ISSN 0973-8975

J.Mech.Cont. & Math. Sci., Vol.-7, No.-2, Januaryy (2013) Pages 1031-1044

BUCKLING OF (2*n*+1) LAYERS PLYWOOD SHELL UNDER TWO WAY COMPRESSIONS

By

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Abstract:

The object of this paper is to obtain all the stress resultants of an anisotropic (2n+1) layers plywood shell. The deferential equations of equilibrium of (2n+1) layers plywood shell under three simultaneous loads are obtained. The solution of the deferential equations for anisotropic (2n+1) layers plywood shell in case of two way compressions is obtained here. The stable region for a plywood shell in this case is obtained. Buckling diagram for five layers plywood shell and seven layers plywood shell are shown graphically as special cases.

Keywords and phrases : *an anisotropic layers, plywood shell, two way compressions, buckling diagram*

বিমূর্ত সার *(Bengali version of the Abstract)*

অসমদৈশিক (2n+1) স্তরের প্লাইউড্ শেলের সকল লব্ধি পীড়ণগুলির নির্ণয় করাই হচ্ছে এই পত্রের উদ্দেশ্য । যুগপৎ তিনটি ভারের অধীন (2n+1) স্তরের প্লাইউড্ শেলের সাম্যাবস্থার অন্তরকলন সমীকরণটি নির্ণয় করা হয়েছে । অসমদৈশিক (2n+1) স্তরের প্লাইউড্ শেলের দ্বিমুখী সংপীড়ণের ক্ষেত্রের জন্য অন্তরকলন সমীকরণের সমাধান নির্ণয় করা হয়েছে । এই ক্ষেত্রে প্লাইউড্ শেলের জন্য স্থির অঞ্চলকে নির্ণয় করা হয়েছে । প্লাইউড্ শেলের পাঁচটি এবং সাতটি স্তরের জন্য আয়তন - আকার নক্শাকে বিশেষ ক্ষেত্র হিসাবে লেখচিত্রের সাহায্যে দেখানো হয়েছে ।

1. **Introduction:**

Woods are anisotropic in nature. Woods display much more rigidity in the direction of the grain than across. So along the cross grain direction the rigidity of the wood is very less. There for to satisfy the need of such wooden material which has approximately same rigidity in both direction, the concept of plywood is introduced.

Shells are used for roof structures and large columnless areas and for storage tanks. A large number of air craft hangers, factory and car sheds, covered markets, planetarium and rail-road terminals etc. have been created with shell constructions. A detailed study of the shell of arbitrary shape is necessary for consideration of variety, economy and architectural showmanship in building construction.

The solution of buckling of cylindrical shells in case of isotropic material is known from the literature on shells, e.g. Flugge [5]. Singer and Fersh-Scher [9] solved the buckling of the orthotropic conical shells under external pressure. Singer [10] solved the buckling of orthotropic and stiffened conical shells. Buckling problem of anisotropic cylindrical shells has occupied the interest of many researchers such as Tasi [11], Cheng and Kuenzi [2], Hess [6], Thieleman, Schnell and Fischer [13], Cheng and Ho [1]. De [3] solved the buckling problem of 3-layer plywood shells under two way compressions.

Anisotropic plywood shell of $(2n+1)$ layers is under consideration of this work. The object of this work is to obtain elastic laws of buckling of $(2n+1)$ layers plywood shell. The solution of the differential equations of the equilibrium for anisotropic plywood shells in case of two way compressions is obtained here. The stable region for a five layer plywood shell and seven layers plywood shell are shown graphically.

2. Theory:

Here we consider a circular cylindrical shell with coordinates x, φ and z. x is the distance of any point from a datum plane, φ is the angular distance of the point from a datum generator and z is the distance from the middle surface. u, v and w are the components of velocity.

Symmetric $(2n+1)$ layers plywood shells are considered here. Wood displays much more rigidity in the direction of the grain than across, so Hooke's law is not symmetric with respect to x and φ here. For the inner layer it takes the form (Flugge [5]),

$$\left.\begin{aligned} \sigma_x &= E_1\varepsilon_x + E_\nu\varepsilon_\varphi \\ \sigma_\varphi &= E_\nu\varepsilon_x + E_2\varepsilon_\varphi \\ \tau_{x\varphi} &= G\gamma_{x\varphi} \end{aligned}\right\} \qquad (1)$$

Here $\sigma_x, \sigma_\varphi, \tau_{x\varphi}$ are stresses and $\varepsilon_x, \varepsilon_\varphi, \gamma_{x\varphi}$ are strains and the four moduli E_1, E_2, E_ν, G are all independent of each other. If the outer symmetric layers are made of the same kind of wood and this we shall assume then their elastic laws are the same except that the moduli E_1 and E_2 change places.

To explain the elastic behavior of plywood, we consider the figure of the 7 layers plywood material.

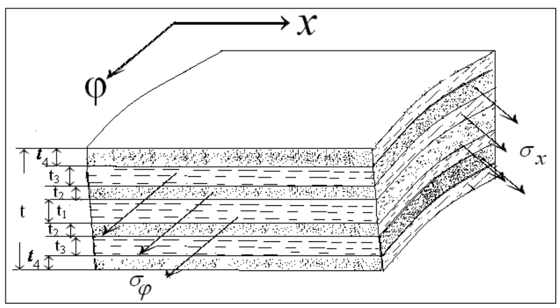

Figure 1 : a 7-layers plywood material.

In the above figure, it has been assumed that the grain of the inner layer is running in the x direction. E_1 is the common modulus of elasticity of wood i.e., the one for stresses in the direction of the grain, while E_2 is the much smaller cross grain modulus. When in a shell the grain runs circumferentially in the middle layer and lengthwise in the adjacent two layers to the middle layer then for two layers adjacent to the middle layer we must identify E_2 with common modulus and E_1 with the cross grain modulus. Similarly for the next two adjacent layers the common modulus of elasticity E_1 and E_2 will be the cross grain modulus and so on.

The strains in this case are given by (Flugge [5]),

$$\left.\begin{aligned}
\varepsilon_x &= \frac{u'}{a} - z\frac{w''}{a^2} \\
\varepsilon_\varphi &= \frac{v^\bullet}{a} - \frac{z}{a}\frac{w^{\bullet\bullet}}{a+z} + \frac{w}{a+z} \\
\gamma_{x\varphi} &= \frac{u^\bullet}{a+z} + \frac{a+z}{a^2}v' - \frac{w'^\bullet}{a}\left(\frac{z}{a} + \frac{z}{a+z}\right)
\end{aligned}\right\} \qquad (2a-c)$$

where $(\ )' = a\dfrac{\partial(\ )}{\partial x}$, $\quad (\ )^\bullet = \dfrac{\partial(\ )}{\partial \varphi}$ respectively and a is the radius of the shell.

The stress resultants for any anisotropic shell are defined by (Flugge [5]),

$$\left.\begin{aligned}
N_x &= \int_{-t/2}^{t/2} \sigma_x (1+\frac{z}{a})dz, & N_\varphi &= \int_{-t/2}^{t/2} \sigma_\varphi dz, \\
N_{x\varphi} &= \int_{-t/2}^{t/2} \tau_{x\varphi} (1+\frac{z}{a})dz, & N_{\varphi x} &= \int_{-t/2}^{t/2} \tau_{\varphi x} dz, \\
M_x &= -\int_{-t/2}^{t/2} \sigma_x (1+\frac{z}{a})zdz, & M_\varphi &= -\int_{-t/2}^{t/2} \sigma_\varphi zdz, \\
M_{x\varphi} &= -\int_{-t/2}^{t/2} \tau_{x\varphi} (1+\frac{z}{a})zdz, & M_{\varphi x} &= -\int_{-t/2}^{t/2} \tau_{\varphi x} zdz
\end{aligned}\right\} \qquad (3a-d)$$

But when we introduce elastic law, we must use it in the form (1) for middle layer and exchange E_1 and E_2 while integrating over the outer middle layer and again the same for the next outer layers and so on. This leads to the definition of following rigidities.

(i) Extensional rigidities: when n is even

$$\left.\begin{aligned}
D_x &= E_1\left(t_1 + 2t_3 + 2t_5 + ... + 2t_{n+1}\right) + 2E_2\left(t_2 + t_4 + t_6 + ... + t_n\right) \\
D_\varphi &= E_2\left(t_1 + 2t_3 + 2t_5 + ... + 2t_{n+1}\right) + 2E_1\left(t_2 + t_4 + t_6 + ... + t_n\right) \\
D_\nu &= E_\nu t
\end{aligned}\right\} \qquad (4a)$$

and when n is odd

$$\left.\begin{aligned} D_x &= E_1\left(t_1 + 2t_3 + 2t_5 + ... + 2t_n\right) + 2E_2\left(t_2 + t_4 + t_6 + ... + t_{n+1}\right) \\ D_\varphi &= E_2\left(t_1 + 2t_3 + 2t_5 + ... + 2t_n\right) + 2E_1\left(t_2 + t_4 + t_6 + ... + t_{n+1}\right) \\ D_\nu &= E_\nu t \end{aligned}\right\} \qquad (4b)$$

(ii) Shear rigidity:

$$D_{x\varphi} = Gt \qquad (4c)$$

(iii)Bending rigidity: when n is even

$$\left.\begin{aligned} K_x &= \frac{1}{12}\left[\begin{array}{l} E_1\left\{t_1^3 - \left(2t_2 + t_1\right)^3 + \left(2t_3 + 2t_2 + t_1\right)^3 - ... - t^3\right\} \\ \quad + E_2\left\{-t_1^3 + \left(2t_2 + t_1\right)^3 - \left(2t_3 + 2t_2 + t_1\right)^3 + ... + (t - 2t_{n+1})^3\right\} \end{array}\right] \\ K_\varphi &= \frac{1}{12}\left[\begin{array}{l} E_2\left\{t_1^3 - \left(2t_2 + t_1\right)^3 + \left(2t_3 + 2t_2 + t_1\right)^3 - ... - t^3\right\} \\ \quad + E_1\left\{-t_1^3 + \left(2t_2 + t_1\right)^3 - \left(2t_3 + 2t_2 + t_1\right)^3 + ... + (t - 2t_{n+1})^3\right\} \end{array}\right] \\ K_\nu &= \frac{1}{12} E_\nu t^3 \end{aligned}\right\} \qquad (4d)$$

when n is odd

$$\left.\begin{aligned} K_x &= \frac{1}{12}\left[\begin{array}{l} E_1\left\{t_1^3 - \left(2t_2 + t_1\right)^3 + \left(2t_3 + 2t_2 + t_1\right)^3 - ... + (t - 2t_{n+1})^3\right\} \\ \quad + E_2\left\{-t_1^3 + \left(2t_2 + t_1\right)^3 - \left(2t_3 + 2t_2 + t_1\right)^3 + ... - (t - 2t_{n+1})^3 + t^3\right\} \end{array}\right] \\ K_\varphi &= \frac{1}{12}\left[\begin{array}{l} E_2\left\{t_1^3 - \left(2t_2 + t_1\right)^3 + \left(2t_3 + 2t_2 + t_1\right)^3 - ... + (t - 2t_{n+1})^3\right\} \\ \quad + E_1\left\{-t_1^3 + \left(2t_2 + t_1\right)^3 - \left(2t_3 + 2t_2 + t_1\right)^3 + ... - (t - 2t_{n+1})^3 + t^3\right\} \end{array}\right] \\ K_\nu &= \frac{1}{12} E_\nu t^3 \end{aligned}\right\} \qquad (4e)$$

(iv)Twisting rigidity :

$$K_{x\varphi} = \frac{1}{12} G t^3 \qquad (4f)$$

where

$$t = t_1 + 2t_2 + 2t_3 + \ldots + 2t_{n+1}.$$

where, t is the total thickness of the plate and t_1 is the thickness of the middle most layer, t_2 is the thickness of two adjacent layers of the middle most layer, t_3 is the thickness of the next two layers and so on.

Substituting the values of σ_x, σ_φ, $\tau_{x\varphi}$ from (1) in (3) and using (2), after simplifications we get the elastic laws for the $(2n+1)$ layers plywood shell in the following form,

$$\left.\begin{aligned}
&N_\varphi = \frac{D_\varphi}{a}(v^\bullet + w) + \frac{D_\nu}{a}u' + \frac{K_\varphi}{a^3}(w + w^{\bullet\bullet}), \quad N_x = \frac{D_x}{a}u' + \frac{D_\nu}{a}(v^\bullet + w) - \frac{K_x}{a^3}u'', \\
&N_{\varphi x} = \frac{D_{x\varphi}}{a}(u^\bullet + v') + \frac{K_{\varphi x}}{a^3}(v' - w'^\bullet), \quad N_{x\varphi} = \frac{D_{x\varphi}}{a}(u^\bullet + v') + \frac{K_{x\varphi}}{a^3}(u^\bullet + w'^\bullet), \\
&M_\varphi = \frac{K_\varphi}{a^2}(w + w^{\bullet\bullet}) + \frac{K_\nu}{a^2}w'', \quad M_x = \frac{K_x}{a^2}(w'' - u') + \frac{K_\nu}{a^2}(w^{\bullet\bullet} - v^\bullet), \\
&M_{\varphi x} = \frac{K_{x\varphi}}{a^2}(2w'^\bullet + u^\bullet - v'), \quad M_{x\varphi} = \frac{2K_{x\varphi}}{a^2}(w'^\bullet - v')
\end{aligned}\right\} \qquad (5)$$

Thus we got all the stress resultants and the rigidities for $(2n+1)$ layers plywood shell. These are the most general form of rigidities for arbitrary number of layers. Also these formulae contain the formulae for isotropic shell as a special case. We only need to replace in (1) the module E_1 and E_2 by $E/(1-\nu^2)$ and E_ν by $E\nu/(1-\nu^2)$ and G by $E/2(1+\nu)$ and $n = 0$, $t = t_1$ and make the necessary changes in the definition of the rigidities.

3. The basic equations:

We consider a $(2n+1)$ layers plywood shell shaped as a circular cylinder of length *l* (Figure 2) and subjected simultaneously to three simple loads:

(1) A uniform normal pressure on its wall, $P_r = p$,

(2) An axial compression applied at its edge, the force per unit circumference being P,

(3) A shear load applied at the edges so as to produce a torque in the cylinder, the shearing force is T.

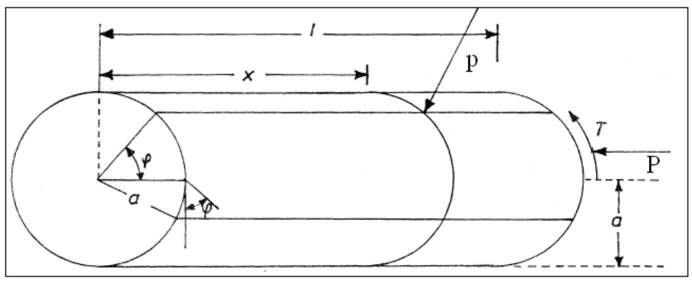

Figure 2: a cylindrical shell

The equation of equilibrium of buckling of circular cylindrical shell (Flugge [5]) are given by,

$$\left.\begin{aligned} &aN'_x + aN^{\bullet}_{\varphi x} - pa\left(u^{\bullet\bullet} - w'\right) - Pu'' - 2Tu'^{\bullet} = 0, \\ &aN^{\bullet}_{\varphi} + aN'_{x\varphi} - M^{\bullet}_{\varphi} - M'_{x\varphi} - pa\left(\mathrm{v}^{\bullet\bullet} + w^{\bullet}\right) - P\mathrm{v}'' - 2T\left(\mathrm{v}'^{\bullet} + w'\right) = 0 \\ &M^{\bullet\bullet}_{\varphi} + M'^{\bullet}_{x\varphi} + M''_x + aN_{\varphi} + pa\left(u' - \mathrm{v}^{\bullet} + w^{\bullet\bullet}\right) + Pw'' - 2T\left(\mathrm{v}' - w'^{\bullet}\right) = 0 \end{aligned}\right\} \qquad (6)$$

Substituting (5) in (6), the differential equations for the buckling problem of a $(2n+1)$ layers plywood shell under three different loads appear in the following form after proper simplification:

$$
\left.\begin{aligned}
& u'' + A_1 u^{\bullet\bullet} + A_2 \mathrm{v}'^{\bullet} + A_3 w' + k_1 \left\{ A_4 \left(u^{\bullet\bullet} + w'^{\bullet\bullet} \right) - w''' \right\} - \\
& \qquad q_1 \left(u^{\bullet\bullet} - w' \right) - q_2 u - 2 q_3 u = 0 \\
& A_5 u'^{\bullet} + \mathrm{v}^{\bullet\bullet} + A_6 \mathrm{v}'' + w^{\bullet} + k_1 \left[3 A_7 \mathrm{v}'' - A_8 w'''^{\bullet} \right] - \\
& \qquad\qquad A_9 \left[q_1 \left(\mathrm{v}^{\bullet\bullet} + w^{\bullet} \right) + q_2 \mathrm{v}'' + 2 q_3 \left(\mathrm{v}'^{\bullet} + w' \right) \right] = 0 \\
& A_{10} u' + \mathrm{v}^{\bullet} + w + k_1 \begin{bmatrix} A_7 u'^{\bullet\bullet} - A_9 u''' - A_8 \mathrm{v}''^{\bullet} + A_9 w'''' + 2 A_{11} w''^{\bullet\bullet} \\ + A_{12} \left(w^{\bullet\bullet\bullet\bullet} + 2 w^{\bullet\bullet} + w \right) \end{bmatrix} \\
& \qquad + A_9 \left[q_1 \left(u' - \mathrm{v}^{\bullet} + w^{\bullet\bullet} \right) + q_2 w'' - 2 q_3 \left(\mathrm{v}' - w'^{\bullet} \right) \right] = 0
\end{aligned}\right\} \qquad (7)
$$

where,

$$
\left.\begin{aligned}
& A_1 = \frac{D_{x\varphi}}{D_x}, \quad A_2 = \frac{D_\nu + D_{x\varphi}}{D_x}, \quad A_3 = \frac{D_\nu}{D_x}, \quad A_4 = \frac{K_{x\varphi}}{K_x}, \\
& A_5 = \frac{D_\nu + D_{x\varphi}}{D_\varphi}, \quad A_6 = \frac{D_{x\varphi}}{D_\varphi}, \quad A_7 = \frac{D_x K_{x\varphi}}{D_\varphi K_x}, \quad A_8 = \frac{D_x \left(K_\nu + 3 K_{x\varphi} \right)}{D_x K_\nu}, \\
& A_9 = \frac{D_x}{D_\varphi}, \quad A_{10} = \frac{D_\varphi}{D_x}, \quad A_{11} = \frac{D_x \left(K_\nu + 2 K_{x\varphi} \right)}{D_\varphi K_x}, \quad A_{12} = \frac{D_x K_\varphi}{D_\varphi K_x}, \\
& k_1 = \frac{K_x}{a^2 D_x}, \quad q_1 = \frac{pa}{D_x}, \quad q_2 = \frac{P}{D_x}, \quad q_3 = \frac{T}{D_x}.
\end{aligned}\right\} \qquad (8)
$$

Equations (7) describe the buckling of a cylindrical shell under the most general homogeneous stress action in the anisotropic case. It is easy to observe that the parameters defined by equations (8) are small quantities. For k_1 it is obvious, since we are interested in thin shells where $t << a$. The three load parameters q_1, q_2 and q_3 are approximately the elastic strains, in the limiting case, caused by the corresponding basic loads. Since all our theory is based on the assumption that such strains are small as compared with unity, we shall neglect the squares and higher order terms of q_1, q_2 and q_3 whenever possible.

4. Solution for shell under two way compressions (without shear load):

We consider that the shell is under two way compressions and there is no shear load. Therefore (T = 0, hence $q_3 = 0$) the equations (7) admit a solution of the form

$$\left.\begin{aligned} u &= A\cos m\varphi\cos\frac{\lambda x}{a}, \\ v &= B\sin m\varphi\sin\frac{\lambda x}{a}, \\ w &= C\cos m\varphi\sin\frac{\lambda x}{a}, \end{aligned}\right\} \qquad (9)$$

where $\lambda = \dfrac{s\pi a}{l}$ (10), l is the length of the shell and s is an integer.

The solution (9) describes a buckling mode with s half waves along the length of the cylinder and $2m$ half waves around its circumference. Although this is far from being the most general solution, it is the one which fulfils reasonable boundary conditions.

It is evident that the solution (9) satisfies the boundary conditions $v = w = 0$ at $x = 0$ and $x = 1$ also $N_x = M_x = 0$ at $x = 0$ and $x = 1$.

This shows that the solution (9) represents the buckling of a shell whose edges are supported in tangential and radial directions, but are neither restricted in the axial direction nor clamped.

Substituting the solution (9) into the differential equation (7) ($q_3 = 0$), the trigonometric functions drop out entirely and we are left with the following equations:

$$\left.\begin{aligned} &A\left[\lambda^2 + (A_1 + k_1 A_4)m^2 - q_1 m^2 - q_2\lambda^2\right] + B\left[-A_2\lambda m\right] + \\ &\qquad C\left[-A_3 - k_1(\lambda^3 - A_4\lambda m^2) - q_1\lambda\right] = 0 \\ &A\left[-A_5\lambda m\right] + B\left[m^2 + (A_6 + 3k_1 A_7)\lambda^2 - q_1 A_9 m^2 - q_2 A_9\lambda^2\right] + \\ &\qquad C\left[m + k_1 A_8\lambda^2 m - q_1 A_9\lambda\right] = 0 \\ &A\left[-A_{10}\lambda - k_1(A_9\lambda^3 - A_7\lambda m^2) - q_1 A_9\lambda\right] + B\left[m + k_1 A_8\lambda^2 m - q_1 A_9\lambda\right] + \\ &\qquad C\left[1 + k_1\left\{A_9\lambda^4 + 2A_{11}\lambda^2 m^2 + A_{12}(m^2 - 1)^2\right\} - A_9(q_1 m^2 + q_2\lambda^2)\right] = 0 \end{aligned}\right\} \qquad (11),$$

The equations (11) are three linear equations with buckling amplitudes *A, B, C* as unknowns and with the brackets as coefficients. Since the equations are homogeneous, they admit, in general, only the solution $A = B = C = 0$, which shows that the shell is not in neutral equilibrium. The non-vanishing solution *A, B, C* is possible if and only if the determinant of the nine coefficients of the equations (11) is equal to zero. Thus the vanishing of this determinant is the buckling condition of the shell. Whenever the buckling condition is fulfilled, any two of the three equations (11) determine the ratios $\frac{A}{C}$ and $\frac{B}{C}$ and thus the buckling mode according to (9). As in all cases of neutral equilibrium, the magnitude of the possible deformation remains arbitrary.

The buckling condition contains four unknowns: the dimensionless loads q_1 and q_2 and the modal parameters m and λ. Also we know that m must be an integer (0, 1, 2, 3, 4, ...) and λ must be an integer multiple of $\pi a/l\left(s=1,2,3,4.....\right)$. Thus we can write the buckling condition separately for every pair $m,\ \lambda$ fulfilling these requirements, and consider it as a relation between q_1 and q_2 which describes those conditions of the two loads for which the shell is in neutral equilibrium.

The coefficients of the equations (11) are linear functions of k_1, q_1 and q_2. The expanded determinant is, therefore, a polynomial of the third degree in these parameters. Since they are very small quantities it is sufficient to keep only the linear terms and to write the buckling condition in the following form:

$$C_1 + C_2 k_1 = C_3 q_1 + C_4 q_2 \qquad (12)$$

The equation (l2) describes a straight line in the $q_1 q_2$ -plane and the limit of the stable domain is a polygon consisting of the sections of straight lines for various pairs of $m,\ \lambda$.

The coefficients $C_1,\ C_2,\ C_3$ and C_4 of the equation (12) can be found by expanding the determinant and putting it equal to zero. Thus we have,

$$C_1 = A_6\left(1 - A_3A_{10}\right)\lambda^4, \qquad (13a)$$

$$C_2 = \left[A_9\lambda^4 + 2A_{11}\lambda^2m^2 + A_{12}m^4\right]\left[A_6\lambda^4 + 2A_{13}\lambda^2m^2 + A_1m^4\right] - A_6\left(A_3A_9 + A_{10}\right)\lambda^6 - 2\lambda^4m^2\left[A_8 + A_{10} - A_5 - A_3\left(A_5A_8 + A_6A_7\right)\right] - \lambda^2m^4\left[2A_1A_8 + 4A_{12}A_{13} + A_4\left(A_5 + A_6 - A_{10}\right)\right] - 2A_1A_{12}m^6 + \left[3A_1A_7 + A_4A_6 + 2A_{12}A_{13}\right]\lambda^2m^2 + A_1A_{12}m^4, \qquad (13b)$$

$$C_3 = m^2\left[A_9\left\{A_1m^4 + A_6\lambda^4 + \left(1 + A_1A_6 - A_5^2\right)\lambda^2m^2\right\}\right] + \lambda^2m^2\left[2\left(A_5 + A_{10}\right) + A_{10}\left(2A_5 - A_{10}\right) + A_6 - A_9\right] - A_1A_9m^4, \qquad (13c)$$

$$C_4 = \lambda^2\left[A_9\left\{A_6\lambda^4 + A_1m^4 + 2A_{13}\lambda^2m^2\right\} + A_1m^2\right]. \qquad (13d)$$

and $\quad A_{13} = 1 + A_1A_6 - A_2A_5. \qquad (14)$

From the formulas (12), (13) and (14) the stability curve may easily be constructed when l and k_1 are given.

5. Numerical calculations:

From the formulae (12) and (13) the stability curve in case of two way compression may easily be drawn when n, l and k_1 are given. We consider the shell to be made of the same material as that of Gaboon (Okoume), so that

$$E_1 = 1.28\times10^6\,psi, \qquad E_2 = 0.11\times10^6\,psi$$
$$E_\nu = 0.014\times10^6\,psi, \qquad G = 0.085\times10^6\,psi$$

vide Timoshenko and Woinowsky-Krieger [14] and $k_1 = 10^{-6}$.

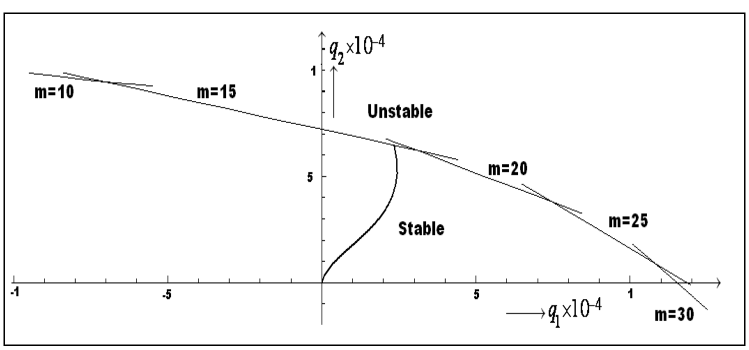

Figure 1: buckling diagram for a 5-layers cylindrical shell λ=20.

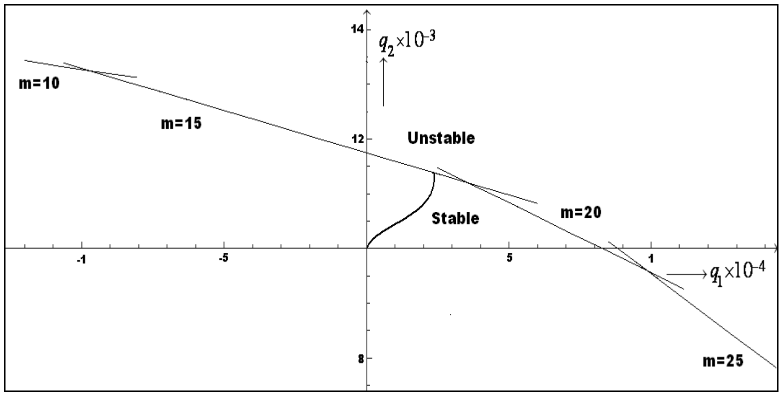

Figure 2: buckling diagram of a 5-layers cylindrical shell for λ=30.

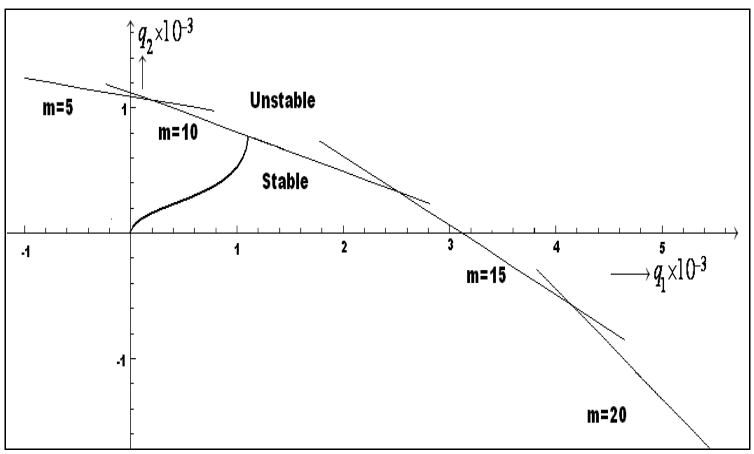

Figure 3: buckling diagram for a 7-layers cylindrical shell λ=30.

6. Conclusions:

When a load is applied, the corresponding diagram point moves along some path, as shown by the bold line in the Figure 1, Figure 2 and Figure 3. As long as it does not meet any of the curves, the shell is in stable equilibrium. But as soon as one of the curves is reached, equilibrium becomes neutral, with the buckling mode defined by the parameters m, λ of each curve. The stable domain in the q_1, q_2

plane is therefore, bounded by the envelope of all the curves which is shown in Figure 1, Figure 2 and Figure 3. The stable and unstable domain for $\lambda = 20$ and $\lambda = 30$ is shown for 5-layers plywood shell and unstable domain for $\lambda = 30$ is shown for 7-layers plywood shell. Stable and unstable domain can be drown from equation (12) along with (13) and (14) for any number of layers.

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