ISSN 0973-8975

J.Mech.Cont.& Math. Sci., Vol.-7, No.-1, July (2012) Pages 931-940

SOME FEATURES OF α-R_0 SPACES IN SUPRA FUZZY TOPOLOGY

By

[1]M. F. Hoque, [2]R. C. Bhowmik, [3]M. R. Kabir, and [4]D. M. Ali

[1]Dept. of Mathematics, Pabna Science and Technology University, Pabna, Bangladesh;

[2]Dept. of Mathematics, Pabna Science and Technology University, Pabna, Bangladesh;

[3]Dept. of Mathematics, Pabna Science and Technology University, Pabna, Bangladesh

[4]Department of Mathematics, Rajshahi University, Rajshahi, Bangladesh

Abstract

This paper introduce and study four concepts of R_0 supra fuzzy topological spaces. We have shown that all these four concepts are 'good extension' of the corresponding concepts of R_0 topological spaces and established relations among them. It has been proved that all the definitions are hereditary, productive and projective. Further some other properties of these concepts are studied.

Keywords and phrases : fuzzy set, topological spaces, supra fuzzy topological spaces.

বিমূর্ত সার (***Bengali version of the Abstract***)

R_0- সুপ্রা ফাজি টপোলজীয় দেশের (R_0 supra fuzzy topological spaces.) চারটি ধারণাকে এই পত্রে উপস্থাপন এবং ভাল ভাবে বিচার বিশ্লেষণ করা হয়েছে । আমরা দেখিয়েছি যে এই চারটি ধারণা হচ্ছে R_0- টপোলজীয় দেশের অনুসঙ্গী ধারণার 'ভাল সংযোজন ' এবং ইহাদের মধ্যে পারস্পরিক সম্পর্ক প্রতিষ্ঠিত করেছি । এটা প্রমাণ করা হয়েছে যে সব সংজ্ঞাগুলিই উত্তরাধিকার সূত্রে প্রাপ্ত , প্রসারণশীল এবং অভিক্ষেপক । অধিকন্তু এই সকল ধারণার আরও কিছু ধর্মকে ভাল ভাবে বিচার বিশ্লেষণ করেছি ।

1. Introduction

The fundamental concept of fuzzy set was introduced first by Zadeh [10] in 1965. Later Chang [4] and Lowen [6] developed the theory of fuzzy topological spaces in the sense of Zadeh. A large number of research papers have been published dealing with various aspects of such spaces. In 1983, Mashhour et al. [7] introduced supra topological spaces and studied s-continuous functions and s*-continuous functions. Abd EL-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and characterized a number of basic concepts. Hossain and Ali [5] generalized on R_0 and R_1 fuzzy topological spaces. The aim of this paper is to introduce α-R_0 supra fuzzy topological spaces and study their basic properties. Here $I=[0,1]$ and $I_1=[0,1)$ have their usual meaning.

1.1 Definition(10): For a set X, a function $u: X \to [0,1]$ is called a fuzzy set in X. For every $x \in X$, u(x) represents the grade of membership of x in the fuzzy set u. Some authors say that u is a fuzzy subset of X.

1.2 Definition(4): Let X and Y be two sets and $f: X \to Y$ be a function. For a fuzzy subset u in X, we define a fuzzy subset v in Y by

$$v(y) = \sup \{ u(x) \} \text{ if } f^{-1}[\{y\}] \neq \phi, \ x \in X$$
$$= 0 \text{ otherwise .}$$

and the inverse image of v under f is the fuzzy subset $f^{-1}(v) = v_o f$ in X is defined by

$$f^{-1}(v)(x) = v(f(x)) \text{ , for } x \in X.$$

1.3 Definition(4): Let $I=[0,1]$, X be a non empty set and I^X be the collection of all mappings from X into I, i.e. the class of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^X, satisfying the following conditions:

(i) $1, 0 \in t$,

(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t$.

(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

The pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t- open (or simply open) fuzzy sets. A fuzzy set v is called a t-closed (or simply closed) fuzzy set if 1-v∈t.

1.4 Definition[6] : A fuzzy topology on a nonempty set X is a collection t of fuzzy subsets of X such that

(i) all constant fuzzy subsets of X belong to t.

(ii) t is closed under formation of fuzzy union of arbitrary collection of members of t.

(iii) t is closed under formation of fuzzy intersection of finite collection of members of t.

1.5 Definition[1]: Let X be a nonempty set. A subfamily t^* of I^X is said to be a supra fuzzy topology on X if and only if

(i) $1, 0 \in t^*$,

(ii) If $u_i \in t^*$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological space. The elements of t^* are called supra open fuzzy sets in (X, t^*) and complement of a supra open fuzzy set is called a supra closed fuzzy set. If (X, t) be a fuzzy topological space and t^* be a supra fuzzy topology on X. Then t^* is called a supra fuzzy topology associated with t if $t \subset t^*$.

1.6 Definition[8]: Let (X, t) and (X, s) be two topological spaces. Let t^* and s^* are associated supra topologies with t and s respectively and $f : (X, t) \longrightarrow (Y, s)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each i.e., if for any $v \in s^*$, $f^{-1}(v) \in t^*$, the function f is called supra fuzzy homeomerphic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous.

1.7 Definition[8]: Let (X, t^*) and (Y, s^*) be two supra topological spaces. If u_1 and u_2 are two supra fuzzy subsets of X and Y respectively then the Cartesian product $u_1 \times u_2$ of two fuzzy subsets u_1 and u_2 is a supra fuzzy subsets of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min(u_1(x), u_2(y))$, for each pair $(x, y) \in X \times Y$.

1.8 Definition[9]**:** Suppose $\{ X_i , i \in \Lambda \}$, be any collection of sets and X denoted the Cartesian product of these sets, ie $X = \Pi_{i\in\Lambda} X_i$. Here X consists of all points $p = \langle a_i , i \in \Lambda \rangle$, where $a_i \in X_i$. For each $j_o \in \Lambda$, we define the projection $\pi_{jo} : X \longrightarrow X_{jo}$ by $\pi_{jo} (\langle a_i : i \in \Lambda \rangle) = a_{jo}$. These projections are used to define the product supra topology.

1.9 Definition[10] **:** Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f : X \longrightarrow R$ is lower semi continuous if and only if $\{x \in X : f(x) > \alpha\}$ is open for all $\alpha \in R$.

1.10 Definition[10] **:** Let (X, T) be a topological space and T^* be associated supra topology with T. Then the lower semi continuous topology on X associated with T^* is $\omega(T^*) = \{\mu : X \to [0,1],\ \mu \text{ is supra lsc}\}$. We can easily show that $\omega(T^*)$ is a supra fuzzy topology on X.

Let P be the property of a supra topological space (X, T^*) and FP be its supra fuzzy topological analogue. Then FP is called a 'good extension' of P " if and only if the statement (X, T^*) has P if and only if $(X , \omega(T^*))$ has FP " holds good for every topological space (X, T).

2. α-R_0 spaces in supra fuzzy topology

2.1 **Definition:** Let (X, t^*) be a supra fuzzy topological space and $\alpha \in I_1$. Then

(a) (X, t^*) is an α –R_0 (i) space if and only if for all x , y $\in$ X with $x \neq y$, whenever there exist $u \in t^*$ with u (x) =1 and u (y)$\leq \alpha$, then there exist $v \in t^*$ with v (x) $\leq \alpha$ and v (y) = 1.

(b) (X, t^*) is an α –R_0 (ii) space if and only if for all x, y $\in$ X , $x \neq y$, whenever $u \in t^*$ with u (x) = 0 and u (y) > α , then there exists $v \in t^*$ with v (x) > α and v (y) = 0 .

(c) (X, t^*) is an α-R_0 (iii) space if and only if for all x, y $\in X$ with $x \neq y$, whenever there exists $u \in t^*$ with $0 \leq u(x) \leq \alpha < u(y) \leq 1$, then there exists $v \in t^*$ with $0 \leq v(y) \leq \alpha < v(x) \leq 1$.

(d) (X, t^*) is an R_0 (iv) space if and only if for all x, y $\in X$ with $x \neq y$, whenever there exists $u \in t^*$ with $u(x) < u(y)$, then there exists $v \in t^*$ with $v(x) > v(y)$.

The following examples show that $\alpha - R_0$ (i), α- R_0 (ii), $\alpha -R_0$ (iii) and R_0 (iv) are all independent.

2.2 Example: Let X = {x, y} and u, v $\in I^X$ are defined by u (x) = 1, u (y) = 0 and v (x) = 0.45, v (y) = 1. Consider the supra fuzzy topology t^* on X generated by {0, u, v, 1, Constants}. Then by definition, for $\alpha = 0.55$, (X, t^*) is $\alpha - R_0$(i), but (X, t^*) is not $\alpha - R_0$ (ii).

2.3 Example: Let X = {x, y} and u, v $\in I^X$ are defined by u (x) = 0, u (y) =1 and v (x) = 0.73, v (y) = 0. Consider the supra fuzzy topology t^* on X generated by {0, u, v, 1, Constants}. Then by definition, for $\alpha = 0.63$, (X, t^*) is $\alpha - R_0$ (ii), but (X, t^*) is not $\alpha - R_0$ (i).

2.4 Example: Let X = { x , y } and u, v $\in I^X$ are defined by u (x) =1 , u (y) = 0 and v (x) = 0.32, v (y) = 0.69. Consider the supra fuzzy topology generated by {0, u, v, 1, Constant}. Then by definition, for $\alpha = 0.52$, (X, t^*) is α-R_0 (iii), but (X, t^*) is not $\alpha - R_0$ (i) and (X, t^*) is not $\alpha - R_0$(ii).

2.5 Example: Let X = { x , y } and u, v $\in I^X$ are defined by u (x) = 1 , u (y) = 0 and v (x) = 0.24, v (y) = 0.48. Consider the supra fuzzy topology t^* on X generated by {0, u, v, 1, Constants}. Then by definition , for α= 0.53, (X, t^*) is $\alpha - R_0$ (iv), but (X, t^*) is not $\alpha - R_0$ (i), (X, t^*) is not $\alpha - R_0$ (ii), and (X, t^*) is not $\alpha - R_0$ (iii).

2.6 Example: Let X = { x ,y , z} and u , v, w $\in I^X$ are defined by u (x) = 1 , u (y) =1, u (z) = 0 and v (x) = 0, v (y) = 0, v (z) =1 and w (x) = 0.92 , w (y) = 0.52 , w (z) = 0. Consider the fuzzy topology t^* on X generated by { 0 , u ,v , w , 1, Constants} .Then for $\alpha = 0.63$, it can easily shown that (X, t^*) is $\alpha - R_0$ (i) and (X, t^*) is $\alpha - R_0$ (ii) . But we observe (X, t^*) is not $\alpha - R_0$ (iii), and (X, t^*)

is not R_0 (iv), Since w (x) > $\alpha \geq$ w (y) but there does not exist $q \in t^*$ such that q (x) $\leq \alpha <$ q (y).

2.7 Example: Let X = { x , y , z } and u , v $\in I^X$ are defined by u (x) = 0.83 , u (y) = 0.41 , u (z) =0.36, and v(x) = 0.36, v (y) = 0.83 , v (z) =0.24. Consider the supra fuzzy topology t^* on X generated by {0, u, v, 1, Constants}. Then by definition, for α =0.5, (X, t^*) is $\alpha - R_0$(iii), but (X, t^*) is not R_0 (iv), since u (y) > u (z) but we have no $q \in t^*$ such that q (y) < q(z) .

2.8 Theorem: (X, t^*) is 0- R_0 (ii) if and only if (X , t^*) is 0- R_0 (iii).

Proof: The proof is trivial.

2.9 Theorem: Let (X, t^*) be a supra fuzzy topological space and $I_\alpha(t^*) = \{u^{-1}(\alpha, 1] / u \in t^*\}$. Then the following is true:

(a) (X , t^*) is $\alpha - R_0$ (iii) space if and only if (X , $I_\alpha(t^*)$) is R_0 space.

(b) if (X , t^*) is $\alpha - R_0$ (i) space, then (X , $I_\alpha(t^*)$) is not R_0 space and conversely.

(c) if (X , t^*) is α - R_0 (ii) space then (X , $I_\alpha(t^*)$) is not R_0 space and conversely.

(d) if (X , t^*) is R_0 (iv), then (X , $I_\alpha(t^*)$) is not R_0 space and conversely.

Proof: Let (X, t^*) be $\alpha - R_0$ (iii). We have to prove that (X, $I_\alpha(t^*)$) is R_0. Let x , y $\in$ X with x $\neq$ y and M $\in I_\alpha(t^*)$ with x $\in$ M , y $\notin$ M or x $\notin$M , y $\in$ M . Suppose that x $\in$ M, y $\notin$ M. We can write, M = $u^{-1}(\alpha, 1]$, for some $u \in t^*$. Then we have u (x) > α, u (y) $\leq \alpha$, i.e., $0 \leq u(y) \leq \alpha < u(x) \leq 1$. Since (X , t^*) is $\alpha - R_0$(iii), $\alpha \in I_1$,then there exists $v \in t^*$ such that $0 \leq v(x) \leq \alpha < v(y) \leq 1$, i.e., v (x) $\leq \alpha$, v (y) > α . It follows that x $\notin v^{-1}(\alpha, 1]$, y $\in v^{-1}(\alpha, 1]$ and also $v^{-1}(\alpha, 1] \in I_\alpha(t^*)$. Thus (X, $I_\alpha(t^*)$) is R_0.

Conversely, suppose that (X , $I_\alpha(t^*)$) is R_0 . We have to prove that (X, t^*) is $\alpha - R_0$ (iii). Let x , y $\in$ X with x $\neq$ y and $u \in t^*$ with $0 \leq u(x) \leq \alpha < u(y) \leq 1$, i.e., u (x) $\leq \alpha$, u (y) > α , it follows that x $\notin u^{-1}(\alpha, 1]$, y $\in u^{-1}(\alpha, 1]$, and $u^{-1}(\alpha, 1] \in I_\alpha(t)$, for every $u \in t^*$. Since (X , $I_\alpha(t^*)$) is R_0, then there exists M $\in I_\alpha(t^*)$ such that x $\in$ M , y $\notin$ M . We can write M = $v^{-1}(\alpha, 1]$, where $v \in t^*$, it follows that $v(x) > \alpha$, $v(y) \leq \alpha$, ie $0 \leq v(y) \leq \alpha < v(x) \leq 1$. Thus (X, t^*) is $\alpha - R_0(t^*)$, i.e., (a) is proved.

2.10 Example: Let $X = \{x, y, z\}$ and $u, v \in I^X$ are defined by $u(x) = 1$, $u(y) = 0$, $u(z) = 0.8$ and $v(x) = 0$, $v(y) = 1$, $v(z) = 0.7$. Consider the supra fuzzy topology t^* on X generated by $\{0, u, v, 1, \text{Constants}\}$. Then for $\alpha = 0.6$, we have, (X, t^*) is $\alpha - R_0$ (i) . Now, $I_\alpha(t^*) = \{X, \Phi, \{x, z\}, \{y, z\}, \{z\}\}$. It is observed that (X, t^*) is not R_0 space , since $y, z \in X$, $y \neq z$ and $\{x, z\} \in I_\alpha(t^*)$, with $z \in \{x, z\}$, $y \notin \{x, z\}$, but no such $U \in I_\alpha(t^*)$ with $x \notin U$, $y \in U$.

2.11 Example: Let $X = \{x, y, z\}$ and $u, v \in I^X$ are defined by $u(x) = 0.3$, $u(y) = 0$, $u(z) = 0.8$, and $v(x) = 0.8$, $v(y) = 1$, $v(z) = 0$. Consider the supra fuzzy topology t^* on X generated by $\{0, u, v, 1, \text{Constants}\}$. Then, for $\alpha = 0.5$, we have, (X, t^*) is $\alpha - R_0$ (ii) and (X, t^*) is also R_0 (iv) . Now $I_\alpha(t^*) = \{X, \Phi, \{z\}, \{y\}, \{y, z\}\}$. It is observed that $(X, I_\alpha(t^*))$ is not R_0 space, since $x, y \in X$, $x \neq y$ and $\{y\} \in I_\alpha(t^*)$ with $x \notin \{y\}$, $y \in \{y\}$, but no such $U \in I_\alpha(t^*)$ with $x \in U$, $y \notin U$.

2.12 Example: Let $X = \{x, y\}$ and $u, v, w \in I^x$ are defined by $u(x) = 1$, $u(y) = 0$, $v(x) = 0.4$, $v(y) = 0.9$, $w(x) = 0.7$, $w(y) = 0.3$. Consider the supra fuzzy topology t^* on X generated by $\{0, u, v, w, 1, \text{Constants}\}$. Then for $\alpha = 0.6$, we have, (X, t^*) is not $\alpha - R_0$ (i) and (X, t^*) is not $\alpha - R_0$ (ii). Now, $I_\alpha(t^*) = \{X, \Phi, \{x\}, \{y\}\}$. Then we see that $I_\alpha(t^*)$ is a topology on X and $(X, I_\alpha(t^*))$ is R_0 .

2.13 Example: Let $X = \{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.4$, $u(y) = 0.5$, $v(x) = 0.3$, and $v(y) = 0.4$. Consider the supra fuzzy topology t^* on X generated by $\{0, u, v, 1, \text{Constant}\}$. Then, for $\alpha = 0.6$, we have (X, t^*) is not $\alpha - R_0$ (iv). Now, $I_\alpha(t^*) = \{X, \Phi\}$. Then $I_\alpha(t^*)$ is a topology on X and $(X, I_\alpha(t^*))$ is R_0 .

Hence the proof is complete.

2.14 Theorem: Let (X, T^*) be a supra topological space. Then (X, T^*) is R_0 , if and only if $(X, w(T^*))$ is $\alpha - R_0(p)$, where p = i , ii , iii , iv .

Proof: Let (X, w (T^*)) be α –R_0 (i). Let x, y $\in$ X with x $\neq$ y and U $\in$ T^* with x $\in$ U, y $\notin$ U. But $1_U \in$ w (T^*) and 1_U (x) = 1, 1_U (y) = 0. Now, we have $1_U \in$ w (T^*) with 1_U (x) = 1, 1_U (y) $\leq \alpha$. Since (X, w (T^*)) is α –R_0 (i) , there exists v $\in$ w (T^*) such that v (x) $\leq \alpha$, v (y) = 1. Then x $\notin$ v^{-1}(α , 1] , y $\in$ v^{-1} (α , 1] as v (x) $\leq$ α , v (y) = 1 and also there exists v^{-1} (α , 1] $\in$ T^*. Thus (X, T^*) is R_0 – space.

Conversely, suppose that (X, T^*) be a R_0 –space. We have to prove that (X, w (T^*)) is α – R_0(i). Let x, y $\in$ X with x $\neq$ y and there exists u $\in$ w (T^*) such that u (x) = 1 , u (y) $\leq \alpha$. Then x $\in$ u^{-1}(α, 1], y $\notin$ u^{-1} (α, 1] as u (x) =1, u (y) $\leq \alpha$. Hence u^{-1} (α, 1] $\in$ T^*. Since (X, T^*) is R_0, then there exists V $\in$ T^* such that x $\notin$ V, y $\in$ V, but 1_V $\in$ w (T^*) and 1_V (x) = 0, 1_V (y) = 1, i.e., there exists $1_V \in$ w (T^*) such that 1_v (x) $\leq \alpha$, 1_V (y) = 1 .Thus (X , w (T^*)) is α – R_0(i) .

Hence (X , T^*) is R_0 if and only if (X , w (T^*)) is α – R_0(i).

In the same way, we can prove that

(a) (X , T^*) is R_0 if and only if (X , w (T^*)) is α – R_0(ii).

(b) (X , T^*) is R_0 if and only if (X , w (T^*)) is α – R_0(iii).

(c) (X , T^*) is R_0 if and only if (X , w (T^*)) is R_0(iv)

Thus it is seen that α – R_0 (p) is a good extension of its topological counter part (p = i, ii, iii, iv).

2.15 Theorem: Let (X , t^*) be a supra fuzzy topological space and A$\subseteq$ X , t^*_A= { u /A : u $\in$ t^*}, then

(a) (X , t^*) is an α –R_0 (i) if and only if (A , t^*_A) is an α – R_0 (i) .

(b) (X , t^*) is an α –R_0 (ii) if and only if (A , t^*_A) is an α – R_0 (ii).

(c) (X , t^*) is an α –R_0 (iii) if and only if (A , t^*_A) is an α – R_0 (iii)

(d) (X , t^*) is an R_0(iv) if and only if (A , t^*_A) is an R_0 (iv).

Proof: Suppose that (X, t^*) is α – R_0 (i). Then for x , y $\in$ A , with x $\neq$ y and u $\in$ t^*_A such that u (x) =1, u (y) $\leq \alpha$, then also x , y $\in$ X , x $\neq$ y. But we can write u = w/A, where w $\in$ t^* and hence w (x) =1, w (y) $\leq \alpha$. Since (X , t^*) is α –R_0 (i), then there exists m $\in$ t^* such that m (x) $\leq \alpha$, m (y) = 1. But from the definition

$m/A \in t^*_A$, for every $m \in t^*$ and $m/A(x) \le \alpha$, $m/A(y) = 1$. Thus (A, t^*_A) is $\alpha - R_0$ (i) . i.e., (a) proved.

Similarly (b), (c) and (d) can be proved.

2.16 Theorem: Given (X_i, t^*_i), $i \in \Lambda$ be supra fuzzy topological spaces and $X = \prod_{i \in \Lambda} X_i$ and t^* be a product supra fuzzy topology on X . Then

(a) $\forall\, i \in \Lambda$, (X_i, t^*_i) is $\alpha - R_0$(i) if and only if (X, t^*) is $\alpha - R_0$(i) .

(b) $\forall\, i \in \Lambda$, (X_i, t^*_i) is $\alpha - R_0$(ii) if and only if (X, t^*) is $\alpha - R_0$(ii) .

(c) $\forall\, i \in \Lambda$, (X_i, t^*_i) is $\alpha - R_0$(iii) if and only if (X, t^*) is $\alpha - R_0$(iii) .

(d) $\forall\, i \in \Lambda$, (X_i, t^*_i) is R_0(iv) if and only if (X, t^*) is R_0(iv) .

Proof: Let (X_i, t^*_i), $i \in \Lambda$ be $\alpha - R_0$ (i) . We have to prove that (X, t^*) is $\alpha - R_0$ (i). Let $x, y \in X$, with $x \neq y$ and $u \in t^*$ such that $u(x) = 1$, $u(y) \le \alpha$. But we have $u(x) = \min\{ u_i(x_i) : i \in \Lambda\}$ and $u(y) = \min\{ u_i(y_i) : i \in \Lambda \}$ and hence we can find an $u_i \in t^*_i$ and $x_i \neq y_i$ such that $u_i(x_i) = 1$ and $u_i(y_i) \le \alpha$. Since (X_i, t^*_i), $i \in \Lambda$ is α- R_0 (i) , $\alpha \in I_1$, then there exist $v_i \in t^*_i$, such that $v_i(x_i) \le \alpha$, $v_i(y_i) = 1$. But $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$ and hence $v_i(\pi_i(x) \le \alpha$, $v_i(\pi_i(y)) = 1$. It follows that there exists $u_i o \pi_i \in t^*$ such that $(v_i o\, \pi_i)(x) \le \alpha$, $(v_i o\, \pi_i)(y) = 1$. Thus (X, t^*) is $\alpha - R_0$ (i).

Conversely, suppose that (X, t^*) is $\alpha - R_0$ (i). We have to prove that (x_i, t^*_i), $i \in \Lambda$, is α– R_0 (i). Let for some $i \in \Lambda$, a_i be a fixed element in X_i, suppose that $A_i = \{ x \in X = \prod_{i \in \wedge} X_i \ / \ x_j = a_j$ for some $i \neq j \}$. So that A_i is the subset of X , and this implies that $(A_i, t^*_{A\,i})$ is also the subspace of (X, t^*). Since (X, t^*) is $\alpha - R_0$(i) , then $(A_i, t^*_{A\,i})$ is also $\alpha - R_0$ (i) and A_i is a homeomorphic image of X_i. Thus (X_i, t^*_i) is $\alpha - R_0$ (i), i.e., (a) is proved.

Similarly, (b), (c) and (d) can be proved.

References

1) Abd EL –Monsef, M. E, and Ramadan, A. E.: On fuzzy supra topological spaces; Indian J. Pure and Appl.Math. 18 (4), 322-329, 1987.
2) Ali, D. M.: A note on T_0 and R_0 fuzzy topological spaces; Pro. Math. Soc. B. H. U. Vol. 3, 165-167, 1987.
3) Azad, K. K: On Fuzzy semi- continuity, Fuzzy almost continuity and Fuzzy weakly continuity; J. Math. Anal . Appl. 82(1), 14-32, 1981.
4) Chang, C. L.: Fuzzy topological spaces; J. Math. Anal Appl. 24, 182-192, 1968.
5) Hossain, M. S. and Ali, D. M.: On R_0 and R_1 fuzzy topological spaces; R U Studies Part-B J Sc. 33, 51-63, 2005.
6) Lowen, R.: Fuzzy topological spaces and fuzzy compactness; J. Math. Anal. Appl. 56, 621-633, 1976.
7) Mashhour, A. S., Allam, A. A., Mahmoud, F. S. and Khedr, F. H.: On fuzzy supra topological spaces; Indian J. Pure and Appl. Math. 14 (4), 502-510, 1983.
8) Ming, P. P., Ming. L. Y.: Fuzzy topology II. Product and Quotient Spaces; J. Math. Anal. Appl. 77, 20-37, 1980.
9) Wong, C. K: Fuzzy points and local properties of Fuzzy topology; J. Math. Anal . Appl. 46, 316-328, 1974.
10) Zadeh, L. A.: Fuzzy sets. Information and control, 8, 338-353, 1965.