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SOME CHARACTERIZATIONS OF N-DISTRIBUTIVE LATTICES

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Abstract.

In this paper, we have included several characterizations of n-distributive lattices. Also we have generalized the prime Separation Theorem for an n-annihilator $I = J^{\perp_n}$ *(where J is a non-empty finite subset of L) and characterized the n-distributive lattices.*

Keywords and phrases : distributive lattices, annihilator, prime Separation Theorem

বিমূর্ত সার *(Bengali version of the Abstract)*

এই পত্রে আমরা n-বন্টিত ল্যাটিসের (n-distributive lattices) বহুবিধ বৈশিষ্ট্যকে অর্ন্তভুক্ত করেছি । n - এনিহিলেটারের (n-annihilator) $I = J^{\perp_n}$ *(যেখানে J, L - এর একটি অশূন্য সসীম উপসেট) জন্য মুখ্য বিচ্ছেদ উপপাদ্যের সাধারণীকরণ এবং n - বন্টিত ল্যাটিসের বিশিষ্টায়ন করেছি ।*

1) Introduction:

J.C Varlet [7] introduced the notion of 0-distributives lattices to generalize the notion of pseudocomplemented lattices. A lattice L with 0 is called 0-distributive if for all $a,b,c \in L$, $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Of course every distributive lattice is a 0-distributive lattice. Moreover, L is 0-distributive if and only if for each

$a \in L$ the set of all elements disjoint with element a forms an ideal. Since a pseudo complemented lattice is characterized by the fact that for each element a, the set of elements disjoint with a is a principal ideal, so every pseudo complemented lattice is 0-distributive. Similarly, if $1 \in L$, then one can describe 1-distributive lattice. For detailed literature on 0-distributive lattices we refer the readers to consult [7], [1] and [6]. Recently [5] have generalized the whole concept and introduced the notion of n-distributive lattice for any neutral element $n \in L$. For an element n of a lattice L, a convex sublattice of L containing n is called an n-*ideal* of L. An element $n \in L$ is called a *standard* element if for $a, b \in L, a \wedge (b \vee n) = (a \wedge b) \vee (a \wedge n)$, while n is called a *neutral* element if (i) it is standard and (ii) $n \wedge (a \vee b) = (n \wedge a) \vee (n \wedge b)$ for all $a, b \in L$. Set of all n-ideals of a lattice L is denoted by $I_n(L)$ which is an algebraic lattice; where $\{n\}$ and L are the smallest and the largest elements. For two n-ideals I and J, $I \cap J$ is the infimum and $I \vee J = \{x \in L / i_1 \wedge j_1 \le x \le i_2 \vee j_2, \textit{ for some } i_1, i_2 \in I \textit{ and } j_1, j_2 \in J\}$. The n-ideal generated by a finite numbers of elements $a_1, a_2, ..., a_m$ is called a *finitely generated n-ideal* denoted by $\langle a_1, a_2, ..., a_m \rangle_n$. Moreover, $\langle a_1, a_2, ..., a_m \rangle_n =$
$\{x \in L / a_1 \wedge a_2 \wedge ... \wedge a_m \wedge n \le x \le a_1 \vee a_2 \vee ... \vee a_m \vee n\}$
$= [a_1 \wedge a_2 \wedge .... \wedge a_m \wedge n, a_1 \vee a_2 \vee .... \vee a_m \vee n]$

Thus, every finitely generated n-ideal is an interval containing n. n-ideal generated by a single element $a \in L$ is called a *principal n-ideal* denoted by $\langle a \rangle_n$ and $\langle a \rangle_n = [a \wedge n, a \vee n]$. Moreover $[a, b] \cap [c, d] = [a \vee c, b \wedge d]$ and $[a, b] \vee [c, d] = [a \wedge c, b \vee d]$. If n is a neutral element, then by [3], $\langle a \rangle_n \cap \langle b \rangle_n = \langle m(a, n, b) \rangle_n$, where $m(x, y, z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$. Set of all finitely generated n-ideals of L is denoted by $F_n(L)$, while the set of principal n-ideals is denoted by $P_n(L)$. Thus $F_n(L)$ is a lattice but $P_n(L)$ is a semilattice when n is neutral element.

For a neutral element $n \in L$, L is called n-*distributive* if for all $a,b,c \in L$, $\langle a\rangle_n \cap \langle b\rangle_n = \{n\} = \langle a\rangle_n \cap \langle c\rangle_n$ imply $\langle a\rangle_n \cap (\langle b\rangle_n \vee \langle c\rangle_n) = \{n\}$. Equivalently, L is n-distributive if for all $a,b,c \in L$, $a \wedge b \le n \le a \vee b$ and $a \wedge c \le n \le a \vee c$ imply $a \wedge (b \vee c) \le n \le a \vee (b \wedge c)$.[5] have shown that for a neutral element $n \in L$, L is n-distributive if and only if for $a \in L$, $\{a\}^{\perp_n} = \{x \in L / m(a,n,x) = n\}$ is an n-ideal. In this paper we will include some more characterizations of n-distributive lattices. Then we extend the separation Theorem for n-ideals given in [5] with the help of annihilator n-ideals. Throughout the paper we will consider n as a neutral element.

Theorem 1: *Let $n \in L$ be neutral. L is n-distributive if and only if $(n]$ is 1-distributive and $[n)$ is 0-distributive.*

Proof: Suppose L is n-distributive. Let $p,q,r \in (n]$ and $p \vee q = n = p \vee r$. Then $p \wedge q \le n = p \vee q$ and $p \wedge r \le n = p \vee r$ imply $p \wedge (q \vee r) \le n \le p \vee (q \wedge r) \le (p \vee q) \wedge (p \vee r) = n$ as L is n-distributive. This implies $p \vee (q \wedge r) = n$, and so $(n]$ is 1-distrivutive. Dually we can show that $[n)$ is 0-distributive. Conversely, suppose $(n]$ is 1-distributive and $[n)$ is 0-distributive. Let $a,b,c \in L$ with $a \wedge b \le n \le a \vee b$ and $a \wedge c \le n \le a \vee c$. Then $(a \vee n) \wedge (b \vee n) = (a \wedge b) \vee n = n$ as n is neutral. Similarly $(a \vee n) \wedge (c \vee n) = n$. Thus $(a \vee n) \wedge (b \vee c \vee n) = n$ as $[n)$ is 0-distributive. This implies $a \wedge (b \vee c) \le n$. Similarly using the 1-distributive property of $(n]$ we see that $n \le a \vee (b \wedge c)$ as n is neutral. Therefore, $a \wedge (b \vee c) \le n \le a \vee (b \wedge c)$, and so L is n-distributive.

A non-empty subset I of a lattice L is called a *down set* if for $a \in I$ and $x \le a$ $(x \in L)$ imply $x \in I$. I is called an *ideal* if it is a down set and for all $a,b \in I$,

$a \vee b \in I$. Dually, a non-empty subset F of L is called an *up set* if for all $a \in F$ and $x \geq a$ ($x \in L$) imply $x \in F$. F is called a *filter* of L if it is an up set and for $a, b \in F$, $a \wedge b \in F$. A subset T of L is called *convex* if for $a \leq x \leq b$ with $a, b \in T$ imply $x \in T$. Of course all the ideals and filters of a lattice are convex sublattices. Moreover, for every convex sublattice C of L, $C = (C] \cap [C)$. A proper filter F of L is called a *maximal filter* if for any filter $M \supseteq F$ implies $M = F$ or $M = L$. A proper filter F is called a *prime filter* if for any $f, g \in L$, $f \vee g \in F$ implies either $f \in F$ or $g \in F$. Similarly, a down set I of L is prime if $a \wedge b \in I$ $(a, b \in L)$ implies either $a \in I$ or $b \in I$. In a lattice L with 0, a prime down (up set) set P is *minimal* if it does not contain any other prime down set (up set). It is very easy to show that F is a maximal filter if and only if $L - F$ is a minimal prime down set. Similarly, I is a maximal ideal if and only if $L - I$ is a minimal prime up set. Moreover, F is a prime filter if and only if $L - F$ is a prime ideal. A convex sublattice P is called a prime convex sublattice if for any $p \in P$, $m(x, p, y) \in P$ implies either $x \in P$ or $y \in P$. By [4] P is a prime convex sublattice if and only if P is either a prime ideal or a prime filter. Thus we have:

Lemma 2: *Let F be a non-empty subset of L not containing n. Then F is a filter (ideal) if and only if $L - F$ is a prime down set(up set) containing n.*

Lemma 3: *Let F be a non-empty subset of L not containing n. Then F is a maximal filter (ideal) if and only if $L - F$ is a minimal prime down set (up set) containing n.*

Let n be neutral in L. For $a \in L$, we define $\{a\}^{\perp_n} = \{x \in L / m(x, n, a) = n\}$, known as n-annihilator of a. For $A \subseteq L$, $A^{\perp_n} = \{x \in L / m(x, n, a) = n\}$ for all $a \in A$. In an n-distributive lattice, [5] have shown that $\{a\}^{\perp_n}$ and $A^{\perp_n}$ are n-ideals. Moreover,

$A^{\perp_n} = \bigcap_{a \in A} \left\{ \{a\}^{\perp_n} \right\}$. If A is an n-ideal, then in a n-distributive lattice $A^{\perp_n}$ is the annihilator n-ideal and so it is the pseudo complement of A in $I_n(L)$. Thus in a n-distributive lattice L, $I_n(L)$ is pseudo complemented.

Lemma 4*:* *For an element* $a \neq n$ *in* L, $\{a\}^{\perp_n}$ *is a convex subset containing n but not containing* a.

Proof: Let $x, y \in \{a\}^{\perp_n}$ and $x \leq t \leq y$. Then $x \wedge a \leq n \leq x \vee a$ and $y \wedge a \leq n \leq y \vee a$ imply $t \wedge a \leq y \wedge a \leq n \leq x \vee a \leq t \vee a$, and so $t \in \{a\}^{\perp_n}$. Thus $\{a\}^{\perp_n}$ is a convex subset. Since $m(n,n,a) = n$ so $n \in \{a\}^{\perp_n}$. Also $m(a,n,a) = a \neq n$ implies $a \notin \{a\}^{\perp_n}$. Hence $\{a\}^{\perp_n}$ is a convex subset containing n but not containing a.

Corollary 5: *If* $A \subseteq L$ *and* $n \notin A$, *then* $A^{\perp_n}$ *is a convex subset containing* n *but disjoint from A.*

Proof: It is trivial by Lemma 4 and $A^{\perp_n} = \bigcap_{a \in A} \{a\}^{\perp_n}$.

Theorem 6: *Let* n *be a neutral element of a lattice* L *and A be a nonempty subset of* L *not containing* n. *Then* $A^{\perp_n}$ *is the intersection of all the minimal prime convex subsets containing n but not containing* A.

Proof: Let $X = \cap(P / A \subsetneq P$, P is a minimal prime convex set containing n). Let $x \in A^{\perp_n}$. Then $m(x,n,a) = n$ for all $a \in A$. This implies for each P, there exists $z \in A - P$ such that $m(x,n,z) = n$. Since P is prime, so $x \in P$ and so $x \in X$. Conversely, let $x \in X$. If $x \notin A^{\perp_n}$ then $m(x,n,a) \neq n$ for some $a \in A$. Then by [5, Lemma 5] either $x \vee n \notin \{a \vee n\}^{\perp}$ or $x \wedge n \notin \{a \wedge n\}^{\perp^d}$. Suppose $x \vee n \notin \{a \vee n\}^{\perp}$. Then $(x \vee n) \wedge (a \vee n) \neq n$ which implies $(x \wedge a) \vee n > n$, and so $x \wedge a \lneqq n$. Let $D = [x \wedge a)$. Then D is a proper filter as $n \notin D$. So by [5, Lemma 2], there exists a maximal filter $M \supseteq D$, and not containing n. Hence by Lemma 3,

$L-M$ is a minimal prime down set containing n. Now $x \in D$ implies $x \in M$ and so $x \notin L-M$. Moreover $A \subsetneq L-M$ as $a \in M$ implies $a \notin L-M$ which is a contradiction to $x \in X$. Similarly if $x \wedge n \notin \{a \wedge n\}^{\perp^d}$, Then $(x \wedge n) \vee (a \wedge n) \neq n$. Implies $(x \vee a) \wedge n < n$ and so $x \vee a \ngeq n$. Consider $I = (x \vee a]$. Clearly $n \notin I$. So there exists a maximal ideal Q containing I. but not containing n. Then by same argument as above $L-Q$ is a minimal prime up-set containing n. But $A \subsetneq L-Q$. Also $x \notin L-Q$, which is again a contradiction to $x \in X$. Therefore $x \in A^{\perp_n}$.

Following characterization of n-distributive lattice is given in [5].

Theorem 7: *For a neutral element n of a lattice L, the following conditions are equivalent.*

(i) L *is n-distributive.*

(ii) For every $a \in L$, $\{a\}^{\perp_n}$ is an n-ideal.

(iii) For any $A \subseteq L$, $A^{\perp_n}$ is an n-ideal.

(iv) $I_n(L)$ is pseudo complemented .

(v) $I_n(L)$ is 0-distributive.

(vi) Every maximal convex sublattice of L not containing n is prime.

Now we give the following characterization:

Theorem 8**:** *For a neutral element n of a lattice L, the following conditions are equivalent.*

(i) L is n-distributive.

(ii) Every maximal convex sublattice not containing n is prime.

(iii) Every minimal prime down set of L containing n and every minimal prime up set of

L containing n is a minimal prime ideal (filter), and so a minimal prime n-ideal.

(iv) Every filter (ideal) not containing n is disjoint from a minimal prime n-ideal.

(v) For each $a \neq n$, there is a minimal prime n-ideal not containing a.

(vi) Each $a \neq n$ is contained in a prime convex sublattice not containing n.

Proof: (i) $\Leftrightarrow$ (ii) holds by Theorem 7.

(ii) implies (iii). Let A be a minimal prime down set (up set) of L containing n. Then

$L - A$ is a maximal filter (ideal) not containing n. Hence by (ii) it is a prime filter (ideal). Hence A is a minimal prime ideal (filter). Since $n \in A$, so it is a minimal prime n-ideal.

(iii) implies (ii). Let F be a maximal convex sublattice of L not containing n. Since $F = (F] \cap [F)$, so either $n \notin (F]$ or $n \notin [F)$. Without loss of generality suppose $n \notin [F)$. Then by the maximality of F, $F = [F)$. Thus F is a filter and so $L - F$ is a minimal prime down set containing n, and so by (iii), it is a minimal prime ideal. Hence F is a prime filter, and so is a prime convex sublattice.

(i) implies (iv). Let F be a filter not containing n. Then by [5, Corollary 7], there is a prime (maximal) filter $Q \supseteq F$ not containing n. Thus $L - Q$ is a minimal prime ideal (n-ideal), which is disjoint from F. Similarly, if I is an ideal not containing n, then it is also disjoint from a minimal prime n-ideal.

(iv) implies (v). Let $a \in L$ and $a \neq n$. Then $[a) \cap \{n\} = \varphi$ or $(a] \cap \{n\} = \phi$. Without loss of generality suppose $[a) \cap \{n\} = \varphi$. Then by (iv), there is a minimal prime ideal P containing n such that $P \cap [a) = \varphi$. Then P is in fact, an n-ideal and $a \notin P$.

(v) implies (vi). Let $a \in L$ and $a \neq n$. Then by (v) there exists a minimal prime n-ideal P such that $a \notin P$. But we know that the prime n-ideals are either prime ideals or prime filters, so without loss of generality suppose P is a prime ideal. This implies $a \in L - P$, which is a prime filter not containing n. That is, $L - P$ is the prime convex sublattice containing a, but not containing n.

(vi) implies (i). Let L be not n-distributive. Then there exist $a, b, c \in L$ such that $\langle a \rangle_n \cap \langle b \rangle_n = \{n\}$ and $\langle a \rangle_n \cap \langle c \rangle_n = \{n\}$ but $\langle a \rangle_n \cap (\langle b \rangle_n \vee \langle c \rangle_n) \neq \{n\}$. Then $a \wedge b \leq n \leq a \vee b$ and $a \wedge c \leq n \leq a \vee c$. Now $\langle a \rangle_n \cap (\langle b \rangle_n \vee \langle c \rangle_n) = [(a \vee (b \wedge c)) \wedge n, (a \wedge (b \vee c)) \vee n] \neq \{n\}$ implies either $a \wedge (b \vee c) \lneqq n$ or $a \vee (b \wedge c) \gneqq n$. Without loss of generality, suppose $a \wedge (b \vee c) \lneqq n$. Then by (vi), $a \wedge (b \vee c) \in Q$, where Q is a prime convex sublattice not containing n. Then Q is either an ideal or a filter. Since $a \wedge (b \vee c) \lneqq n$, so Q can not be considered as an ideal. For if $a \wedge (b \vee c) > n$, then it would imply $n \in Q$. Therefore Q must be a filter. Now, $a \in Q$ and $b \vee c \in Q$ implies either $a \wedge b \in Q$ or $a \wedge c \in Q$ as Q is prime. In either case, $n \in Q$ as $a \wedge b, a \wedge c \leq n$, which is a contradiction. Similarly by considering $a \vee (b \wedge c) \gneqq n$ we will get another contradiction. Therefore, L is n-distributive.

Theorem 9: *Let L be n-distributive and $x \in L$. Then a prime ideal P containing $\{x\}^{\perp_n}$ is a minimal prime ideal containing $\{x\}^{\perp_n}$ if and only if for all $p \in P$ there is a $q \in L - P$ such that $m(p, n, q) \in \{x\}^{\perp_n}$.*

Proof. Let P be a prime ideal containing $\{x\}^{\perp_n}$ such that the given condition holds. Let K be a prime ideal containing $\{x\}^{\perp_n}$ such that $K \subseteq P$. Let $p \in P$. Then there exists $q \in L - P$ such that $m(p, n, q) \in \{x\}^{\perp_n}$. Hence $m(p, n, q) \in K$. Since K is prime and $q \notin K$, so $p \in K$. This implies $P \subseteq K$ and so $K = P$. Therefore, P is minimal.

Conversely, let P be a minimal prime ideal containing $\{x\}^{\perp_n}$. Let $p \in P$. Suppose for all $q \in L - P$, $m(p, n, q) \notin \{x\}^{\perp_n}$. Set $D = (L - P) \vee [p)$. We claim that

$\{x\}^{\perp_n} \cap D = \varphi$. If not, let $y \in \{x\}^{\perp_n} \cap D$. Then $y \geq r \wedge p$ for some $r \in L - P$. Then $n \leq m(p, n, r \vee n) = (p \wedge r) \vee n \leq y \vee n$ implies $m(p, n, r \vee n) \in \{x\}^{\perp_n}$, by convexity of $\{x\}^{\perp_n}$. This gives a condition to the assumption, as $r \vee n \in L - P$. Then by [5, Theorem9], there exists a maximal (prime) filter $Q \supseteq D$ and disjoint from $\{x\}^{\perp_n}$. By the same proof of [5, Theorem 9], $x \in Q$. Let $M = L - Q$. Then M is a prime n-ideal. Since $x \in Q$, so $x \notin M$. Let $t \in \{x\}^{\perp_n}$. Then $m(t, n, x) = n$ implies $t \in M$ as M is prime. Thus $\{x\}^{\perp_n} \subseteq M$. Now $M \cap D = \varphi$. Therefore $M \cap (L - P) = \varphi$, and hence $M \subseteq P$. Also $M \neq P$, because $p \in D$ implies $p \notin M$, but $p \in P$. Hence M is a prime n-ideal containing $\{x\}^{\perp_n}$, which is properly contained in P. This gives a contradiction to the minimal property of P. Therefore, the given condition holds.

To generalize the separation Theorem for n-ideals in a distributive lattice given in [2], [5, Theorem9] have given such a separation property in a n-distributive lattice with respect to $\{x\}^{\perp_n}$ for any $x \in L$. We now improve this result for an n-annihilator $I = J^{\perp_n}$ for some finite subset J of L.

Theorem 10: (The Separation Theorem). *Let n be a neutral element of L. Then L is n-distributive if and only if for a filter F and an n-annihilator $I = J^{\perp_n}$ (where J is a non-empty finite subset of L) with $F \cap I = \varphi$, there exists a prime filter Q containing F such that $Q \cap I = \varphi$.*

Proof. Suppose L is n-distributive and $I = J^{\perp_n}$ for some non-empty finite subset J of L. Let $\mathcal{F}$ be the set of all filters containing F and disjoint from I. Then using Zorn's Lemma, there exists a maximal filter Q containing F and disjoint from I. Now $J \neq \{n\}$, as then $J^{\perp_n} = L \supset F$.Suppose $j_1, j_2, \dots, j_k$ are the elements in J which are different from n. We claim that at least one of $j_i \in Q$, $i = 1, 2, \dots, k$. If not, then for each i, $(Q \vee [j_i)) \cap I \neq \varphi$ by the maximality of Q. Let $t_i \in (Q \vee [j_i)) \cap I$. Then $t_i \geq q_i \wedge j_i$ for some $q_i \in Q$ and $t_i \in I$ implies

$m(t_i, n, x) = n$ for all $x \in J$. Thus in particular, $m(t_i, n, j_i) = n$. This implies $t_i \wedge j_i \leq n$ and so, $q_i \wedge j_i \leq t_i \wedge j_i \leq n$. Then $(q_i \vee n) \wedge j_i = (q_i \wedge j_i) \vee (n \wedge j_i) \leq n$ implies $m(q_i \vee n, n, j_i) = n$ for each $i = 1, 2, ..., k$. Thus we obtain the elements $q_1 \vee n, q_2 \vee n, ..., q_k \vee n$ in Q. Choose $q = (q_1 \vee n) \wedge (q_2 \vee n) \wedge ... \wedge (q_k \vee n) = (q_1 \wedge q_2 \wedge ... \wedge q_k) \vee n \in Q$. Then $q \wedge j_i \leq q_i \wedge j_i \leq n$ and $q \wedge n = n$ imply $m(q, n, j_i) = n$ for each i and so $q \in I$, which contradicts that $Q \cap I = \varphi$. Therefore $j_i \in Q$ for some $i = 1, 2, ..., k$. Now let $z \notin Q$. Then by the maximality of Q, $(Q \vee [z)) \cap I \neq \varphi$. Let $t \in (Q \vee [z)) \cap I$. Then $t \geq q \wedge z$ for some $q \in Q$ and $m(t, n, j) = n$ for all $j \in J$. So, $m(t, n, j_i) = n$. Then $t \wedge j_i \leq n \leq t \vee j_i$ and so $q \wedge j_i \wedge z \leq t \wedge j_i \leq n$ which implies $m(z, n, (q \wedge j_i) \vee n) = n$. Then by [5, Lemma3] Q is a maximal filter not containing n. Hence Q is prime as L is n-distributive.

Conversely, let $\langle x \rangle_n \cap \langle y \rangle_n = \{n\}$ and $\langle x \rangle_n \cap \langle z \rangle_n = \{n\}$. We need to prove that $\langle x \rangle_n \cap (\langle y \rangle_n \vee \langle z \rangle_n) = \{n\}$. That is $x \wedge (y \vee z) \leq n \leq x \vee (y \wedge z)$. If not, let $x \wedge (y \vee z) \nleq n$. Then $[y \vee z) \cap \{x\}^{\perp_n} = \varphi$. For otherwise $t \in [y \vee z) \cap \{x\}^{\perp_n}$, implies $t \wedge x \leq n \leq t \vee x$ and $t \geq y \vee z$, which implies $x \wedge (y \vee z) \leq t \wedge x \leq n$, a contradiction. So, there exists a prime filter Q containing $[y \vee z)$ disjoint from $\{x\}^{\perp_n}$. As $y, z \in \{x\}^{\perp_n}$, so $y, z \notin Q$. Thus $y \vee z \notin Q$, as Q is prime. This implies $[y \vee z) \not\subset Q$, a contradiction. Dually by taking $x \vee (y \wedge z) \ngeq n$, we would have another contradiction. Therefore, $x \wedge (y \vee z) \leq n \leq x \vee (y \wedge z)$, and so L is n-distributive.

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