

UNCHARGED PARTICLE TUNNELING FROM NON-ACCELERATING AND ROTATING BLACKHOLES WITH ELECTRIC AND MAGNETIC CHARGES

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Abstract

By applying Parikh-Wilczek's semi-classical tunneling method we obtain the emission rate of massless uncharged particle at the event horizon of non-accelerating and rotating blackhole with electric and magnetic charges. We consider the spacetime background dynamical and incorporate the self-gravitation effect of the emitted particles when energy conservation and angular momentum conservation are taken into account. We find that the emission rate at the event horizon is equal to the difference of Bekenstein-Hawking entropy before and after emission. We also find the Hawking temperature .

Keywords and phrases : uncharged particle, emission rate, self-gravitation effect, Bekenstein-Hawking entropy, Hawking temperature

বিমূর্ত সার (Bengali version of the Abstract)

বৈদ্যুতিক এবং চৌম্বক চার্জযুক্ত ত্বরান্বিত ও আবর্তিত কৃষ্ণ বিবরের ঘটনা সংঘটিত দিগন্তে ভরহীন চার্জহীন কণার বিকিরণের হারকে আমরা নির্ণয় করেছি পেরিখ - উইলচেঙ্ক - এর (Parikh-Wilczek) অর্ধ - চিরায়ত সুরঙ্গ পদ্ধতির মাধ্যমে । শক্তি সংরক্ষণ এবং কৌণিক ভরবেগ সংরক্ষণকে হিসাবের মধ্যে রেখে , দেশ - সময়কে গতিশীলতার পটভূমিতে এবং স্ব - মধ্যাকর্ষণের প্রভাবকে সংযুক্ত করে বিকিরিত কণাগুলিকে আমরা বিচার করেছি । আমরা দেখিয়েছি যে ঘটনা সংঘটিত দিগন্তে বিকিরণের হার বেকেনস্টাইন - হকিং এনট্রপির (Bekenstein-Hawking entropy) পূর্বে এবং পরের বিকিরণের পার্থক্য সমান । হকিং - এর উষ্ণতাকে ও আমরা নির্ণয় করেছি ।

1. Introduction:

The classical ‘no hair’ theorem stated that all the information about the collapsing body was lost except three conserved quantity: the mass, the angular momentum and the electric charge. So the only solutions of Einstein-Maxwell equations in four dimensions is the stationary and rotating Kerr-Newman blackhole solutions. In classical theory , the loss of information is not a serious problem since it could be thought that the information is preserved inside the blackhole but just not very accessible. Even , once Hawking thought that the loss of information never recovered. But recently he change his opinion about information loss paradox. However , taking quantum effect into consideration , the situation is changed due to Hawking discovery that blackholes radiates thermally[1]

Due to the emission of thermal radiation blackhole could loss energy, shrink and eventually evaporate away completely. Since the radiation with a precisely thermal spectrum carries no information , so the information carried by a physical system falling toward blackhole singularity has no way to be recovered after a blackhole has disappeared completely. This is known as so called “ information loss paradox”[2] which means that pure quantum states (the original matter that forms the blackhole) can evolve into mixed states (the thermal spectrum at infinity). This type of evolution violates the fundamental principle of quantum theory, as these prescribe a unitary time evolution of basis states[3].

The information loss paradox can perhaps be attributed to the semi-classical nature of the investigations of Hawking radiation. However, researches in string theory indeed support the idea that Hawking radiation can be described within a manifestly unitary theory, but it still remains a mystery how information is recovered. Although a complete resolution of the information loss paradox might be within a unitary theory

of quantum gravity or string/ M-theory , it is argued that the information could come out if the outgoing radiation were not exactly thermal but had subtle corrections[2].

There is some degree of mystery remains in the mechanism of blackhole radiation. In the original derivation of blackhole evaporations, Hawking described the thermal radiation as a quantum tunneling process created by vacuum fluctuation near the event horizon [4]. In this process , the radiation is like electron-positron pair creation in a constant electric field. The energy of a particle can change its sign after crossing the event horizon. So a pair created by vacuum fluctuations just inside or outside the horizon can materialize with zero total energy, after one member of the pair has tunneled to the opposite side. But in [1] Hawking did not proceed in this way. He considered the creation of a blackhole in the cotext of a collapse geometry, calculating the Bogoliubov transformations between the initial and final states of incoming and outgoing radiation. However , there were two difficulties to overcome this problem. The first was to find a well – behaved coordinate system at the event horizon. The second was where is the barrier.

Recently , a method to describe Hawking radiation as tunneling process was developed by Krause and Wilczek [5] and elaborated by Parikh and Wilczek[6,7,8,9]. This method involves calculating the imaginary part of the action for the (classically forbidden) process of s-wave emission across the horizon, which in turns is related to the Boltzmann factor for emission at the Hawking temperature. Using the (Wentzel-Kramers –Brillouin) WKB approximation¹ the tunneling probability for the classically forbidden trajectory of the s-wave coming from inside to outside the horizon is given by

$$\Gamma \propto \exp(-2 \text{Im} S) \tag{1}$$

where S is the classical action of the trajectory to leading order in \hbar (set equal to unity). Expanding the action in terms of the particle energy , the Hawking temperature is recovered at linear order. In other words for

$$2S = \beta E + O(E^2) \text{ this gives}$$

$$\Gamma \sim \exp(-2S) \cong \exp(-\beta E) \quad (2)$$

which is the regular Boltzmann factor for a particle of energy E and β is the inverse temperature of the horizon. Besides treating the Hawking radiation as a tunneling process Krause-Parikh-Wilczek also took the tunneling particles back reaction into account. They obtained the corresponding modified spectrum. The most interesting result was that they found this modified spectrum was implicitly consistent with the unitary theory and could support the conservation of information[5,6,7,8]. Following this tunneling method , there have been many generalizations , such as its application to other spacetimes. The Hawking radiation as tunneling from various spherically symmetric blackholes were found in [10,11,12,13,14,15,16,17,18,19,20,21,22,23]. Also , there are some attempts to extend this method to the case of stationary axisymmetric blackholes [24,25,26,27,28,29,30,31,32]. Recently, some people investigated the massive charged particles tunneling from the static spherically symmetric as well as stationary axisymmetric blackholes [33,34,35,36,37,38,39]. They all found a satisfying result.

[¹ For large values of the quantum numbers or of the masses of the particles in the system the quantum mechanics gives results closely similar to classical mechanics. For intermediate cases it is found that the old quantum theory often gives good results. It is therefore pleasing that there has been obtained an approximation method of solution of the wave equation based on an expansion the first term of which leads to the classical result , the second term to the old-quantum theory result , and the higher terms to corrections which bring in the effects characteristic of the new mechanics. This method is usually called the Wentzel-Kramers-Brillouin method (precisely the WKB approximation method)]

In this paper we follow the Krause-Parikh -Wilczek method to obtain the tunneling rate of the massless particles at the event horizon of a non accelerating and rotating blackholes with electric and magnetic charges. Our paper is organized as follows:

In section-2 we give the metric of non-accelerating and rotating blackhole with electric and magnetic charges. The horizon area and Bekenstein-Hawking entropy formula are also given in this section. In section-3 we introduce the dragging coordinate system in order to infinite red shift surface coincide with the event horizon surface , so that the geometrical optical limit can be applied. In section-4 we transform the metric in general Painleve-like transformation and obtain the equation of null geodesics. In section-5 we discuss the tunneling rate and a concluding remarks is given in section-6.

2 . Non-accelerating and rotating blackholes with electric and magnetic charge:

The Plebanski-Demianski [34,35,36] metric covers a large family of spacetimes which include, among others, the well known blackhole solutions like Schwartzschild, Reissner-Nordström , Kerr, Kerr-Newman, Kerr-NUT, Kerr-Newman-NUT and many others. Here we study a special case of this family – blackholes with rotation but non-accelerating with electric and magnetic charges. The metric of this such kind of blackhole is given by [37]

$$ds^2 = -\frac{(\Delta - a^2 \sin^2 \theta)}{\rho^2} dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\sin^2 \theta [(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta]}{\rho^2} d\phi^2 - \frac{2a \sin^2 \theta [(r^2 + a^2) - \Delta]}{\rho^2} dt d\phi \tag{3}$$

where $\Delta = r^2 + a^2 + e^2 + g^2 - 2Mr$, $\rho^2 = r^2 + a^2 \cos^2 \theta$. Here M is the mass of the blackhole, e and g are the electric and magnetic charges respectively, a is the angular momentum per unit mass. The event horizon equations are given by

$$\Delta = 0 \text{ which gives } r_{\pm} = M \pm \sqrt{M^2 - a^2 - e^2 - g^2} \quad (4)$$

The event horizon area of this blackhole is given by

$$A = 4\pi(r_+^2 + a^2) \quad (5)$$

and Bekenstein-Hawking entropy

$$S_{BH} = \frac{A}{4} = \pi(r_+^2 + a^2) = \pi[2M^2 + 2M\sqrt{M^2 - a^2 - e^2 - g^2} - e^2 - g^2] \quad (6)$$

3. Dragging coordinate system and infinite red- shift surface:

The infinite red shift surface is given by $g_{00} = 0$ which gives

$$r_{\pm} = M \pm \sqrt{M^2 - a^2 \cos^2 \theta - e^2 - g^2} \quad (7)$$

Obviously the infinite red shift surface does not coincide with the event horizon surface, which means that there is an energy layer exists between them. So the geometrical optical limit cannot be applied. Also there exist a frame dragging effect in the stationary rotating spacetime, the matter field in the ergosphere near the horizon must be dragged by the gravitational field also, so a reasonable physical picture should be depicted in the dragging coordinate system. This hints that we must transform the metric (3) into a dragging coordinate system.

$$\text{Let } \Omega = \frac{d\phi}{dt} = -\frac{g_{03}}{g_{00}} \quad (8)$$

where Ω is the angular velocity.

For the metric (3) we have,

$$g_{00} = \frac{-(\Delta - a^2 \sin^2 \theta)}{\rho^2}, \quad g_{11} = \frac{\rho^2}{\Delta}, \quad g_{22} = \rho^2, \quad g_{33} = \frac{\sin^2 \theta [(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta]}{\rho^2}$$

$$g_{03} = \frac{-a \sin^2 \theta [(r^2 + a^2) - \Delta]}{\rho^2}$$

From (8),
$$\Omega = \frac{d\phi}{dt} = \frac{a[(r^2 + a^2) - \Delta]}{(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta} \tag{9}$$

At the horizon the angular velocity becomes,

$$\Omega_h = \frac{a}{r_+^2 + a^2} \tag{10}$$

The line element (3) in the dragging coordinate system becomes,

$$ds^2 = \hat{g}_{00} dt^2 + g_{11} dr^2 + g_{22} d\theta^2 \tag{11}$$

where
$$\hat{g}_{00} = g_{00} - \frac{g_{03}^2}{g_{33}} = \frac{-\Delta \rho^2}{[(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta]} .$$

The line element (11) represents a 3-dimensional hypersurface of 4-dimensional spacetime. The infinite red-shift surface now coincide with the event horizon surface in the dragging coordinate system. So the geometrical optical limit can be applied now.

4. Painleve-like coordinate transformation and null geodesics:

To investigate the Hawking radiation as tunneling process it is necessary to eliminate coordinate singularity at the event horizon. In the expression (11) , there still exists coordinate singularity at the event horizon in the dragging coordinate system. So we continue performing a general Painleve coordinate transformation[38]. This transformation can be done by

$$dt \rightarrow dt + F(r, \theta) dr + G(r, \theta) d\theta \tag{12}$$

where $F(r, \theta)$ and $G(r, \theta)$ are two determined functions of r and θ , and satisfy the integrability condition,

$$\frac{\delta F(r, \theta)}{\delta \theta} = \frac{\delta G(r, \theta)}{\delta r} \tag{13}$$

Thus from (11) we obtain,

$$ds^2 = \hat{g}_{00} dt^2 + \left\{ \hat{g}_{00} F^2(r, \theta) + g_{11} \right\} dr^2 + \left\{ \hat{g}_{00} G^2(r, \theta) + g_{22} \right\} d\theta^2 + 2G(r, \theta) \hat{g}_{00} dt d\theta + 2F(r, \theta) G(r, \theta) \hat{g}_{00} dr d\theta + 2F(r, \theta) \hat{g}_{00} dt dr \tag{14}$$

We demand that constant time- slices are flat Euclidean space in radial. So we set,

$$\begin{aligned} \hat{g}_{00} F^2(r, \theta) + g_{11} &= 1 \\ \Rightarrow F(r, \theta) &= \pm \sqrt{\frac{1 - g_{11}}{\hat{g}_{00}}} \end{aligned} \tag{15}$$

From equation (13), $G(r, \theta) = \int \frac{\delta F(r, \theta)}{\delta \theta} dr + C(\theta)$ (16)

where $C(\theta)$ is an arbitrary analytic function of θ .

Substituting the value of $F(r, \theta)$ into equation (14) we get,

$$ds^2 = \hat{g}_{00} dt^2 + dr^2 + \left\{ \hat{g}_{00} G^2(r, \theta) + g_{22} \right\} d\theta^2 + 2\sqrt{\hat{g}_{00}(1 - g_{11})} G(r, \theta) dr d\theta + 2\hat{g}_{00} G(r, \theta) dt d\theta \pm 2\sqrt{\hat{g}_{00}(1 - g_{11})} dt dr \tag{17}$$

The positive sign (+) denotes the spacetime line element of the outgoing particle and the minus sign (-) denotes the spacetime line element of the ingoing particles at the horizon.

According to Landau's theory of the coordinate clock synchronization[45] in a spacetime decomposed in 3+1 dimension, the difference of coordinate times of two events taking place simultaneously in different place is

$$\Delta T = -\int \frac{g_{0i}}{g_{00}} dx^i \quad (i = 1,2,3) \quad (18)$$

If the simultaneity of coordinate clocks can be transmitted from one place to another and has nothing to do with the integration path, components of the metric should satisfy[46]

$$\frac{\delta}{\delta x^j} \left(-\frac{g_{0i}}{g_{00}} \right) = \frac{\delta}{\delta x^i} \left(-\frac{g_{0j}}{g_{00}} \right) \quad , \quad (i, j = 1,2,3) \quad (19)$$

Now the metric (17) in the new coordinate system , has a number of attractive features : (1) the metric is well-behaved at the event horizon; (2) the time coordinate t represents the local proper time for radially free-falling observers; (3) the hypersurfaces of constant time-slices are just flat Euclidean space in the oblate spheroidal coordinates; (4) by substituting the components of the metric (17) into equation (19), we see that the metric satisfy the Landau's condition of the coordinate clock synchronization $\frac{\delta F(r, \theta)}{\delta \theta} = \frac{\delta G(r, \theta)}{\delta r}$; (5) the infinite red-shift surface coincide with the event horizon surface so that the WKB approximation can be used. These attractive features are very advantageous for us to discuss Hawking radiation as tunneling and to do an explicit computation of the tunneling probability at the event horizon.

Now in order to investigate the tunneling process we evaluated the radial null geodesics described by equation (17) . Since the tunneling processes take place near the event horizon, so we may consider a particle tunneling from the event horizon as an ellipsoid shell . To conserve the symmetry of the spacetime , we think the particle should be still an ellipsoid shell during the tunneling process i.e. the particle does not

have motion in θ -direction[41]. Under these condition we obtain the radial null geodesics from equation (17) ($ds^2 = 0 = d\theta^2$) as

$$\dot{r} = \frac{[\pm\rho^2 - \sqrt{\rho^2(\rho^2 - \Delta)}]}{\sqrt{(r^2 + a^2)^2 - a^2\Delta\sin^2\theta}} \quad (20)$$

where a dot denotes differentiation with respect to t and the positive sign (+) represents an outgoing geodesics and the negative sign (-) represents an ingoing geodesics.

5. Tunneling process:

We adopt the picture of a pair of virtual particles spontaneously created just inside the horizon. The positive energy virtual particle can tunnel out and materialize as a real particle escaping classically to infinity. The negative energy particle is absorbed by the blackhole resulting in a decrease in the mass and the angular momentum of the blackhole. We consider the particle as an ellipsoid shell of energy ω and angular momentum ωa . When the particle's self-gravitation is taken into account, then equation (17) and (20) should be modified. To ensure the conservation of energy and angular momentum, we fix the total mass and angular momentum of the blackhole and allow the hole mass and angular momentum to fluctuate. When particle tunnels out, the blackhole mass and angular momentum will become $M - \omega$ and $a(M - \omega)$ respectively.

The shell of energy will move along the modified null geodesic in the radial direction

$$\dot{r} = \frac{[\pm\rho^2 - \sqrt{\rho^2(\rho^2 - \bar{\Delta})}]}{\sqrt{(r^2 + a^2)^2 - a^2\bar{\Delta}\sin^2\theta}} \quad (21)$$

where

$\bar{\Delta} = r^2 - 2(M - \omega) + a^2 + e^2 + g^2$ is the horizon equation after the emission of the particle with energy ω .

Now the coordinate ϕ does not appear in the dragged Painleve-Gullstrand metric (17)

So ϕ is an ignorable coordinate in the Lagrangian function L . To eliminate this degree of freedom completely, the action should be written as

$$L = \int_{t_{in}}^{t_{out}} (L - P_{\phi} \dot{\phi}) dt \tag{22}$$

So the imaginary part of the action is

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}(P_r, P_{\phi})} \left[\int_{(0,0)}^{(P_r, P_{\phi})} (r dP'_r - \dot{\phi} dP'_{\phi}) \right] \frac{dr}{\dot{r}} \tag{23}$$

where P_r and P_{ϕ} are two canonical momentum conjugate to r and ϕ respectively.

$r_{in} = M + \sqrt{M^2 - a^2 - e^2 - g^2}$ and $r_{out} = (M - \omega) + \sqrt{(M - \omega)^2 - a^2 - e^2 - g^2}$ are the locations of the event horizon before and after a particle tunnels out, they are just inside and outside the barrier through which the particle tunnels.

We now eliminate the momentum in favor of energy by using Hamilton's equations

$$\dot{r} = \frac{dH}{dP_r} \Big|_{(r; \phi, P_{\phi})} = \frac{d(M - \omega)}{dP_r} \tag{24}$$

$$\dot{\phi} = \frac{dH}{dP_{\phi}} \Big|_{(\phi; r, P_r)} = a\Omega \frac{d(M - \omega)}{dP_{\phi}} \tag{25}$$

where $dH_{(\phi,r,P_r)} = \bar{\Omega} dJ = a \bar{\Omega} d(M - \omega)$ represents the energy change of the blackhole because of the loss of the angular momentum when a particle tunnels out [42] , and the dragging angular velocity is given by

$$\bar{\Omega} = \frac{a[(r^2 + a^2) - \bar{\Delta}]}{(r^2 + a^2)^2 - a^2 \bar{\Delta} \sin^2 \theta} \tag{26}$$

Substituting equations (20), (24) (25) and (26) into equation (23) and noting that we must choose the positive sign in equation (21) as the particle is propagating from inside to outside the horizon, then we have

$$\begin{aligned} \text{Im}S &= \text{Im} \left[\int_{r_{in}}^{r_{out}^{M-\omega}} \int_M (1 - a\Omega') \frac{dr}{r} d(M - \omega') \right] \\ &= \text{Im} \int_{r_{in}}^{r_{out}^{M-\omega}} \int_M (1 - a\Omega') \frac{\sqrt{(r^2 + a^2)^2 - a^2 \Delta' \sin^2 \theta}}{[\rho^2 - \sqrt{\rho^2(\rho^2 - \Delta')}]} dr d(M - \omega') \end{aligned} \tag{27}$$

where

$$\begin{aligned} \bar{\Delta} &= r^2 + a^2 + e^2 + g^2 - 2(M - \omega')r = (r - r'_+)(r - r'_-) \\ r'_\pm &= (M - \omega') \pm \sqrt{(M - \omega')^2 - a^2 - e^2 - g^2} \end{aligned}$$

We multiply and divide the integrand with $\rho^2 + \sqrt{\rho^2(\rho^2 - \Delta')}$ to obtain

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}^{M-\omega}} \int_M (1 - a\Omega) \frac{[\rho^2 + \sqrt{\rho^2(\rho^2 - \Delta')}] \sqrt{(r^2 + a^2)^2 - a^2 \Delta' \sin^2 \theta}}{(r - r'_+)(r - r'_-) \rho^2} dr d(M - \omega) \tag{28}$$

We see that $r = r'_+$ is a pole of order one. The integral can be evaluated by deforming the contour around the pole, so as to ensure that the positive energy solution decay in

time. Note that all real parts , divergent or not , can be discarded since they only contribute a phase. Doing the r integral first we obtain,

$$\text{ImS} = -2\pi \int_M^{M-\omega} \frac{(M-\omega)^2 - \frac{1}{2}(a^2 + e^2 + g^2) + (M-\omega)\sqrt{(M-\omega)^2 - a^2 - e^2 - g^2}}{\sqrt{(M-\omega)^2 - a^2 - e^2 - g^2}} d(M-\omega) \quad (29)$$

Finishing the integral we have

$$\text{ImS} = -\pi[(M-\omega)^2 - M^2 + (M-\omega)\sqrt{(M-\omega)^2 - a^2 - e^2 - g^2} - M\sqrt{M^2 - a^2 - e^2 - g^2}] \quad (30)$$

The tunneling rate is therefore

$$\Gamma \sim e^{-2\text{ImS}} = e^{2\pi[(M-\omega)^2 - M^2 + (M-\omega)\sqrt{(M-\omega)^2 - a^2 - e^2 - g^2} - M\sqrt{M^2 - a^2 - e^2 - g^2}]} \quad (31)$$

Using Bekenstein-Hawking entropy formula $S_{BH} = \pi(r_+^2 + a^2)$,

we have

$$S_{BH}(M) = \pi[2M^2 + 2M\sqrt{M^2 - a^2 - e^2 - g^2} - e^2 - g^2]$$

$$S_{BH}(M - \omega) = \pi[2(M - \omega)^2 + 2(M - \omega)\sqrt{(M - \omega)^2 - a^2 - e^2 - g^2} - e^2 - g^2]$$

$$\begin{aligned} \Delta S_{BH} &= S_{BH}(M - \omega) - S_{BH}(M) \\ &= 2\pi[(M - \omega)^2 + (M - \omega)\sqrt{(M - \omega)^2 - a^2 - e^2 - g^2} - M^2 - M\sqrt{M^2 - a^2 - e^2 - g^2}] \end{aligned}$$

where ΔS_{BH} is the difference of entropies of the blackhole before and after the emission. From equation (31) we have

$$\Gamma \sim e^{\Delta S_{BH}} \quad (32)$$

The result is obviously consists with an underlying unitary theory. Following the reference [10] , expanding ΔS_{BH} in $(\omega - \omega_0)$ and neglecting the higher order terms we have

$$\Gamma \sim e^{\Delta S_{BH}} = e^{-\beta(\omega - \omega_0) \left[1 - \frac{r_+^2 + a^2}{r_+^4} (M + \sqrt{M^2 - a^2 - e^2 - g^2}) - \frac{M(a^2 + e^2 + g^2)}{2(M^2 - a^2 - e^2 - g^2)} (\omega - \omega_0) \right]} \quad (33)$$

where $\omega_0 = a\omega\Omega$ and β is the inverse Hawking temperature given by

$$\beta = \frac{1}{T_H} = \frac{2\pi}{\kappa} = \frac{2\pi \left[M + \sqrt{M^2 - a^2 - e^2 - g^2} \right]^2}{\sqrt{M^2 - a^2 - e^2 - g^2}}.$$

Also in equation (31) we see that the first term gives the thermal Boltzmann factors $e^{-\beta(\omega-\omega_0)}$ for the emanating radiation. The second term represents correction from the responds of the background geometry to the emission of a quantum. Furthermore equation (32) indicates that when the energy conservation and the angular momentum conservation as well as the particle's self-gravitation are taken into account , the tunneling rate is related to the change of blackhole entropy during the process of the particle's emission and the radiant spectrum is not precisely thermal.

6. Concluding remarks:

In this paper, we have presented the Hawking radiation as tunneling from non-accelerating and rotating with electric and magnetic charged blackhole by applying Krause-Parikh-Wilczek's semi-classical quantum tunneling method[5,6,7,8]. We find that the emission rate at the event horizon is equal to the difference of Bekenstein-Hawking entropy before and after the emission of a particle. The Hawking temperature of this type of blackhole recovered by expanding ΔS_{BH} in $(\omega - \omega_0)$ and neglecting the higher order terms.

In special case , if we put $a = e = g = 0$ then the result reduces for the Schwarzschild blackhole and if $a = 0, g = 0$ then the result reduces to the Reissner-Nordstrom blackhole and supports the Parikh-Wilczek's result[6]. Also if we assume the equivalent charge $Q^2 = e^2 + g^2$ then the result is similar for the tunneling of uncharged particle from Kerr-Newman blackhole[41].

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