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THERMAL STRESSES IN AN AEOLOTROPIC THIN ROTATING ANNULAR DISC HAVING TRANSIENT SHEARING STRESS APPLIED ON THE OUTER EDGE.

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Abstract

In this paper thermal stresses in an aeolotropic thin rotating annular disc under transient shearing stress applied on the outer edge are derived when the modulus of elasticity and the coefficient of thermal expansion vary exponentially as the n^{th} power of the radial distance from the centre of the circular disk. Corresponding results for homogeneous case are deduced as a special case and found in agreement with the previous results. Numerical results are presented in a tabular form and graphically.

Keyword and phrases : *thermal stresses, thermal expansion, aeolotropic, shearing stress*

সংক্ষিপ্তসার

ক্ষণস্থায়ী কৃন্তন পীড়ণাধীন আবর্তনরত অসমদৈশিক পাতলা বলয়াকার চাকতির বাইরের প্রান্তের উপর প্রযুক্ত তাপজ পীড়ণ কে নির্ণয় করা হয়েছে যখন স্থিতিস্থাপক গুণাঙ্ক এবং তাপজ প্রসারণ সহগ বৃত্তাকার চাকতির কেন্দ্র থেকে অরীয় (ব্যাসার্ধ) দূরত্বে n - তম ঘাতের সূচকীয় ভেদে থাকে। বিশেষ ক্ষেত্র হিসাবে সমসত্ত্ব ক্ষেত্রের জন্য অনুরূপ ফল নির্ণয় করা হয়েছে এবং এটা দেখা গেছে যে পূর্ব নির্ধারিত ফলাফলের সঙ্গেঁ ইহা সঙ্গঁতিপূর্ণ। সাংখ্যমান তালিকাকারে দেখানো হয়েছে এবং লেখ-চিত্রের সাহায্যে প্রকাশ করা হয়েছে।

1. Introduction:

Ghosh [3] has obtained the expression for stresses and displacements due to surface loading of an isotropic half space in the absence of temperature field, where the Poisson's ratio is an arbitrary function of depth. Timoshenko and Goodier [9] in the theory of elasticity, had considered thermal stress on homogeneous circular discs, cylinders and spheres. Mollah [5] obtained the thermal stress in non-homogeneous circular disc of varying thickness rotating about a central axis. Wankhede [10] has investigated the quasi-static thermal stress in a thin circular plate. Gogulwar and Deshmukh [2] have obtained thermal stress in a thin circular plate with heat sources. De and Choudhury [1] has obtained the thermal stress in a homogeneous thin rotating annular disc having transient shearing stress applied on the outer edge when coefficient of thermal expansion vary as the n^{th} power of the radial distance.

The object of this paper is to obtain thermal stresses in an aeolotropic thin rotating annular disc having transient shearing stress applied on the outer edge, when both the modulus of elasticity and coefficient of thermal expression vary exponentially as the n^{th} power of the radial distance from the centre. The thermo elastic stresses in a homogenous disc under similar boundary conditions are deduced as a special case, which are found in agreement with the previous results. Finally numerical results are represented in a tabular form and shown graphically.

2. Fundamental equations:

We assume that displacement, stress and temperature do not vary across the thickness of the disk and that its lateral surfaces are free from any stress. Here we consider plain polar co-ordinates (r,θ) with reference to the center of the annular disk as origin. From symmetry we see that the radial displacement 'u' and the tangential displacement 'v' are independent of θ. The strain displacement relations are given by,

$$e_r=\frac{\partial u}{\partial r},\ e_\theta=\frac{u}{r},\ e_{r\theta}=\frac{\partial u}{\partial r}-\frac{u}{r} \tag{2.1}$$

where e_r, e_θ are the radial and tangential strains and $e_{r\theta}$ is the shearing strain. The stress displacement equation in case of plane stress reduces to

$$\left.\begin{aligned}\sigma_r &= \frac{E}{(1-\nu^2)}\left[\frac{\partial u}{\partial r}+\nu\frac{u}{r}\right]-\frac{E\alpha T}{(1-\nu)}\\ \sigma_\theta &= \frac{E}{(1-\nu^2)}\left[\nu\frac{\partial u}{\partial r}+\frac{u}{r}\right]-\frac{E\alpha T}{(1-\nu)}\\ \tau_{r\theta} &= \frac{E}{2(1+\nu)}\left[\frac{\partial \mathrm{v}}{\partial r}-\frac{\mathrm{v}}{r}\right]\end{aligned}\right\} \tag{2.2}$$

Where

σ_r = radial stress,
σ_θ = tangential stress,
$\tau_{r\theta}$ = shearing stress,
E = modulus of elasticity,
α = coefficient of thermal expasion,
ν = Poisson's ratio,
T = temperature at any point and at any time t.

For non homogeneity we assume $E = E_0 e^{nr}, \alpha = \alpha_0 e^{nr}$ where E_0 and α_0 are non zero positive constants, n = any integer and the temperature

$$T = T_0(1+r^2)e^{-2pt},$$

where T_0 = temperature at the centre of the disk at time $t = 0$ and $r = 0$, p = constant.

Then from (2.2) we get,

$$\left.\begin{aligned}\sigma_r &= \frac{E_0}{(1-\nu^2)}\left[e^{nr}\left(\frac{\partial u}{\partial r}+\nu\frac{u}{r}\right)-\alpha_0 T_0(1+\nu)(1+r^2)e^{2(nr-pt)}\right]\\ \sigma_\theta &= \frac{E_0}{(1-\nu^2)}\left[e^{nr}\left(\nu\frac{\partial u}{\partial r}+\frac{u}{r}\right)-\alpha_0 T_0(1+\nu)(1+r^2)e^{2(nr-pt)}\right]\\ \tau_{r\theta} &= \frac{E_0}{2(1+\nu)}e^{nr}\left[\frac{\partial \mathrm{v}}{\partial r}-\frac{\mathrm{v}}{r}\right]\end{aligned}\right\} \tag{2.3}$$

The equations of motion are given by,

$$\left.\begin{aligned}&\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}+\rho F_r=\rho f_r\\&\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2}{r}\tau_{r\theta}+\rho F_\theta=\rho f_\theta\end{aligned}\right\}\tag{2.4}$$

where ρ = density of the material of the disk that varies exponentially with the distance from the centre.

$\rho=\rho_0 e^{nr}$ and ρ_0 = density at $r = 0$.

Let us assume that the angular velocity vary exponentially with time, so that

$$\Omega=\Omega_0 e^{-pt}\tag{2.5}$$

where Ω_0 = non- zero positive constant.

We assume for a rotating disc that the radial velocity of a particle in the disc is negligible in comparison with the angular velocity. So, the Coriolis component of body force f is neglected.

Since the problem is one of the relative equilibrium, the body forces and the acceleration components at a point whose distance from the axis of rotation is r, are respectively

$$F_r=\Omega^2 r,\qquad F_\theta=-r\frac{d\Omega}{dt},\qquad f_r=\frac{\partial^2 u}{\partial t^2},\qquad f_\theta=\frac{\partial^2 \mathrm{v}}{\partial t^2}\tag{2.6}$$

We assume

$$u=U(r)e^{-2pt},\qquad \mathrm{v}=\mathrm{V}(r)e^{-pt}\tag{2.7}$$

From (2.4) using (2.3), (2.5), (2.6) and (2.7) we get,

$$\begin{aligned}&r^2\frac{d^2U}{dr^2}+(nr+1)r\frac{dU}{dr}-\left[(1-n\nu r)+\frac{4(1-\nu^2)\rho_0 p^2 r^2}{E_0}\right]U\\&=2\alpha_0 T_0(1+\nu)\left\{nr^2(1+nr)r^3\right\}e^{nr}-\frac{(1-\nu^2)}{E_0}\rho_0\Omega_0^2 r^3\end{aligned}\tag{2.8}$$

$$r^2\frac{d^2V}{dr^2}+(nr+1)r\frac{dV}{dr}-\left[(1+nr)+\frac{2(1+\nu)\rho_0 p^2 r^2}{E_0}\right]V=\frac{2(1+\nu)}{E_0}p\rho_0\Omega_0^2 r^3\tag{2.9}$$

3. Solution of the problem:

Solution of the equation (2.8) is given by,

$$U = A_1F_1(r) + B_1F_2(r) + \frac{\Omega_0}{4p^2}r + \left[\frac{m_1(2n^2-k_1-3)}{(4+\nu)(2n^2-k_1)}r + \frac{m_1 n}{(2n^2-k_1)}r^2\right]e^{nr} \quad (3.1)$$

where,

$$F_1(r) = C_1 r + C_2 r^2 + C_3 r^3 + C_4 r^4 + \ldots\ldots\ldots\ldots, \quad (3.1.1)$$

$$F_2(r) = F_1(r)\log r + \frac{1}{r} + d_0 + d_1 r + d_2 r^2 + d_3 r^3 + d_4 r^4 + \ldots\ldots\ldots\ldots, \quad (3.1.2)$$

and

$$C_1 = -\frac{n^2\nu(\nu-1)}{2}, \quad (3.1.1a)$$

$$C_2 = \frac{n^3\nu(\nu^2-1)}{6} - \frac{nk_1(\nu+1)}{6}, \quad (3.1.1b)$$

$$C_3 = \frac{n^2k_1(\nu+1)(\nu+2)}{48} - \frac{n^4(\nu^2-1)\nu(\nu+2)}{48} - \frac{\nu n^2 k_1(\nu+1)}{16} + \frac{k_1}{16}, \quad (3.1.1c)$$

$$C_4 = \frac{n^5(\nu^2-1)\nu(\nu+2)(\nu+3)}{720} - \frac{n^3k_1(\nu+1)(\nu+2)(\nu+3)}{720} - \frac{nk_1^2(\nu+1)}{90}, \quad (3.1.1d)$$

$$d_0 = (\nu-1)n, \quad (3.1.2a)$$

$$d_1 = -\frac{(2-5\nu+\nu^2)n^2}{4} - \frac{k_1}{4}, \quad (3.1.2b)$$

$$d_2 = \frac{(3\nu^2-1)n^3}{6} - \frac{11\nu(1-\nu^2)n^3}{36} + \frac{(10\nu+4)nk_1}{36} + \frac{(\nu-1)nk_1}{3}, \quad (3.1.2c)$$

$$d_3 = \frac{31(\nu^2-1)\nu(\nu+2)n^4}{576} - \frac{(\nu^2+\nu-1)\nu(\nu+2)n^4}{24} - \frac{n^2k_1(2\nu+3)}{48} - \frac{43(\nu+1)(\nu+2)n^2k_1}{576} - \frac{(\nu-1)(\nu+2)n^2k_1}{24} - \frac{(2\nu-1)n^2k_1}{16} \quad (3.1.2d)$$

$$\frac{5(\nu-1)(\nu+2)nk_1}{24}-\frac{5k_1^2}{64},$$

$$d_4=\frac{(\nu^4+4\nu^3+3\nu^2-2\nu)n^5}{144}+\frac{187(\nu^2-1)\nu(\nu+2)(\nu+3)n^5}{43200}-\frac{(3\nu^2+12\nu+11)n^3k_1}{720}+\frac{187(\nu+1)(\nu+2)(\nu+3)n^3k_1}{43200}+\frac{(\nu-1)(\nu+2)(\nu+3)nk_1^2}{360}+\frac{71(\nu+1)nk_1^2}{2700}+\frac{(\nu-1)nk_1^2}{45}-\frac{n^5}{120}-\frac{nk_1^2}{90}, \quad (3.1.2e)$$

$$k_1=\frac{4\rho_0p^2\left(1-\nu^2\right)}{E_0} \quad (3.1.3)$$

$$m_1=2\alpha_0T_0\left(1+\nu\right) \quad (3.1.4)$$

Solution of the equation (2.9) is given by,

$$V=A_2F_3(r)+B_2F_4(r)+\frac{\Omega_0}{p}r \quad (3.2)$$

where

$$F_3(r)=\frac{k_2}{2}r+\frac{k_2^2}{16}r^3-\frac{nk_2^2}{120}r^4............, \quad (3.2.1)$$

$$F_4(r)=F_3(r)\log r+\frac{1}{r}-\frac{k_2}{4}r-\frac{nk_2}{6}r^2+\left(\frac{5k_2^2}{64}+\frac{n^2k_2}{48}\right)r^3+\left(\frac{77k_2^2}{7200}-\frac{nk_2^2}{72}-\frac{n^3k_2}{360}\right)r^4+............., \quad (3.2.2)$$

$$k_2=\frac{2\rho_0p^2\left(1+\nu\right)}{E_0}. \quad (3.2a)$$

Substituting the values of u and v *in* (2.3) we get,

$$\sigma_r=\frac{E_0}{\left(1-\nu^2\right)}e^{(nr-2pt)}[(A_1+B_1\log r)G_1(r)+B_1G_2(r)+\frac{\Omega_0^2}{4p^2}(1+\nu)+e^{nr}\{E_1\left(1+\nu+nr\right)+E_2\left(2r+\nu r+nr^2\right)\}-\alpha_0T_0\left(1+\nu\right)\left(1+r^2\right)e^{nr}], \quad (3.3)$$

$$\sigma_\theta = \frac{E_0}{(1-\nu^2)} e^{(nr-2pt)} [(A_1 + B_1 \log r) G_3(r) + B_1 G_4(r) + \frac{\Omega_0^2}{4p^2}(1+\nu)$$
$$+ e^{nr}\{E_1(1+\nu+nr) + E_2(2r+\nu r+nr^2)\}$$
$$-\alpha_0 T_0 (1+\nu)(1+r^2) e^{nr}], \qquad (3.4)$$

$$\tau_{r\theta} = \frac{E_0}{(1-\nu^2)} e^{(nr-pt)} [(A_2 + B_2 \log r) G_5(r) + B_2 G_6(r)], \qquad (3.5)$$

Where

$$E_1 = \frac{m_1(2n^2-k_1-3)}{(4+\nu)(2n^2-k_1)} \qquad (3.3a)$$

$$E_2 = \frac{m_1 n}{(2n^2-k_1)}, \qquad (3.3b)$$

And

$$G_1(r) = (1+\nu)C_1 + (2+\nu)C_2 r + (3+\nu)C_3 r^2 + (4+\nu)C_4 r^3 + \ldots\ldots, \qquad (3.3.1)$$

$$G_2(r) = \frac{(\nu-1)}{r^2} + \frac{\nu d_0}{r} + (C_1 + d_1 + \nu d_1) + (C_2 + 2d_2 + \nu d_2)r + (C_3 + 3d_3 + \nu d_3)r^2$$
$$+ (C_4 + 4d_4 + \nu d_4)r^3 \ldots\ldots, \qquad (3.3.2)$$

$$G_3(r) = (1+\nu)C_1 + (2\nu+1)C_2 r + (3\nu+1)C_3 r^2 + (4\nu+1)C_4 r^3 + \ldots\ldots, \qquad (3.4.1)$$

$$G_4(r) = \frac{(\nu-1)}{r^2} + \frac{d_0}{r} + (\nu C_1 + d_1 + \nu d_1) + (\nu C_2 + 2\nu d_2 + d_2)r + (\nu C_3 + 3\nu d_3 + d_3)r^2$$
$$+ (\nu C_4 + 4\nu d_4 + d_4)r^3 \ldots\ldots, \qquad (3.4.2)$$

$$G_5(r) = \frac{k_2^2 r^2}{8} - \frac{n k_2^2 r^3}{40} \ldots\ldots, \qquad (3.5.1)$$

$$G_6(r) = -\frac{2}{r^2} - \frac{k_2}{2} - \frac{n k_2}{6} r + \left(\frac{k_2^2}{16} + \frac{5k_2}{32} - \frac{n^2 k_2}{24}\right) r^2$$
$$- \left(\frac{n k_2^2}{120} + \frac{n k_2^2}{24} - \frac{77 k_2^2}{2400} + \frac{n^3 k_2}{120}\right) r^3 + \ldots\ldots \qquad (3.5.2)$$

Boundary conditions are given by,

$$\left.\begin{aligned} &\tau_{r\theta} = -ke^{-pt} \quad \text{and} \quad \sigma_r = 0, && on\ r = a \\ &\tau_{r\theta} = \sigma_r = 0, && on\ r = b \end{aligned}\right\} \tag{3.6}$$

where a and b are the inner and outer radii of the disc respectively.

Using the boundary conditions (3.6) in (3.3), (3.4) and (3.5) we can calculate A_1, B_1, A_2 and B_2.

Thus $A_1, B_1,\ A_2$ and B_2 are completely known. Hence σ_r, σ_θ and $\tau_{r\theta}$ are all known. For n=0, the equation (2.8), (2.9) and the corresponding solutions are in agreement with the result of Mollah [5].

4. Numerical result and discussion:

For numerical calculation we take the parameter as follows (Love [4] for copper on the inner boundary):

$E_0 = 1.234\text{x}10^{12}$ dynes/cm^2,

$\alpha_0 = 16\text{x}10^{-86}$ cm/°C,

$\nu = 0.378,\quad \rho_0 = 8.843$ gm/cc,

$T_0 = 500$ °C, $\quad p = 0.5,$

$\Omega_0 = 1$ radian/sec, $\quad a = 1$ cm, $\quad b = 2$ cm.

The adjoining table exhibits the value of

$$\mathrm{R} = 10\frac{1-\nu^2}{E_0}e^{2pt}\sigma_r, \qquad \mathrm{X} = \frac{1-\nu^2}{E_0}e^{2pt}\sigma_\theta;$$

The values of R and X for different radii are shown if table 1. The variation of R and X with radius r are represented graphically in figure 1 and figure 2 respectively.

Table 1

r	R	X
1.000	0	-4.8242
1.025	0.9521	-4.5317
1.050	1.8226	-4.25
1.075	2.6069	-4.0026
1.100	3.3002	-3.7739
1.125	3.8975	-3.5744
1.150	4.3947	-3.4080
1.175	4.7450	-3.2787
1.200	5.0675	-3.1903
1.225	5.2342	-3.1465
1.250	5.2822	-3.1509
1.275	5.2076	-3.2072
1.300	5.0071	-3.3184
1.325	4.6784	-3.4877
1.350	4.2197	-3.7177
1.375	3.6300	-4.0108
1.400	2.9104	-4.3687
1.425	2.0622	-4.7928
1.450	1.0891	-5.2836
1.475	-0.0037	-5.8410
1.500	-1.2085	-6.4640
1.525	-2.5158	-7.1505
1.550	-3.9125	-7.8972
1.575	-5.3831	-8.6995
1.600	-6.9078	-9.5512
1.625	-8.4636	-10.4444
1.650	-10.0222	-11.3691
1.675	-11.5510	-12.3132
1.700	-13.0105	-13.2619
1.725	-14.3569	-14.1979
1.750	-15.5375	-15.1001
1.775	-16.4929	-15.9446
1.800	-17.1544	-16.7028
1.825	-17.4447	-17.3423
1.850	-17.2749	-17.8255
1.875	-16.5445	-18.1091
1.900	-15.1404	-18.1445
1.925	-12.9362	-17.8758
1.950	-9.7879	-17.2397
1.975	-5.5360	-16.1652
2.000	0	-14.5721

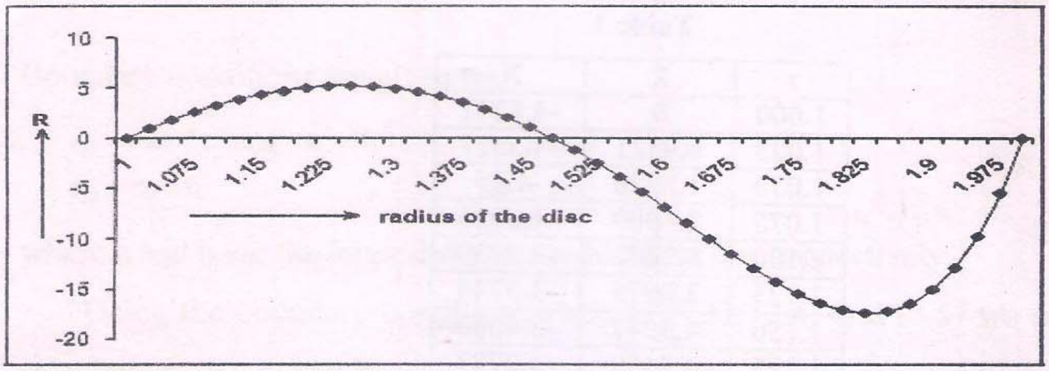

Figure1: The curve shows the variation of radial stress with the radial distances.

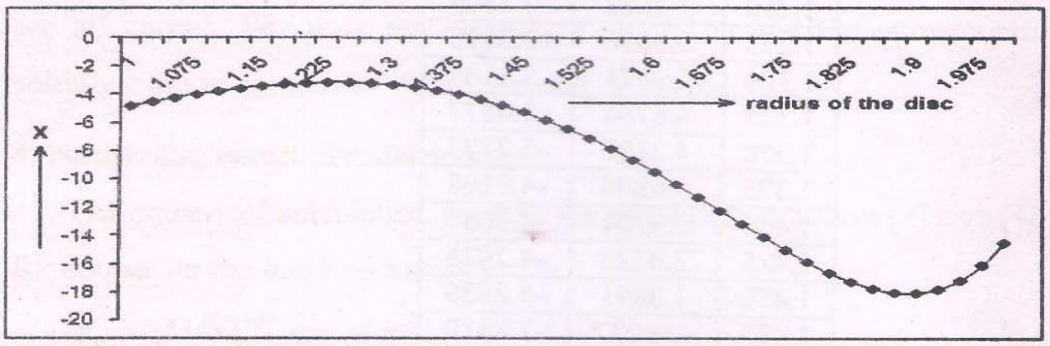

Figure2: The curve shows the variation of tangential stress with radial distances.

5. Conclusion:

For the aeolotropic thin annular disk having transient shearing stress applied on the outer edge and the inner edge is assumed to be stress free, the radial stress gradually increases from the inner edge ($r = 1$) and attains a maximum value and then gradually decreases and attains a minimum value and again gradually increases and become zero on the outer edge ($r = 2$). The tangential stress for the same disk gradually increases from the inner edge ($r = 1$) and attain a maximum, value and then gradually decreases and attain a minimum value and again gradually increases. In case of tangential stress, tangential stress on the inner wall also assumed to the same pattern of the curve as in the case of radial stress.

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