frac{1}{2} \Pr f \theta' = \Pr \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \tag{25}$$

With the boundary conditions

$$\begin{aligned}
&f = f' = 0; \quad \theta = 1 \quad \text{at}\,\eta = 0 \\
&f' \to 0; \quad \theta \to 0 \quad \text{as}\,\eta \to \infty \\
&f = \theta = 0; \quad \text{at} \quad \xi = 0, \eta \rangle 0
\end{aligned} \tag{26}$$

Here skin friction and the rate of heat transfer are also calculated for large values of ξ.

6. Result and Discussion

In this section we discuss the results obtained from the solution of the equations governing the MHD free convection flow of a viscous incompressible and electrically conducting fluid with uniform viscosity and uniform thermal conductivity, in the presence of uniform transverse magnetic field along an impermeable vertical flat plate. The solutions of the governing non-similar equations are obtained by series solution method of order two and order three for small ξ and the implicit finite difference method or simply Keller box method which is well-documented by Cebeci and Bradshaw (1984) for wide range of ξ. Here we consider the low Prandl number (Pr) liquid metals. We have pursued solutions for Pr equals 0.05 for lithium, 0.02 foe mercury.

Wilks (1976) solved the same problem for the case of small and large ξ for Pr=0.72 and the result for the skin friction and the rate of heat transfer that are obtained by Wilks are entered in the fifth column of tables 1 and 2 respectively. The result that we obtained for skin friction and the rare of heat transfer for Pr = 0.72 in the method of series solution for two and three terms and the implicit finite difference are entered in the first, second and in the third column of the tables 1 and 2 respectively. The values of skin friction and he rate of heat transfer by the method of series solution for two and three terms and by the finite difference method that are entered in the sixth, seventh and eighth column for Pr= 0.05. Comparing these results with the result of Wilks for Pr=0.72, we observed an excellent agreement between these two results. Also we have the results among the three methods for Pr = 0.72 and 0.05. From the result of series solution of two and three terms it can be noted that the result for three terms is more accurate than with two terms. The result with the symbol 'a' in the column three and seven represent the solution for large ξ. From the tables we see that increasing values of the magnetic field parameter ξ the skin friction increase and the rate of heat transfer decrease while the increasing values of Pr both the skin friction and the rate of heat transfer decrease.

The velocity profiles for f_0', f_1' and f_2' are shown in the Figure 2 and the temperature profiles for θ_0, θ_1 and θ_2 are shown in the Figure 3 for Pr = 0.05 and 0.72. The dotted lines represent the curves for f_0' and θ_0 dash-dotted lines represent for f_1' and θ_1 and solid lines represent the curves for f_2' and θ_2 in the Figure 2 and Figure 3 respectively. In thè Figure 2 we see that the curves of f_0', f_2', f_4' etc. are in the upper half side of x-axis and f_1', f_3' etc. are in the lower side of x-axis whose effects are significant near the surface of the plate. Also for increasing values of Pr f_0', f_2', f_4' etc. decrease while f_1', f_3' increase. In the Figure 3 it can be observed that the values of $\theta_0, \theta_1, \theta_2$ etc. are in descending order for different values of Pr. The increasing values of Pr θ_0, θ_2 etc decrease and θ_1, θ_3 etc. increase. The velocity profile and temperature profiles that we obtained is similar to that of Wilks. The effects of different Pr are significant near the surface of the plate.

7. Conclusion

In this paper, the problem of magnetohydrodynamic free convection flow along a vertical flat plate is investigated. The local non-similarity equations governing the flow for the case of uniform viscosity and thermal conductivity are developed. To establish the accuracy of the solutions of the present problem three methods, namely (i) the extended series solution method (ii) the finite difference method together with Keller box elimination technique (iii) the asymptotic solution methods are employed. The numerical computations were carried out only for the case of assisting flow for the fluids having low Prandtl number appropriate for liquid metals (Pr = 0.05, 0.02 and 0.01 for lithium, mercury and sodium respectively).

The results thus we obtained for skin friction and the rate of heat transfer coefficient are presented in tabular form in the case of different properties of the liquid metals. The velocity profiles and the thermal conductivity profiles are given graphically in the in the case of constant viscosity. Finally, followings may be concluded from the throughout present investigations:

1. For increasing values of Prandtl number, the local skin friction decreases monotonically and the rate of heat transfer decreases.
2. The skin friction increase and the rate of heat transfer decrease at the increasing values of the magnetic field parameter, ξ.
3. Profiles for the velocity as well as the thermal conductivity decrease due to the increasing values of the Prandtl number, Pr.
4. Both the velocity profiles and temperature profiles decreases due to increasing values of pseudo similarity variable, η.

Table 1: Numerical values of skin friction coefficient τ obtained by different methods for different Pr.

	Pr=0.72				Pr=0.05		
	Series solution				Series solution		
ξ	2 terms	3 terms	Finite differ.	Wilks	2 terms	3 terms	Finite differ.
0.05	0.4923	0.4952	0.4958	0.4953	2.4552	2.4718	2.4885
0.10	0.5655	0.5725	0.5784	0.5722	2.7913	2.8308	2.8510
0.20	0.6390	0.6558	0.6571	0.6544	3.1034	3.1974	3.2044
0.30	0.6788	0.7066	0.7054		3.2510	3.4071	3.4066
0.40	0.7036	0.7435	0.7400		3.3272	3.5509	3.5434
0.50	0.7200	0.7726	0.7666	0.7654	3.3633	3.6579	3.6439
0.60	0.7309	0.7970	0.7882		3.3736	3.7449	3.7217
0.70	0.7379	0.8180	0.8062		3.3662	3.8164	3.7840
0.80	0.7421	0.8368	0.8216		3.3457	3.8777	3.8351
0.90	0.7441	0.8538	0.8350		3.3153	3.9317	3.8779
1.00	0.7443	0.8695	0.8469	0.8459	3.2772	3.9803	3.9143
2.00		0.9197a	0.9225	0.9184		4.1018a	4.1078
3.00		0.9575a	0.9598	0.9559		4.1831a	4.1867
4.00		0.9812a	0.9820	0.9800		4.2283a	4.2206
5.20		1.0019a	1.0025	1.0000		4.2624a	4.2637
6.00		1.0125a	1.0130	1.0101		4.2786a	4.2795
8.00		1.0323a	1.0326	1.0288		4.3070a	4.3073
10.0		1.0462a	1.0464	1.0418		4.3257a	4.3238
12.0		1.0567a	1.0574	1.0515		4.3393a	4.3392
14.0		1.0651a	1.0652	1.0591		4.3496a	4.3495
16.0		1.0720a	1.0722	1.0651		4.3600a	4.3598

where 'a' stands for the solutions of large ξ equations.

Table 2: Numerical values of rate of heat transfer Q obtained by different methods for different Pr.

	Pr=0.72				Pr=0.05		
	Series solution				Series solution		
ξ	2 terms	3 terms	Finite differ.	Wilks	2 terms	3 terms	Finite differ.
0.05	0.8405	0.8422	0.8430	0.8412	1.0666	1.0674	1.0679
0.10	0.6898	0.6926	0.6956	0.6919	0.8730	0.8745	0.8757
0.20	0.5598	0.5646	0.5660	0.5641	0.7058	0.7083	0.7103
0.30	0.4919	0.4983	0.4992		0.6181	0.6214	0.6232
0.40	0.4467	0.4547	0.4535		0.5598	0.5639	0.5765
0.50	0.4133	0.4228	0.4235	0.4227	0.5166	0.5215	0.5238
0.60	0.3870	0.3979	0.3986		0.4824	0.4881	0.4909
0.70	0.3654	0.3776	0.3783		0.4544	0.4607	0.4640
0.80	0.3471	0.3606	0.3613		0.4306	0.4376	0.4415
0.90	0.3313	0.3460	0.3468		0.4101	0.4177	0.4221
1.00	0.3174	0.3341	0.3341	0.3335	0.3920	0.4003	0.4052
2.00		0.2527a	0.2585	0.2581		0.3027a	0.3043
3.00		0.2197a	0.2202	0.2201		0.2527a	0.2536
4.00		0.1955a	0.1959	0.1956		0.2210a	0.2216
5.20		0.1752a	0.1754	0.1752		0.1950a	0.1954
6.00		0.1648a	0.1650	0.1647		0.1719a	0.1822
8.00		0.1454a	0.1455	0.1452		0.1579a	0.1581
10.0		0.1316a	0.1317	0.1314		0.1421a	0.1415
12.0		0.1212a	0.1213	0.1209		0.1291a	0.1292
14.0		0.1129a	0.1126	0.1127		0.1195a	0.1196
16.0		0.1060a	0.1057	0.1059		0.1096a	0.1098

where 'a' stands for the solutions of large ξ equations.

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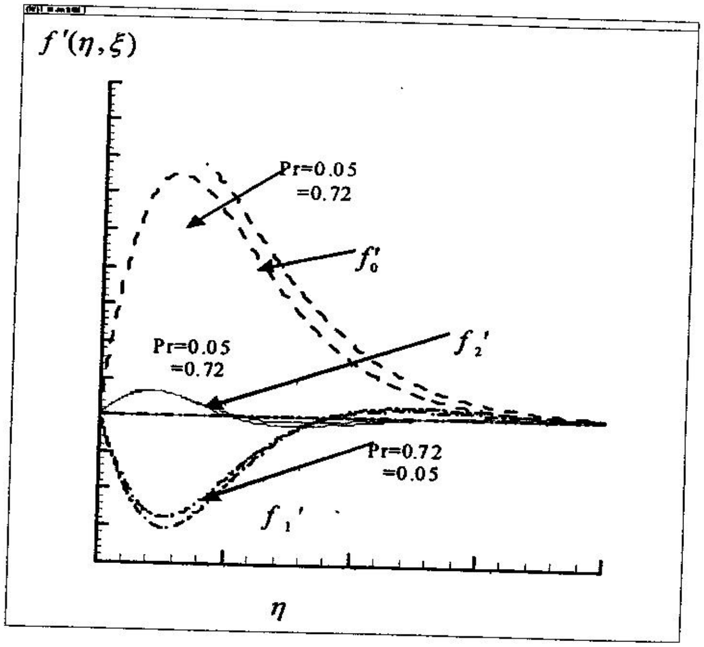

Figure 2: Velocity profile for different values of Pr against η

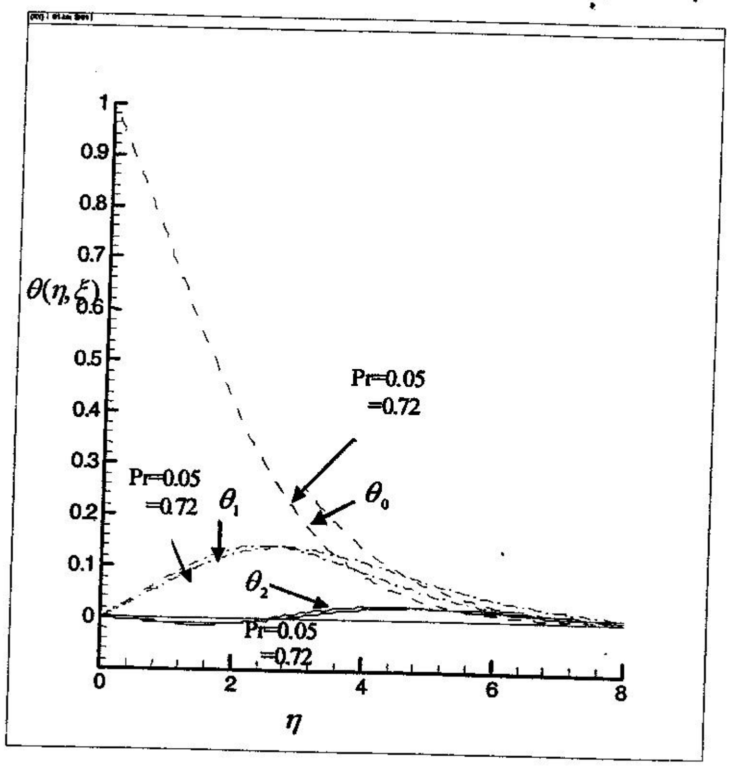

Figure 3: Temperature Profile for different values of Pr against η.

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