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TIME SERIES ANALYSIS AND MATHEMATICAL MODELING OF GENERAL INDEX OF DSE

By

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Abstract

In our study we have analyzed the market volatility in stock prices in the Dhaka Stock Exchange (DSE) during 2005-2008. First of all the raw data is collected form DSE (Dhaka Stock Exchange Department). Then we have analyzed the data in two way, one is based on statistical measure and the other is curve fitting. We also explore the trend of general index of DSE in the form of differential equation with the help of least square method.

Keyword and phrases : *time series, market volatility, stock price, statistical measure.*

সংক্ষিপ্তসার

আমরা গভীর অনুসন্ধানের মাধ্যমে ২০০৫ - ২০০৮ সালের স্থিতিকালে ঢাকা সংভার বিনিময় কেন্দ্রে (DSE) শেয়ার মূল্যের বাজার উদ্বায়িত্বের বিশ্লেষণ করেছি। এরপর কাঁচা রাশিতথ্যকে দু'ভাগে বিশ্লেষণ করেছি। একটিকে পরিসংখ্যান পরিমাপের উপর ভিত্তি করে এবং অপরটিকে বক্র সুসমঞ্জস্য করণের (curve fitting) মাধ্যমে। লঘিষ্ঠ বর্গ পদ্ধতির (least square method) সাহায্যে অবকল সমীকরণের আকারে DSE -- এর সাধারণ সূচকের গতি- প্রকৃতিকেও পুঙ্খানুপুঙ্খভাবে পরীক্ষা করেছি।

1. Introduction

The Dhaka Stock Exchange was a physical stock exchange in its initial days when trading took place in the open outcry system. But with the advent of new technology, the traditional mode of trading was abolished and was replaced by a fully automated computerized Stock Exchange. The trading session occurs in four parts- the pre-opening session, opening session, continuous or regular trading session and closing session or post-closing session. The changing of share price is regular phenomenon. The changing of share price is measured by different indexes. For example DSE general index, DSE all share index, DSE 20 index etc.

Chowdhury and Rahman (2004) have studied the relationship between the predicted volatility of DSE returns and that of selected macroeconomic variables of Bangladesh economy. They have followed the methodology of Schwert (1989; 1990) to calculate the predicted volatility of the variables used in the study. They have calculated volatility from errors after using an autoregressive and seasonality adjusted forecasting model. The volatility series derived from such process has some limitations, which have been corrected in Generalized Conditional Auto Regressive Heteroskedasticity (GCARH) models developed by Bollerslev (1986). A number of studies found that stock price has a significant positive relationship with the dividend payment [Gordon (1959), Ogden (1994), Stevens and Jose (1989), Kato and Loewenstein (1995), Ariff and Finn (1986), and Lee (1995)], while others found a negative relationship [Loughlin (1982) and Easton and Sinclair (1989)]. A negative relationship between dividend announcement stock returns is expected due to tax effect, but researchers tended to relate the positive relationship between the stock returns and dividend announcement with the information effect of dividend. The theoretical literature on dividend effects has been well

developed. In this study we try to established mathematical model from the trend of general index of DSE from our collected raw data.

2. A Brief Historical Background

The stock exchange in Bangladesh was incorporated in April 1954 as the East Pakistan Stock Exchange Ltd. However, formal trading in the Exchange did not commence until 1956. The Exchange remained suspended from 1971 to 1975 due to the liberation war. After liberation, the Exchange opened up in 1976 with only 9 listed companies. In 1976, there were nine listed companies in Dhaka Stock Exchange with a paid up capital of Tk137.52 million. The Dhaka Stock Exchange actually witnessed high growth in 1983 when the market capitalization reached Tk.812 million. By 1987, there was a spurt in the market size with the number of listed companies shooting up to 92. With the opening up of the economy in the 90s the Dhaka Stock Exchange also rapidly developed. As of June 1999, the Exchange has 210 listed companies with 230 listed securities and a market capitalization of 1,046 million US dollars. As in most other developing countries, the capital market in Bangladesh has a relatively recent beginning. It is gradually evolving as an economic institution in response to the internal requirements of a fledgling modern economy, which has emerged as a result of economic development and industrialization efforts. Bangladesh is still a predominantly agricultural economy. However, the industrialization process over the last three decades since independence has diversified the economic base of the country at least to some degree.

3. Materials and Methods

Share market index

Index consists of group of shares. Index denotes the direction of the entire market. Like when people say market is going up or down then that means Index is going up or down. Index consists of high market capitalization and high liquidity shares. High Market capitalization shares - Companies having highest number of shares and highest price of each share. Market capitalization is calculated by multiplying current share price and number of shares in the market. High Liquidity shares - Shares in the market with high volumes

Index Calculation Algorithm

$$\text{Current Index} = \frac{\text{Yesterday's Closing Index} \times \text{Current Market capitalization}}{\text{Opening Market capitalization}}$$

$$\text{Closing Index} = \frac{\text{Yesterday's Closing Index} \times \text{Closing Market Capitalization}}{\text{Opening Market capitalization}}$$

Current Market capitalization = ∑(Last Traded Price × Total no. of indexed shares)

Closing Market capitalization = ∑ (Closing Price × Total no. of indexed shares)

There are three Dhaka Stock Exchange general indexes such as DSE general index, DSE all share index and DSE 20 index.

Curve Fitting as a straight line

Let $(x_i, y_i); i = 1,2,3,.....,n$ be a given set of n pairs of values, x being independent variable and y the dependent variable. The general problem in curve fitting is to find, if possible, an analytic expression of the form $y = f(x)$, for the functional relationship suggested by the given data

Let us consider the fitting of a straight line as

$$y = a + bx \tag{1}$$

To a set of n points $(x_i, y_i); i = 1,2,3,.....,n$ equation (1) represents a family of straight lines for different values of the arbitrary constants a and b. The problem is to determine a and b so that the line (1) is of the line "best fit".

The term "best fit" is interpreted in accordance with Legender's principle of least squares which consists in minimizing the sum of the squares of the deviations of the actual values of y from their estimated values as given by the line best fit.

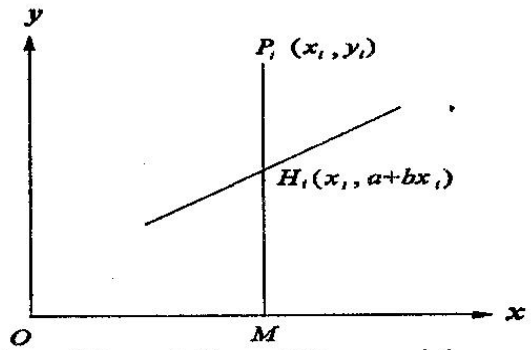

Figure1 Curve fitting model.

Let $P_i(x_i, y_i)$ be any general points in the scatter diagram. Draw $P_iM \perp x$-axis meeting the line in H_i. Abscissa of H_i is x_i and since H_i lies on (1), its ordinate is $a + bx_i$. Hence the co-ordinates of H_i are $(x_i, a + bx_i)$. From Figure1

$$P_iH_i = P_iM - H_iM = y_i - (a + bx_i)$$

which is called the error of estimation or the residual for y_i.

According to the principle of least squares, we have to determine a and b so that $E = \sum_{i=1}^{n} P_iH_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2$ is minimum. From the maxima and minima, the partial derivatives of E, with respect to a and b should vanish separately, i.e.

$$\frac{\partial E}{\partial a} = 0 = -2\sum_{i=1}^{n}[y_i - a - bx_i] \Rightarrow \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} a + b\sum_{i=1}^{n} x_i \Rightarrow \sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i \qquad (2)$$

$$\frac{\partial E}{\partial b} = 0 = -2\sum_{i=1}^{n}[y_i - a - bx_i](-x_i) \Rightarrow \sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} ax_i + \sum_{i=1}^{n} bx_i^2$$
$$\Rightarrow \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2. \qquad (3)$$

Equation (2) and (3) are known as the normal equations for estimating a and b.

Mathematical model of DSE General Index for the year 2005

Table1

Monthly DSE general index for 2005

From our collected data we can find the Table 1 for finding a and b

x	y	x^2	xy	x^3	x^4	x^2y
1	1893	1	1893	1	1	1893
2	1818	4	3636	8	16	7272
3	1932	9	5796	27	81	17388
4	1825	16	7300	64	256	29200
5	1587	25	7935	125	625	39675
6	1682	36	10092	216	1296	60552
7	1649	49	11543	343	2401	80801
8	1548	64	12384	512	4096	99072
9	1644	81	14796	729	6561	149604
10	1666	100	16660	1000	10000	166600
11	1701	121	18711	1331	14641	205821
12	1660	144	19920	1728	20736	239040
$\sum x =$ 78	$\sum y =$ 20605	$\sum x^2 =$ 650	$\sum xy =$ 130666	$\sum x^3 =$ 6084	$\sum x^4 =$ 60710	$\sum x^2y =$ 1096918

x, y represents month and general index respectively.

From (2) and Table 1 we get $20605 = 12a + 78b$ (4)

From (3) and Table 1 we get, $130666 = 78a + 650b$ (5)

Solving (4) and (5) we get

$a = 1865.56$, $b = -22.84$ and $y = 1865.56 - 22.84x$, (6)

which is our mathematical model as a straight line.

4. Mathematical model as an algebraic equation

If the curve represented by a 2nd degree equation, then we consider

$$y = a + bx + cx^2$$

$$E = \sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2\right]^2$$

$$\frac{\partial E}{\partial a} = 0 = -2\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2\right] \Rightarrow \sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i + c\sum_{i=1}^{n} x_i^2 \qquad (7)$$

$$\frac{\partial E}{\partial b} = 0 = -2\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2\right]x_i \Rightarrow \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i^3 \qquad (8)$$

$$\frac{\partial E}{\partial c} = 0 = -2\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2\right]x_i^2 \Rightarrow \sum_{i=1}^{n} x_i^2 y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^4 \qquad (9)$$

From (7) and Table 1 we get, $20605 = 12a + 78b + 650c$

From (8) and Table 1 we get, $130666 = 78a + 650b + 6084c$

From (9) and Table 1 we get, $1080478 = 650a + 6084b + 60710c$

Solving above all those equations we get

$$a = 2020.98,\ b = -89.44,\ c = 5.1236$$

$$y = 2020.98 - 89.44x + 5.1236x^2 \qquad (10)$$

which is the mathematical model as a second degree equation.

Mathematical model as a 2nd order differential equation

Differentiating (10) with respect to x we get,

$$\frac{dy}{dx} = -89.44 + 2 \times 5.1236x \qquad (11)$$

$$\frac{1}{2}\frac{d^2y}{dx^2} = 5.1236 \qquad (12)$$

Using (11) and (12) we get,

$$\frac{dy}{dx} - x\frac{d^2y}{dx^2} = -89.44 \qquad (13)$$

Putting the value (12) and (13) in (10) we get

$$y = 2020.98 + (\frac{dy}{dx} - x\frac{d^2y}{dx^2})x + \frac{1}{2}\frac{d^2y}{dx^2}x^2$$

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 4041.96 \qquad (14)$$

which is the mathematical model of the DSE general index for 2005.

One can see the model of DSE general index for in 2006, 2007 and 2008 (Rahman, 2009).

Mathematical model of DSE General Index for the year 2005 to 2008

Table 2

Monthly DSE general index for 2005 to 2008

x	y	x^2	xy	x^3	x^4	x^2y	x^5	x^6	x^3y
1	1893	1	1893	1	1	1893	1	1	1893
2	1682	4	3364	8	16	6728	32	64	13456
3	1660	9	4980	27	81	14940	243	729	44820
4	1692	16	6768	64	256	27072	1024	4096	108288
5	1328	25	6640	125	625	33200	3125	15625	166000
6	1570	36	9420	216	1296	56520	7776	46656	339120
7	1644	49	11508	343	2401	80556	16807	117649	563892
8	2050	64	16400	512	4096	131200	32768	262144	1049600
9	2941	81	26469	729	6561	238221	59049	531441	2143989
10	2961	100	29610	1000	10000	296100	100000	1000000	2961000
11	3090	121	33990	1331	14641	373890	161051	1771561	4112790
12	2530	144	30360	1728	20736	364320	248832	2985984	4371840
$\sum x =$ 78	$\sum y =$ 25041	$\sum x^2 =$ 650	$\sum xy =$ 181402	$\sum x^3 =$ 6084	$\sum x^4 =$ 60710	$\sum x^2y =$ 1624640	$\sum x^5 =$ 630708	$\sum x^6 =$ 6735950	$\sum x^3y =$ 15876688

x, y represents month and general index respectively.

From (2) and Table 2 we get,

$$25041 = 12a + 78b \qquad (15)$$

From (3) and Table 2 we get

$$181402 = 78a + 650b \qquad (16)$$

Solving (15) and (16) we get

$a = 1239.68$ and $b = 130.32$.

So,

$$y = 1239.68 + 130.32x \tag{17}$$

5. Mathematical model as a 3rd degree equation

If the curve represented by a 3rd degree equation, then we consider,

$$y = a + bx + cx^2 + dx^3$$

$$E = \sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2 - dx_i^3\right]^2$$

$$\frac{\partial E}{\partial a} = 0 = -2\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2 - dx_i^3\right]$$

$$\Rightarrow \sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i + c\sum_{i=1}^{n} x_i^2 + d\sum_{i=1}^{n} x_i^3 \tag{18}$$

$$\frac{\partial E}{\partial b} = 0 = -\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2 - dx_i^3\right]x_i$$

$$\Rightarrow \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i^3 + d\sum_{i=1}^{n} x_i^4 \tag{19}$$

$$\frac{\partial E}{\partial c} = 0 = -2\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2 - dx_i^3\right]x_i^2$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^4 + d\sum_{i=1}^{n} x_i^5 \tag{20}$$

$$\frac{\partial E}{\partial d} = 0 = -2\sum_{i=1}^{n}\left[y_i - a - bx_i - cx_i^2 - dx_i^3\right]x_i^3$$

$$\Rightarrow \sum_{i=1}^{n} x_i^3 y_i = a\sum_{i=1}^{n} x^3 + b\sum_{i=1}^{n} x_i^4 + c\sum_{i=1}^{n} x_i^5 + d\sum_{i=1}^{n} x_i^6 \tag{21}$$

From (18) we get, $25041 = 12a + 78b + 650c + 6084d$

From (19) we get, $181402 = 78a + 650b + 6084c + 60710d$

From (20) we get, $1624640 = 650a + 6084b + 60710c + 630708d$

From (21) we get, $15876688 = 6084a + 60710b + 630708c + 6735950d$

Solving above all those equations we get

$a = 2820.02$, $b = -887.723$, $c = 160.85$ and $d = -7.25006$.

So,

$$y = 2820.02 - 887.72x + 160.85x^2 - 7.25x^3 \tag{22}$$

which is the mathematical model of DSE general index as 3rd degree equation.

6. Mathematical model as a 3rd order differential equation

Differentiating (22) with respect to x we get,

$$\frac{dy}{dx} = -887.72 + 2\times 160.85x - 3\times 7.25x^2 \tag{23}$$

$$\frac{d^2y}{dx^2} = 2\times 160.85 - 6\times 7.25x \tag{24}$$

$$\frac{d^3y}{dx^3} = -6\times 7.25 \tag{25}$$

Using (24) and (25) we get

$$\frac{d^2y}{dx^2} = 2\times 160.85 + x\frac{d^3y}{dx^3} \Rightarrow \frac{1}{2}\left(\frac{d^2y}{dx^2} - x\frac{d^3y}{dx^3}\right) = 160.85 \tag{26}$$

Using (25), (26) in (23) we get,

$$\frac{dy}{dx} = -887.72 + x\left(\frac{d^2y}{dx^2} - x\frac{d^3y}{dx^3}\right) + 3\times\frac{1}{6}x^2\frac{d^3y}{dx^3}$$

$$\frac{dy}{dx} - x\frac{d^2y}{dx^2} + \frac{1}{2}x^2\frac{d^3y}{dx^3} = -887.72. \tag{27}$$

Putting the value (25), (26), (27) in (22) we get,

$$y = 2820.02 + x\left(\frac{dy}{dx} - x\frac{d^2y}{dx^2} + \frac{1}{2}x^2\frac{d^3y}{dx^3}\right) + \frac{1}{2}x^2\left(\frac{d^2y}{dx^2} - x\frac{d^3y}{dx^3}\right) + \frac{1}{6}x^3\frac{d^3y}{dx^3}$$

$$x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y + 16920.12 = 0 \tag{28}$$

which is the mathematical model of the DSE general index for 2005 to 2008 as a 3rd order differential equation.

Statistical measure for DSE general index (From 2005 to 2008)

Table 3

Some statistical measure

	DSE General Index for 2005	DSE General Index for 2006	DSE General Index for 2007	DSE General Index for 2008
Min	1523.6	1294	1579.2	2480.6
Max	1999.7	1711.5	2990.5	3207.9
Range	476.15	417.48	1411.3	727.27
Mean	1717.1	1501.7	2220	2877.3
SD	125.18	22877	477.68	182.3
Med	1688.7	1531.8	2160.9	2905.3
Skew	0.62216	-0.1415	0.33743	-0.48169
Kur	-0.44082	-1.1144	-1.3698	-0.6757
CV	7.30%	7.70%	21.50%	6.30%

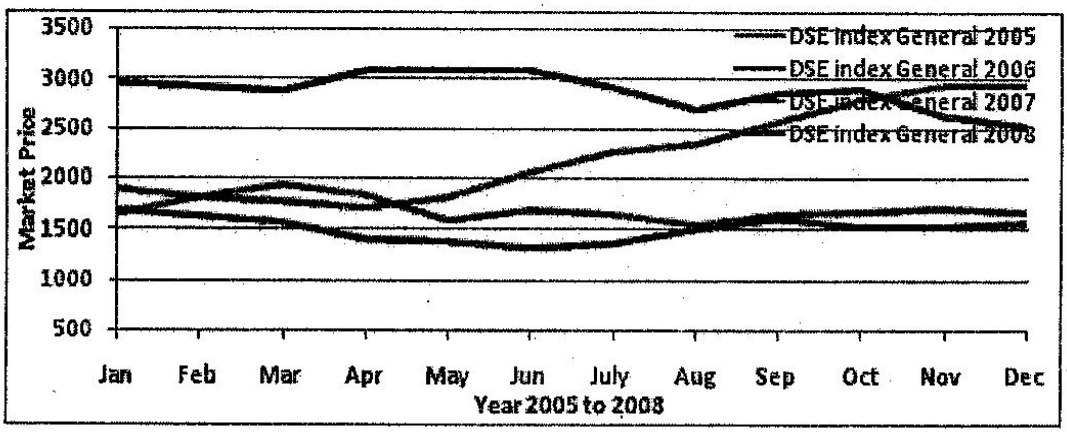

Figure2(a) Yearly Trends of DSE General Index

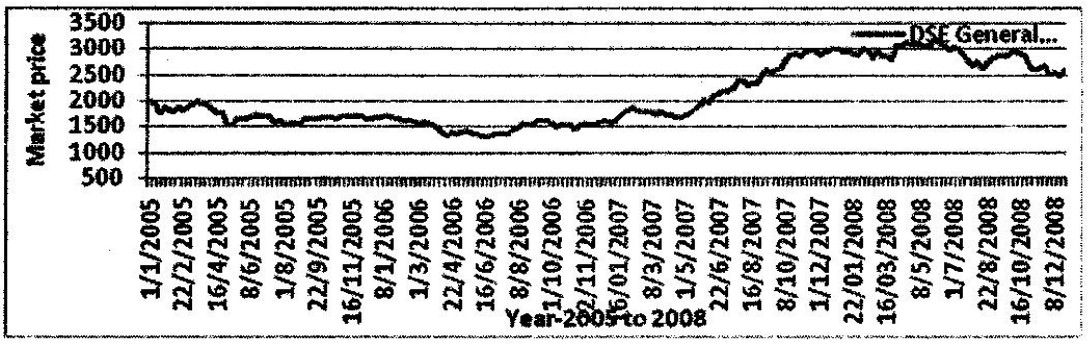

Figure2(b) Yearly Trends of DSE General Index

Graphical presentation of average price form the different months of DSE General Index from 2005 to 2008.

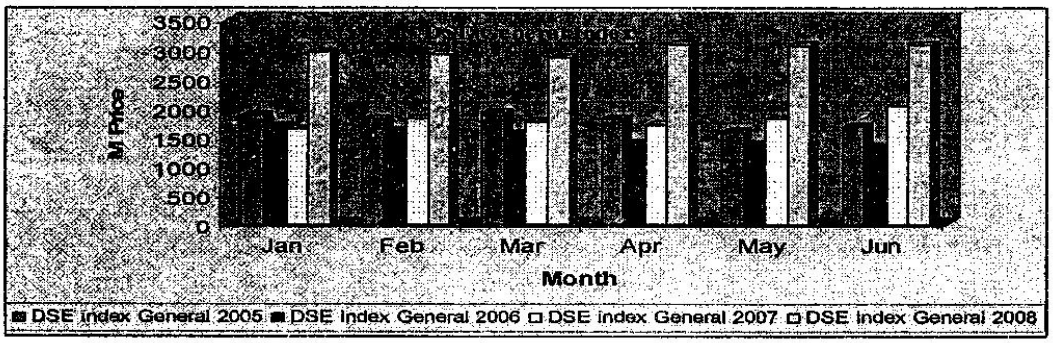

Figure3(a) From January to June.

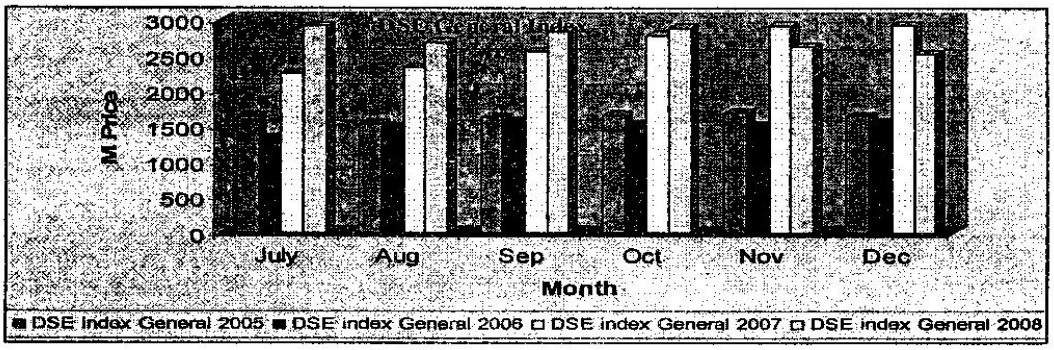

Figure3(b) From July to December.

7. Results and Discussions

From the Table 3 and Figures2(a) & 2(b) we have seen that the DSE general index is fluctuating over the mentioned years. The minimum and

maximum indexes for the years are 1294 and 3207.9 respectively. There is a big difference between highest and lowest index which is not good for the share market. From the table we can say that all types of statistical measure depend on the index.

Figures2(a), 2 (b), 3(a) and 3(b) indicates that the index is highly increasing from March 2007 to the end of November 2008, after that the trend almost similar up to December 2008. The highest index is in April and May 2008.So at that the investment is very risky.

8. Conclusions

Volatility in stock prices is a common phenomenon in the equity market. In the absence of price volatility, potential investors lose interest to participate in the stock market. However, careful monitoring of volatility by the concerned authority is needed in DSE which is yet to achieve maturity especially when high volatility exists in the market. If necessary, there should be effective intervention when the market experiences excess volatility. During unpredictable movements of individual stock prices, it would be useful for the authority to identify the factors behind such price movements and quickly disseminate the information to interested stockholders. In addition, the authority may take measures to make available all relevant information relating to real worth of the companies experiencing excess volatility in stock prices, especially to the investors. It is also important to ensure adequate supply of stocks through active participation of the government in the capital market particularly to dampen the excess demand. Our study result can help the investors and the concerned authority to make right decision in right time.

References:

1) Ariff, M. and Finn, F. J., "Announcement Effects and Market Efficiency in a Thin Market: An Empirical Application to the Singapore Equity Market", *Asia Pacific Journal of Management,* Vol. 6, 1986, pp. 243-267.

2) Bollerslev, T., "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics,* Vol. 31, 1986, pp. 307-27.

3) Chowdhury, S. S. H. and Rahman, M. A., "On the Empirical Relation between Macroeconomic Volatility and Stock Market Volatility of Bangladesh", *The Global Journal of Finance and Economics,* Vol. 1, No. 2, 2004, pp.209-225.

4) Easton, S. A. and Sinclair, N. A., "The Impact of Unexpected Earnings and Dividends on Abnormal Returns to Equity", *Journal of Accounting & Finance,* Vol. 29, 1989, pp.1-19.

5) Gordon, M. J., "Dividend, Earning, and Stock Prices", *The Review of Economics and Statistics,* Vol. 41, 1959, pp. 99-105.

6) Kato, K. and Loewenstein, U., "The Ex-Dividend-Day Behavior of Stock Prices: The Case of Japan", *The Review of Financial Studies,* Vol. 8, 1995, pp. 816-847.

7) Lee, B. S., "The Response of Stock Prices to Permanent and Temporary Shocks to Dividends", *Journal of Financial and Quantitative Analysis,* Vol. 30, 1995, pp. 1-22.

8) Loughlin P.H., "The Effect of Dividend Policy on Changes in Stockholders, Wealth", A PhD Thesis, Graduate School of Saint Louis University, 1982, USA.

9) Ogden, J. P., "A Dividend Payment Effect in Stock Returns", *Financial Review,* Vol. 29, 1994, pp. 345-369.

10) Rahman, M. M., "Trend Analysis and Mathematical Modeling of General Index of Dhaka Stock Exchange", M.Sc. Thesis, Mathematics Discipline, Khulna University, 2009, pp. 64-73.

11) Schwert, G. W. and Stambaugh, R., "Expected Stock Returns and Volatility", *Journal of Financial Economics,* Vol. 19, 1987.

12) Stevens, J. L. and Jose, M. L., "The Effect of Dividend Payout, Stability, and Smoothing on Firm Value", *Journal of Accounting Auditing & Finance,* Vol. 7, 1992, pp.195-216.