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BUCKLING AND VIBRATION OF AN ANNULAR PLATE WITH EXPONENTIALLY VARYING THICKNESS AND DENSITY

By

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Abstract:

The natural frequencies of an annular plate of exponentially varying thickness under the action of a hydrostatic in-plane force have been studied on the basis of the classical theory of plates. The governing differential equation has been obtained and solved. The effects of in-plane force parameter, radii ratio and taper constants on the frequency parameter have been investigated for two different boundary conditions. Critical buckling loads have been computed for different values of taper constant and radii ratio for both the plates.

Keywords and phrases : *annular plate, critical buckling loads, taper constant*.

বিমূর্ত সার (*Bengali version of the Abstract*)

প্লেটের মৌলিক তত্ত্বের উপর ভিত্তি করে উদ্‌স্থৈতিক সমতলস্থ বলের ক্রিয়াধীন সূচকীয় ভেদের পুরুত্বের বলয়াকৃতি প্লেটের স্বাভাবিক কম্পনাংককে অনুসন্ধান করা হয়েছে। নিয়ন্ত্রক অন্তরকলন সমীকরণ এবং ইহার সমাধান নির্ণয় করা হয়েছে।দুটি ভিন্ন প্রান্তিক সর্তের জন্য কম্পনাংক প্রাচলের (Parameter) উপর সমতলস্থ বল প্রাচল, অরীয় (ব্যাসার্ধ) অনুপাত এবং টেপার ধ্রুবকগুলির প্রভাব অনুসন্ধান করা হয়েছে। টেপার ধ্রুবক এবং অরীয় অনুপাতগুলির বিভিন্ন মানের জন্য উভয় প্লেটের ক্ষেত্রে ক্রান্তিক আয়তাংক ভার (Critical bulking load) গণনা করা হয়েছে।

1. Introduction:

In the past years there has been growing interest in the study of buckling and vibration of plates of non-uniform thickness because of their applications in various engineering structure. Circular annular plates are extensively used as structural elements in the construction of aircrafts, ships, automobiles and other vehicles. Annular plates used in naval and aerospace structures are often subjected to in-plane forces. Soni and Amba-Rao[9] studied the axisymmetric vibrations of annular plates of variable thickness. Rosen and Libai[8] analysed the transverse vibrations of uniformly compressed annular plate free at the inner and simply supported at the outer boundary. Gupta and Lal [4] investigated the buckling and vibration of circular annular plates of paraebolically varying thickness.

The object of the present work is to extend the work of Gupta and Lal [4]. Here authors investigate the effect of an in-plane force on the frequency parameter of thin annular circular plate of exponentially varying thickness on the basis of classical theory. For axisymmetric motions, the governing fourth order linear differential equation with variable coefficients has been solved by Frobenius method. Frequencies for the first mode of vibration have been computed for two different boundary conditions and for various values of in-plane force parameter, taper constant. By allowing the frequency to approach zero, the critical buckling loads have also been determined.

2. Equation of motion and Solution:

The small deflection of a thin circular plate of radius a, thickness $h(r)$ and density $\rho(r)$ in the presence of in-plane forces is governed by the equation (Jain [6])

$$D\frac{\partial^4 w}{\partial r^4}+\frac{2}{r}\left(D+r\frac{\partial D}{\partial r}\right)\frac{\partial^3 w}{\partial r^3}+\frac{1}{r^2}\left\{-D+(2+\nu)r\frac{\partial D}{\partial r}+r^2\left(\frac{\partial^2 D}{\partial r^2}-N\right)\right\}\frac{\partial^2 w}{\partial r^2}$$
$$+\frac{1}{r^3}\left\{D-r\frac{\partial D}{\partial r}+r^2\left(\nu\frac{\partial^2 D}{\partial r^2}-N\right)\right\}\frac{\partial w}{\partial r}+\rho h\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

where w is the transverse deflection, N is the uniform in-plane tensile force, D is the flexural rigidity and other symbols have their usual meanings.

The thickness variation can be of any type (Conway [1], Gupta and Lal[4], Tomar and Gupta[10], Gallegojuarez[2]). For free transverse vibration of the plate, we consider $\frac{w}{a} = W(x)\exp(i\omega t)$.

If the thickness of the plate varies exponentially in the redial direction then, $h/a = h_0 e^{nR}$. The density also varies exponentially as $\rho/a = \rho_0 e^{2nR}$, where $R = r/a$.

By the use of these non-dimensional quantities, equation (1) now reduces to,

$$\frac{d^4W}{dR^4} + (\frac{2}{R} + B_1)d\frac{d^3W}{\partial R^3} + (-\frac{1}{R^2} + \frac{B_2}{R} + B_3)\frac{d^2W}{dR^2} + (\frac{1}{R^3} + \frac{B_4}{R^2} + \frac{B_5}{R})\frac{dW}{dR} - \Omega^2 W = 0 \tag{2}$$

where, $\Omega^2 = \frac{12\alpha^2 p^2 \rho_0}{H_0^2}$,

$B_1 = 6n,\ \ B_2 = 3n(1+\nu),\ \ B_3 = 9n^2 - \overline{N},\ \ B_4 = -3n,\ \ B_5 = 9n^2\nu - \overline{N}$,

$\overline{N} = N/aD_0$, also $a = \text{inner redius}, b = \text{outer rediusand and } \varepsilon = \text{radii ratio}$.

3. Solution:

A series solution for W is assumed in the form,

$$W = \sum_{k=0}^{\infty} a_k R^{c+k},\ \ a_0 \neq 0 \tag{3}$$

By Frobenius method the series solution of equation (2) is given by,

$$W = AF_1 + BF_2 \tag{4}$$

where,

$$F_1 = a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4 + a_5R^5 + a_6R^6 + a_7R^7 + ...$$

$$F_2 = F_1\log(R) + b_1R + b_3R^3 + b_4R^4 + b_5R^5 + b_6R^6 + b_7R^7 + ...$$

For equation (2) the indicial roots are 0, 0, 2 and 2. a_2 becomes indeterminate for $c = 0$ .Also

$$a_1 = -\frac{\{B_1(c-1)(c-2) + B_2(c-1) + B_4\}c}{(c+1)^2(c-1)^2} a_0,$$
$$a_3 = -\frac{\{B_1(c+1)c + B_2(c+1) + B_4\}(c+2)}{(c+3)^2(c+1)^2} a_2 - \frac{(B_2 c + B_4)(c+1)}{(c+3)^2(c+1)^2} a_1$$

The remaining coefficients of F_1 are determined in terms of a_0 and a_2 by the following recurrence relation:

$$a_{m+4}(c+m+4)(c+m+2) + \{B_1(c+m+2)(c+m+1) + B_2(c+2+m) + B_4\}(c+m+3)a_{m+3} + \{B_3(c+m+1) + B_5\}(c+m+2)a_{m+2} - \Omega^2 a_m = 0$$

and $b_i = \frac{\partial a_i}{\partial c}$, $i = 1,2,3,4,5,6,7$................

4. Boundary Conditions:

The following two sets of boundary conditions have been considered:

(i) Both the inner and outer edges are clamped (C-C).

(ii) Clamped at the inner and simply supported at the outer edge (C-S).

The edge which is clamped satisfies the condition, $W = \frac{dW}{dR} = 0$ and the simply supported edge satisfies the condition, $W = (\frac{d^2W}{dR^2} + \frac{\nu}{R}\frac{dW}{dR}) = 0$.

Applying these boundary conditions to the equation (4), we get, the frequency equation after eliminating the arbitrary constants.

5. Numerical Calculations:

Critical values of $\overline{N}$ have been computed for various taper constant n and the radii ratio ε . The effect of taper constant n, the radii ratio ε and the in-plane force parameter $\overline{N}$ on the frequencies have been investigated for $\nu = 0.3$.

TABLE 1

Comparison of frequency parameter Ω, for $n = 0$, $\overline{N} = 0$ and $\nu = 0.3$.

Boundary condition			**Obtained by present authors**	**Obtained by Gupta and Lal[4]**	**Obtained by Leissa[7]**
C-C	$\varepsilon = 0.3$	First mode	45.3462	45.3457	45.2
		Second mode	125.362	125.3502	125
	$\varepsilon = 0.5$	First mode	89.2508	89.2507	89.2
		Second mode	246.199	246.3235	246
C-S	$\varepsilon = 0.3$	First mode	29.9777	29.9783	29.9
		Second mode	100.423	100.4211	100
	$\varepsilon = 0.5$	First mode	59.8199	59.82	59.8
		Second mode	198.054	198.0512	198

TABLE 2

Values of $\overline{N}$ for the critical buckling load in compression for $\nu = 0.3$.

Boundary condition		$\varepsilon = 0.2$	$\varepsilon = 0.3$	$\varepsilon = 0.4$	$\varepsilon = 0.5$	$\varepsilon = 0.6$	$\varepsilon = 0.7$	$\varepsilon = 0.8$	$\varepsilon = 0.9$
C-C	$n = 0$	-63.20	-73.29	-77.79	-83.61	-80.41	-68.27	-58.18	-49.01
	$n = 0.1$	-80.85	-70.88	-72.27	-74.46	-80.93	-67.58	-57.41	-49.35
C-S	$n = 0$	-28.73	-37.89	-51.38	-63.40	-76.92	-65.82	-56.95	-48.63
	$n = 0.1$	-28.14	-37.16	-49.68	-58.26	-85.15	-65.47	-56.13	-48.69

TABLE 3

Values of $\overline{N}$ for the critical buckling load in compression $\overline{N}$ for $\nu = 0.3$ and $\varepsilon = 0.5$

Boundary condition	$n = 0.1$	$n = 0.2$	$n = 0.3$	$n = 0.4$	$n = 0.5$	$n = 0.6$	$n = 0.7$	$n = 0.8$	$n = 0.9$
C-C	-74.46	-70.14	-67.76	-67.12	-70.14	-65.68	-45.54	-41.27	-40.18
C-S	-58.26	-55.29	-53.37	-52.69	-55.31	-46.81	-37.92	-35.39	-35.54

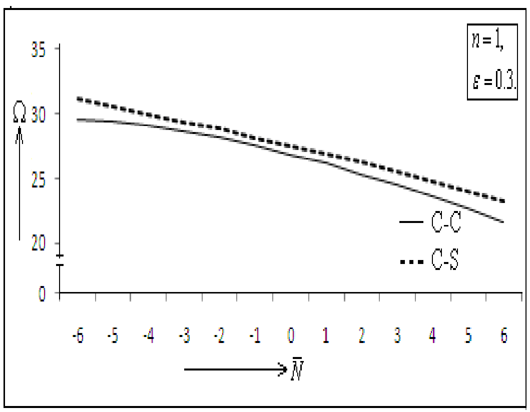

Figure1: Variation of Ω with varying $\overline{N}$ for $n = 1$, for first mode.

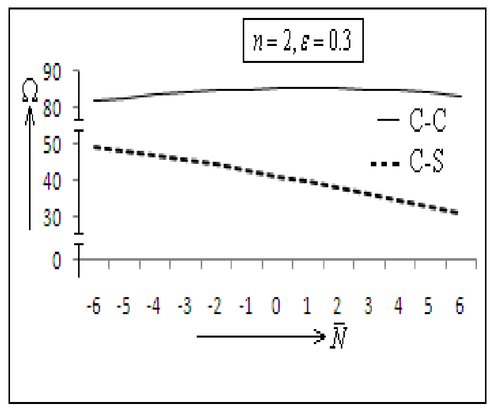

Figure2: Variation of Ω with varying $\overline{N}$ for $n = 2$, for first mode.

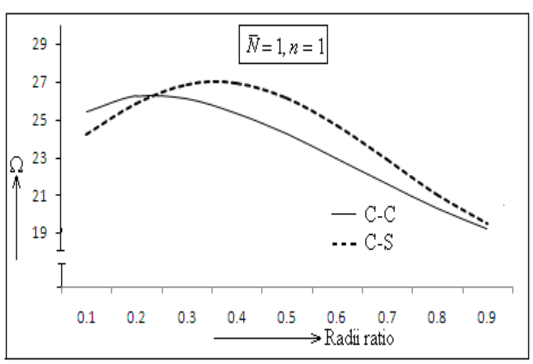

Figure 3: Variation of Ω with varying ε for first mode.

6. Conclusion:

Frequency equations are solved for plates having different thickness and density. In the present work, the effect of in-plane force parameter, the radii ratio and the taper constant on the frequency parameter have been investigated. Figure 1 and figure 2 show the variation of frequency parameter with $\overline{N}$. From these two figures (Fig. 1 and Fig. 2) we observe that for C-C plate, the frequency parameter Ω initially increases and then gradually decreases but for C-S plate, the frequency parameter Ω gradually decreases. For $n=1$ the values of frequency parameter for C-C plate are lesser as compared to that for C-S plate. But for $n=2$ the values of frequency parameter for C-C plate is higher then that of C-S plate. Figure 3 shows the variation of frequency parameter with radii ratio for $n=1$ and $\overline{N}=1$. From figure 3 we see that the frequency of the C-C plate is initially higher then the C-S plate, but after a certain point frequency of the C-S plate become higher then the C-C plate.

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