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GENERALIZED MAGNETOHYDRODYNAMIC COUETTE FLOW OF A BINARY MIXTURE OF VISCOUS FLUIDS THROUGH A HORIZONTAL CHANNEL UNDER SORET EFFECT

BY

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Abstract:

The Soret effect of temperature gradient on separation in generalized magnetohydrodynamic (MHD) Couette flow of a binary mixture of incompressible conducting viscous fluids between two parallel plates has been investigated analytically in the case when one plane is subjected to zero heat flux while the other has prescribed temperature. The expressions for velocity, temperature and the concentration are obtained analytically and the behaviour of concentration is shown graphically. It is observed that the temperature gradient separates the binary mixture components and the lighter component gets collected near the moving wall.

Keywords and phrases : magnetohydrodynamic, Couette flow, viscous fluids, heat flux, temperature gradient

বিমূর্ত সার (*Bengali version of the Abstract*)

দু'টি প্লেটের মধ্যে অসংনম্য পরিবাহী সান্দ্র প্রবাহী পদার্থের দ্বিপদ মিশ্রনের সামান্যীকৃত চৌম্বকীয় জলগতিবিদ্যায় **(MHD)** *কুট্টি প্রবাহে* **(Couette flow)** *বিভাজনের উপর উষ্ণতা নতির সরেট* **(Soret)** *প্রভাবকে বিশ্লেষণাত্ব ভাবে অনুসন্ধান করা হয়েছে এরূপ ক্ষেত্রে যখন একটি সমতলেকে শূন্য তাপ অভিবাহ (zero heat flux) অধীন এবং অপরটিকে নির্দেশিত উষ্ণতায় রাখা হয়েছে। গতিবেগ, উষ্ণতা এবং গাঢ়তার জন্য অভিব্যক্তিগুলি বিশ্লেষণাত্ব ভাবে নির্ণয় করা হয়েছে এবং গাঢ়তার আচরণকে লেখচিত্রের সাহায্যে দেখানো হয়েছে। এটা লক্ষ্য করা গেছে যে উষ্ণতার নতি দ্বিপদ মিশ্রনের উপাদানগুলিকে বিভাজিত করেছে এবং হাল্কা উপাদানটি গতিশীল প্রাচীরের কাছে জড়ো হয়েছে।*

Notations:

(x,y,z)	: Cartesian coordinates,
d	: Distance between the two plates,
c_1	: Ratio of mass of rarer and lighter component to the total mass of the mixture in given volume,
c_2 $(=1-c_1)$	: Concentration of heavier and more abundant component,
c_0	: Saturation concentration near the surface of the body that dissolves in the fluid by diffusion,
$\mathbf{V}$	: Mass average velocity of the mixture,
ρ	: Density of the mixture ,
$\mathbf{V}_1$, ρ_1	: Velocity and density of the rarer abundant components respectively,
$\mathbf{V}_2$, ρ_2	: Velocity and density of more abundant components respectively,
m_1 , m_2	: Masses of the rarer and more abundant components respectively,
p	: Pressure of the mixture ,
U	: Uniform velocity parallel to x-axis for the plate at y=d,
T_1	: Temperature for the plate at y=d,
μ	: Coefficient of viscosity of the mixture,
B_0	: Uniform transverse magnetic field ,
$\mathbf{J}$	: Current density vector ,
$\mathbf{B}$	: Magnetic field vector,
$\mathbf{H}$	: Magnetic intensity vector,

E	: Electric field vector,
σ	: Electric conductivity,
μ_e	: Magnetic permeability,
c_p	: Specific heat at constant pressure,
b	: Electrical characteristic of the medium,
T	: Temperature,
κ	: Thermal conductivity of the fluid mixture,
φ	: Heat due to viscous dissipation,
j^2/σ	: Heat due to Joulean dissipation,
D	: Diffusion or mass transfer coefficient,
k_pD	: Baro-diffusion coefficient,
k_TD	: Thermal-diffusion coefficient,
P_∞	: Working pressure of the medium,
S_T	: Soret coefficient,
i	: Diffusion flux density,
n	: Unit normal vector at the solid surface directed outwards,
M^2 $(=\sigma B_0^2 d^2/\mu)$	: Hartmann number,
N $(= -(\partial p/\partial x)d^2/(\mu U))$	: Reynolds number,
E_c $(= U^2/(c_p T_1))$	: Eckert number,
P_r $(= \mu c_p/\kappa)$	: Prandtl number,
t_d $(=S_T T_1)$	: Thermal diffusion number.

1. Introduction

In recent years a lot of works has been done on the flow of viscous, electrically conducting fluids in channels under a uniform magnetic field. The effect of a magnetic field on the natural convection in inclined layers was investigated by Alchaar et al [1] and Bian et al [2]. A numerical study of hydromagnetic thermal convection in a visco-elastic dusty fluid was carried out by Goel and Agrawal [3]. Chauhan and Vyas [4] examined convection effects on the magnetohydrodynamic Couette flow [5] past a highly porous bed. Yen and Chang [6] discussed the heat transfer for steady laminar flow of an incompressible electrically conducting viscous fluid between two parallel straight insulated walls.

Now the study of separation of a binary fluid mixture in which one component is present in an extremely small portion got its importance due to various applications in engineering, e.g., separation of isotopes in its naturally occurring mixture, separation of gases in the air by the pressure and temperature gradient present in the atmosphere. Zimmermann et al [7] analyzed the flow of a binary mixture of liquids of unequal molecular weights through a horizontal channel. In the analysis, it is found that the fluid mixture displays Soret effects. The effect of temperature gradient and pressure gradient on the separation of a binary mixture of incompressible fluids between two parallel plates has been considered by Shah [8] when the motion is steady. In a recent paper Sharma and Singh [9] generalized the problem of Shah for conducting fluids.

In the present paper, we consider the Soret effect in generalized steady Couette flow of a binary mixture of conducting fluids through horizontal infinite parallel plates. It is found that the effect of the temperature gradient is to separate the components of the binary mixture. The lighter component gets collected near the moving wall.

2. Mass transfer equations

Consider the binary mixture of incompressible, thermally and electrically conducting viscous fluids of which one of the components is very small, so that the density and viscosity of the mixture is independent of the distribution of the components. Here

$$\mathbf{V} = (\rho_1 \mathbf{V}_1 + \rho_2 \mathbf{V}_2)/\rho \quad \text{and} \quad \rho = \rho_1 + \rho_2 .$$

The constitutive equations for steady motion are

$$\rho\,(\mathbf{V}.\,\nabla)\mathbf{V} = -\nabla p + \mu\,\nabla^2\mathbf{V} + \mathbf{J}\times\mathbf{B}\,, \tag{1}$$

and

$$\nabla.\mathbf{V} = 0. \tag{2}$$

The Maxwell equations are

$$\nabla \times \mathbf{H} = 4\pi \mathbf{J}, \tag{3}$$

$$\nabla \times \mathbf{E} = \mathbf{0}, \tag{4}$$

$$\nabla . \mathbf{H} = 0, \tag{5}$$

and Ohm's law, on neglecting Hall current , is given by

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B}), \tag{6}$$

where

$$\mathbf{B} = \mu_e \mathbf{H}. \tag{7}$$

The energy equation for steady case leads to

$$\rho c_p \mathbf{V} . \nabla T = \kappa \nabla^2 T + \phi + j^2 / \sigma . \tag{8}$$

The equation for the species conservation of the first component is given by (Landau and Lifshitz [10])

$$\rho (\mathbf{V} . \nabla) c_1 = - \nabla . \mathbf{i}. \tag{9}$$

For binary mixtures of electrically conducting fluids, the diffusion of individual species is considered in four ways viz., the concentration gradient, the pressure gradient, the temperature gradient and current densities. The diffusion flux density **i** is given by (Landau and Lifshitz [10])

$$\mathbf{i} = b \mathbf{J} - \rho D(\nabla c + k_p \nabla p + k_T \nabla T). \tag{10}$$

The coefficients k_p and k_T may be determined from the thermodynamic properties alone. Landau and Lifshitz [11] have given the explicit expression for k_p as

$$k_p = (m_2 - m_1)\{(c_1/m_1) + (c_2/m_2)\} c_1 c_2 / P_\infty . \tag{11}$$

The expression for k_T is given by (Hurle and Jakeman [12])

$$k_T = S_T c_1 c_2 . \tag{12}$$

Substituting the expression for **i** from equation (10) in (9) and using the result

$$\nabla . \mathbf{J} = 0, \tag{13}$$

we get

$$(\mathbf{V} . \nabla) c_1 = D\{\nabla^2 c_1 + \nabla .(k_p \nabla p) + \nabla .(k_T \nabla T)\}. \tag{14}$$

3. Boundary conditions

The boundary conditions for c_1 are different in different cases, but those for flow field, temperature field and electromagnetic field are the same as in the usual magnetohydrodynamic problems. According to Srivastava [13], the total mass flux and the individual species flux normal to the surface vanish at the surface of the insoluble body i.e.,

$$\rho\, c_1 \mathbf{V}.\mathbf{n} + \mathbf{i}.\mathbf{n} = 0. \tag{15}$$

Substituting the expression for **i** from (10) into (15), we get

$$\rho\; c_1 \mathbf{V}.\mathbf{n} - \rho D\,(\nabla c_1.\mathbf{n} + k_p \nabla p.\mathbf{n} + k_T \nabla T.\mathbf{n}) + b\, \mathbf{J}.\mathbf{n} = 0. \tag{16}$$

The boundary condition near the surface of the body which dissolves in the fluid by diffusion, is given by

$$c_1 = c_0. \tag{17}$$

4. Formulation of the problem

Consider the steady flow of a binary mixture of thermally and electrically conducting incompressible viscous fluids in presence of a uniform transverse magnetic field B_0. Also suppose that the fluids are confined between two non-magnetic parallel plates y=0 and y=d. The plate y=0 subjected to zero heat flux is at rest while the plate y=d has the prescribed uniform temperature T_1 and it moves with uniform velocity U parallel to the x-axis. The concentration c_1 of the first component of the binary mixture is maintained at constant value c_0 at y=0 and the plate at y=d is considered to be impervious.

Clearly, by equation (2), $v_y=v_z=0$ and $v_x=v_x(y)$, where v_x,v_y, and v_z are components of **V** along the axes. Also p=p(x) from the momentum equations in y and z directions. Thus the equation of motion (1) and the energy equation (8) in Cartesian coordinates respectively reduce to

$$\frac{d^2 v_{v_x}}{dy^2} (\sigma B_0^2 v_x)/\mu = (1/\mu)\frac{dp}{dx} \tag{18}$$

and

$$\frac{d^2T}{dy^2}(\mu/\kappa)\left(\frac{dv_x}{dy}\right)+(\sigma v_x^2 B_0)/\kappa = 0 \tag{19}$$

In deriving the equation (18) it is assumed that the magnetic Reynolds number is small so that the induced electric and magnetic fields are taken to be zero.

The equation (14) for the species conservation takes the form

$$\frac{d^2c_1}{dy^2}+S_T\frac{d}{dy}\left(c_t\frac{dT}{dy}\right)=0 \tag{20}$$

since $\frac{\partial p}{\partial y} = 0$ and $c_1 = c_1(y)$.

The boundary conditions are given by

$$v_x = 0, \frac{dT}{dy} = 0 \quad \text{at} \quad y = 0 \tag{21}$$

$$v_x = U, T = T_1 \quad \text{at} \quad y = d \tag{22}$$

The boundary condition for the concentration function c_1 at the plate y = d can be written from the equation (15) by putting **n** = **y** (being normal to the surface), and noting that the surface is impermeable, as

$$\frac{dc_1}{dy}+c_1S_T\frac{dT}{dy}=0 \quad \text{at} \quad y = d \tag{23}$$

Since the concentration of the binary mixture at the plate y = 0 is constant we have,

$$c_1 = c_0. \tag{24}$$

For convenience, we introduce the following non-dimensional quantities:

$$u(\eta) = v_x/U, \quad \theta(\eta) = (T-T_1)/T_1, \quad f(\eta) = c_1/c_0, \quad \eta = y/d. \tag{25}$$

Substituting these in equations (18), (19) and (20), we get respectively,

$$\frac{d^2u}{d\eta^2} - M^2u = -N, \tag{26}$$

$$\frac{d^2\theta}{d\eta^2} + P_1 E_c \left\{ \left(\frac{du}{d\eta} \right)^2 + M^2 u^2 \right\} = 0 \tag{27}$$

$$\frac{d^2 f}{d\eta^2} + t_d \frac{d}{d\eta} \left(\frac{d\theta}{d\eta} \right) f = 0, \tag{28}$$

while the boundary conditions (21)-(24) become

$$u = 0, \frac{D\theta}{D\eta} = 0, f = 1 \quad \text{at} \quad \eta = 0 \tag{29}$$

$$u = 1, \theta = 0, \frac{df}{d\eta} + t_d f \frac{d\theta}{d\eta} = 0 \quad \text{at} \quad \eta = 0 \tag{30}$$

5. Solutions

The solutions of the equations (26) to (28) subject to the boundary conditions (29) and (30) are given by

$$u(\eta) = b_1\{1 - \cosh(M\eta)\} + b_2 \sinh(M\eta), \tag{31}$$

$$\theta(\eta) = c_1 \cosh(2M\eta) + c_2 \sinh(2M\eta) + c_3 \cosh(M\eta) + c_4 \sinh(M\eta)$$
$$+ c_5 \eta^2 + c_6 \eta + c_7, \tag{32}$$

$$f(\eta) = \exp\{ t_d [\theta(0) - \theta(\eta)]\}, \tag{33}$$

where

$b_1 = N/M^2$, $b_2 = \{1 - b_1 + b_1 \cosh(M)\}/\sinh(M)$, $c_1 = -0.25\, P\, (b_1^2 + b_2^2)$, $c_2 = 0.5\, P\, b_1 b_2$, $c_3 = 2Pb_1^2$, $c_4 = -2Pb_1b_2$, $c_5 = -0.5PM^2b_1^2$, $c_6 = PMb_1b_2$, $c_7 = 0.25P\, \{ (b_1^2 + b_2^2) \cosh(2M) - 2b_1b_2 \sinh(2M) + 8b_1b_2 \sinh(M) - 8b_1^2\cosh(M) + 2b_1^2M^2 - 4Mb_1b_2\}$, $P = P_r E_c$, $\theta(0) = c_1 + c_3 + c_7$.

6. Discussions

The equation (33) reveals that if we neglect the effect of temperature gradient then the species separation ceases to occur.

Putting $B_0 = 0$ in equations (26) to (28) and solving them under the boundary conditions (29) and (30) we get

$$f_0(\eta) = \exp\{t_d (P_r E_c/24)[12\eta^2 + (12\eta^2 - 8\eta^3)N + (3\eta^2 - 4\eta^3 + 2\eta^4)N^2]\}. \tag{34}$$

So the equation (33) gives a singular value in absence of magnetic field.

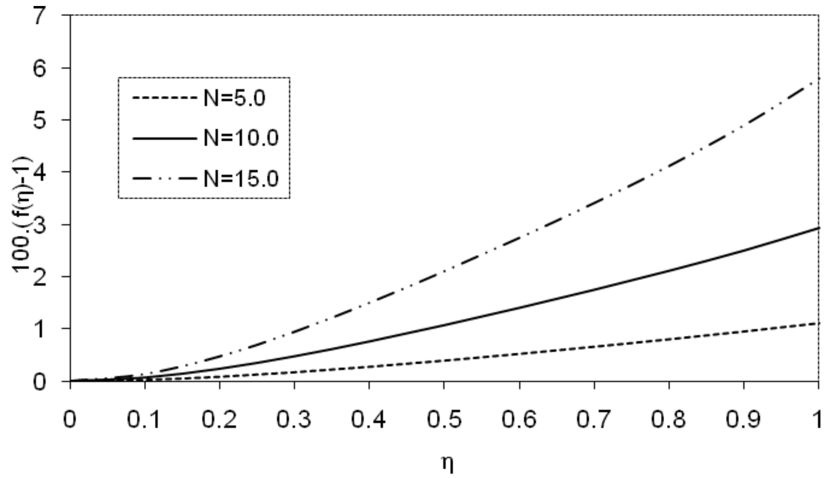

Fig 1 : Variation of concentration for various values of Reynolds number (N) when Pr.Ec=0.05, M=1.0, td=0.1

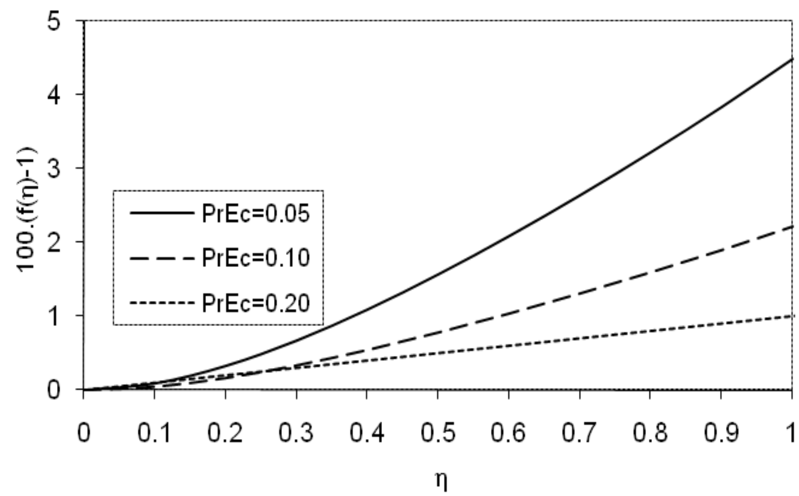

Fig2 :Variation of concentration for various values of the product of Prandtl number and Eckert number (Pr.Ec) when M=1.0, N=5.0, td=0.1

The effects of parameters N, P_r, E_c, t_d and M on the separation process are shown graphically. Figures 1 and 3 show that $f(\eta)$ increases with the increase of the parameters N and t_d. But it is observed from figures 2 and 4, $f(\eta)$

decreases with the increase of parameters $P_r.E_c$ and M. Thus increasing the temperature of the moving plate enhances the species separation of the binary mixture. Reduction of the intensity of the applied magnetic field can also enhance the action of separation of species.

Figures 1 to 4 also show that the concentration of the rarer or lighter species of the binary mixture is more on the surface of the moving plate. Thus the heavier elements are thrown away from the moving plate towards the plate at rest and this process decreases with the increase of the intensity of the transverse magnetic field.

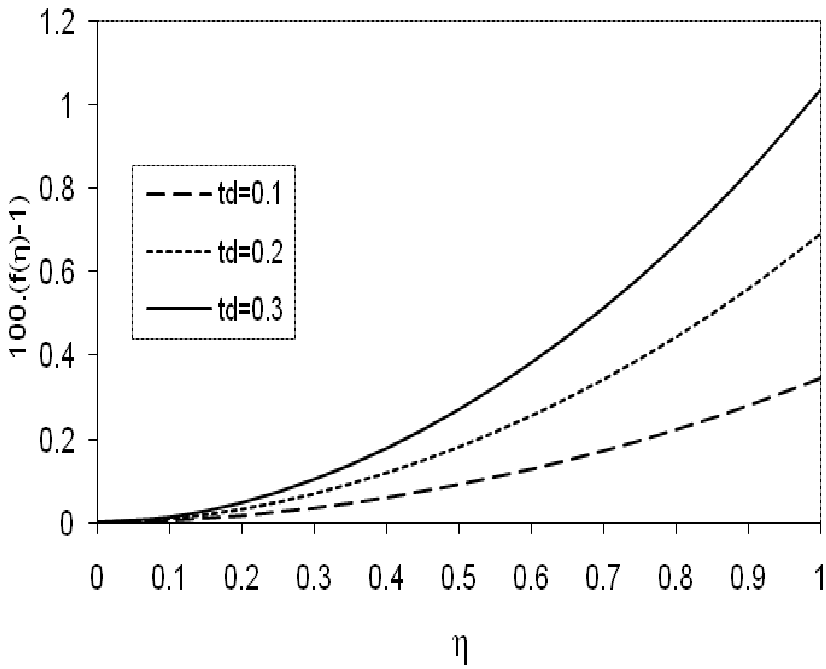

Fig 3 :Variation of concentration for various values of Thermal diffusion number (td) when Pr.Ec=0.05, N=1.0, M=1.0

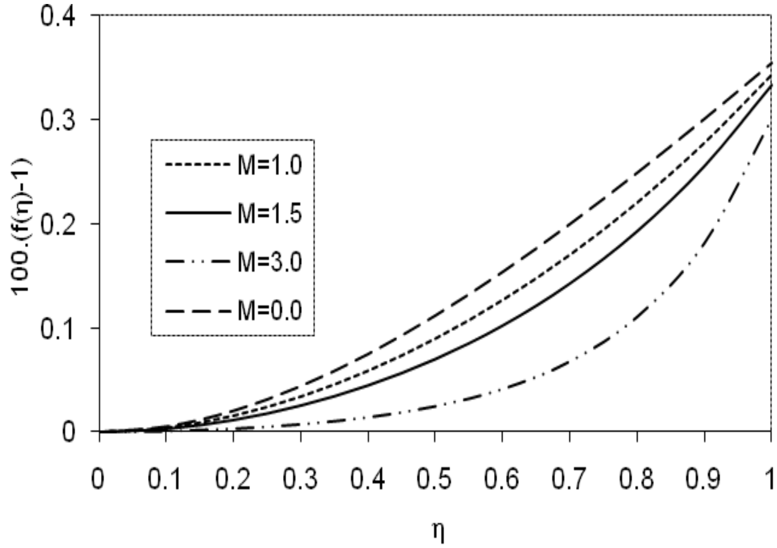

Fig 4 :Variation of concentration for various values of square-root of Hartmann number (M) when Pr.Ec=0.05, N=1.0, td=0.10

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