NATURAL CONVECTIVE HEAT TRANSFER TRANSITORY FLOW IN PRESENCE OF INDUCED MAGNETIC FIELD

By

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Abstract

The effects of induced magnetic field on a free convective heat transfer transient flow of fluid past an infinite vertical plate through a porous medium have been investigated numerically. A mathematical model of the problem is developed from the basis of studying magneto-fluid dynamics(MFD) and the equations are solved by the finite difference method. The numerical values of non-dimensional velocity, induced magnetic field and temperature are computed for the different values of associated parameters in different times. In order to discuss the results, the obtained numerical values of flow variables are plotted in graphs. Finally the important findings of this work are concluded here.

Keywords and phrases : magnetic field, free convective heat transfer, magneto-fluid dynamics, non-dimensional velocity

বিমূর্ত সার (Bengali version of the Abstract)

একটি অসীম উলম্ব প্লেটের সছিদ্র মাধ্যম দিয়ে অতিক্রান্ত ফ্লুইডের একটি মুক্ত পরিচালনী তাপ সঞ্চালন প্রবাহের উপর আবিষ্ট চৌম্বক ক্ষেত্রের প্রভাবকে সাংখ্য রীতিতে অনুসন্ধান করা হয়েছে । চৌম্বকীয় প্রবাহগতিবিদ্যার অনুসন্ধানের ভিত্তিতে এই সমস্যার একটি গাণিতিক মডেল তৈরী করা হয়েছে এবং সমীকরণগুলিকে সসীম - অন্তর (finite difference) পদ্ধতিতে সমাধান করা হয়েছে । অ-মাত্রিক গতিবেগ,আবিষ্ট চৌম্বক ক্ষেত্র এবং উষ্ণতার সাংখ্যমান গণনা করা হয়েছে

বিভিন্ন সময়ে সহযোগী প্রাচলের বিভিন্ন মানের জন্য । ফলাফল সম্পর্কে আলোচনা করার জন্য প্রবাহ - চলের নির্ণিত সাংখ্যমানগুলিকে লেখচিত্রে অংকন করা হয়েছে । শেষত: এই কাজের গুরুত্বপূর্ণ ফলাফলগুলিকে দেখানো হয়েছে।

1. Introduction

The convective heat transfer flows play a decisive role in many engineering applications as distillation, condensation, evaporation, rectification and absorption of a fluid as well as in fluids condensing or boiling at a solid surface. The heat transfer processes are of great interest in power engineering, metallurgy, astrophysics and geophysics. A natural convective heat transfer flow of fluid was first studied by *Finston*(1956). *Sparrow* and *Gregg*(1958) computed a similar solution for laminar free convection from a non-isothermal vertical plate. A finite difference solution of transient free convective flow over an isothermal plate has been obtained by *Soundalgekar* and *Ganesan*(1981). A numerical study on the natural convective cooling problem of a vertical plate is completed by *Camargo et al.*(1996).

The natural convective fluid flows through a porous medium are of great interest in many industrial applications as to insulate the heated body to maintain its temperature. A steady free convective flow through a porous medium bounded by an infinite surface by use of the model of *Yamamoto* and *Iwamura*(1976) for the flow near the surface has been observed by *Raptis et al.*(1981). A free convective flow with heat transfer through a porous medium has been studied by *Ahmed* and *Sarma*(1997). Recently, the analytic solutions of unsteady free convective fluid flow in porous medium have been obtained by *Magyari et al.*(2004).

All the above problems are studied in the absence of induced magnetic field. However, the flow under the action of a strong magnetic field that induced another magnetic field has a great interest in geophysics and astrophysics. Hence, our main goal is to investigate a free convective heat transfer unsteady flow through a porous medium in the presence of an induced magnetic field.

2. Mathematical Model

A natural convective heat transfer unsteady flow of an electrically conducting, incompressible, viscous fluid past an electrically non-conducting infinite vertical plate surrended by a porous medium is considered here. The flow is assumed to be in the *x*-direction, which is chosen along the plate in upward direction and *y*-axis is normal to it. A strong magnetic field is applied normal to the flow region that induced an induced magnetic field. Initially, it is considered that the plate as well as the fluid particles are at rest at the same temperature $T(=T_\infty)$ at all points, where T_∞ be the uniform temperature of fluid.

Within the framework of the above stated assumptions, we have the following system of coupled non-linear partial differential equations in accordance with the Boussinesq's approximation,

Continuity Equation
$$\frac{\partial v}{\partial y} = 0$$

Momentum Equation
$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = g\beta(T-T_\infty) + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\mu_e}{\rho}H_0\frac{\partial H_x}{\partial y} - \frac{\upsilon}{K}u$$

Magnetic Induction Equation
$$\frac{\partial H_x}{\partial t} + v\frac{\partial H_x}{\partial y} = H_0\frac{\partial u}{\partial y} + \frac{1}{\sigma\mu_e}\frac{\partial^2 H_x}{\partial y^2}$$

Energy Equation
$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p \sigma}\left(\frac{\partial H_x}{\partial y}\right)^2 + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

under also the boundary layer phenomena, the appropriate initial and boundary conditions of the problem are as given below,

$t \le 0, \quad u = 0, \quad H_x = 0, \quad T \to T_\infty, \quad$ everywhere

$t > 0, \quad u = 0, \quad H_x = H_w, \quad T = T_w, \quad$ as $y \to 0$

$\quad\quad\quad u = 0, \quad H_x \to 0, \quad T \to T_\infty, \quad$ as $y \to \infty$

where x & y are cartesian coordinates in two directions, u & v are velocity components of flow, g is the local acceleration due to gravity, β is the thermal expansion coefficient, υ is the kinematic viscosity, μ_e is the magnetic permeability, ρ is the density of the fluid, K is the permeability of porous medium, H_0 is the constant induced magnetic field, H_x be the induced magnetic field component, σ is the electrical conductivity, κ is the thermal conductivity, c_p is the specific heat at constant pressure and H_w is the induced magnetic field at the wall.

From the continuity equation, we get $v = \text{constant} = -V_0$ (Constant Suction Velocity). To find the solution of the problem, it is required to transfer the system of equations into a non-dimensional system, so we take the following dimensionless quantities,

$$Y = \frac{yV_0}{\upsilon}, \qquad U = \frac{u}{V_0}, \qquad \tau = \frac{tV_0^2}{\upsilon}, \qquad \bar{H}_x = \sqrt{\frac{\mu_e}{\rho}} \frac{H_x}{V_0}, \qquad \text{and} \quad \bar{T} = \frac{T - T_\infty}{T_w - T_\infty}.$$

Using the above quantities, we have the following model interms of dimensionless variables,

Non-dimensional Momentum Equation $\dfrac{\partial U}{\partial \tau} - \dfrac{\partial U}{\partial Y} = G_r \bar{T} + \dfrac{\partial^2 U}{\partial Y^2} + M \dfrac{\partial \bar{H}_x}{\partial Y} - \gamma U$

Non-dimensional Magnetic Induction Equation $\dfrac{\partial \bar{H}_x}{\partial \tau} - \dfrac{\partial \bar{H}_x}{\partial Y} = M \dfrac{\partial U}{\partial Y} + \dfrac{1}{P_m} \dfrac{\partial^2 \bar{H}_x}{\partial Y^2}$

Non-dimensional Energy Equation $\dfrac{\partial \bar{T}}{\partial \tau} - \dfrac{\partial \bar{T}}{\partial Y} = \dfrac{1}{P_r} \dfrac{\partial^2 \bar{T}}{\partial Y^2} + \dfrac{E_c}{P_m} \left(\dfrac{\partial \bar{H}_x}{\partial Y} \right)^2 + E_c \left(\dfrac{\partial U}{\partial Y} \right)^2$

where τ represents the dimensionless time, Y is the dimensionless cartesian co-ordinate, U is the dimensionless velocity component, $\bar{T}$ be the dimensionless temperature and the non-dimensional parameters are as given below,

$$G_r = \frac{\upsilon g \beta (T_w - T_\infty)}{V_0^3} = \textbf{Grashof Number}, \quad M = \frac{H_0}{V_0} \sqrt{\frac{\mu_e}{\rho}} = \textbf{Magnetic Force Number},$$

$P_m = \upsilon \sigma \mu_e =$ **Magnetic Diffusivity Number**, $\qquad P_r = \dfrac{\upsilon \rho c_p}{\kappa} =$ **Prandtl Number**,

$E_c = \dfrac{V_0^2}{c_p (T_w - T_\infty)} =$ **Eckert Number**, $\qquad \gamma = \dfrac{\upsilon^2}{K V_0^2} =$ **Permeability Number**

also the non-dimensional boundary with initial conditions are as follows,

$\tau \leq 0, \quad U = 0, \quad \bar{H}_x = 0, \quad \bar{T} = 0 \qquad$ everywhere

$\tau > 0, \quad U = 0, \quad \bar{H}_x = 1, \quad \bar{T} = 1, \qquad$ as $Y \to 0$

$\qquad\qquad U = 0, \quad \bar{H}_x \to 0, \quad \bar{T} = 0, \qquad$ as $Y \to \infty$.

3. Numerical Solutions

In order to solve the mathematical model of nonlinear coupled dimensionless partial differential equations with associated initial and boundary conditions, finite difference method has been used in this section. To obtain the difference equations, the region of the flow within the boundary layer is divided into a grid or mesh of lines parallel to X-axis where Y-axis is normal to the plate. Here it is considered that $Y_{max} (= 20)$ as corresponding to $Y \to \infty$ i.e. Y varies 0 to 20. It is also assumed that $\Delta Y = 0.125 (0 \leq y \leq 20)$ is a constant mesh size along Y direction with a smaller time step $\Delta \tau (= 0.01)$.

Let U', $\bar{H}_x'$, $\bar{T}'$ denote the values of U, $\bar{H}_x$, $\bar{T}$ at the end of a time-step respectively. Using the finite difference approximations we obtain the following appropriate set of finite difference equations,

Finite Difference Momentum Equation

$$\frac{U'_j - U_j}{\Delta \tau} - \frac{U_{j+1} - U_j}{\Delta Y} = G_r \bar{T}'_j + \frac{U_{j+1} - 2U_j + U_{j-1}}{(\Delta Y)^2} + M \frac{\bar{H}_{x\,j+1} - \bar{H}_{x\,j}}{\Delta Y} - \gamma U_j$$

Finite Difference Magnetic Induction Equation

$$\frac{\bar{H}'_{x\,j} - \bar{H}_{x\,j}}{\Delta\tau} - \frac{\bar{H}_{x\,j+1} - \bar{H}_{x\,j}}{\Delta Y} = M\frac{U_{j+1} - U_j}{\Delta Y} + \frac{1}{P_m}\frac{\bar{H}_{x\,j+1} - 2\bar{H}_{x\,j} + \bar{H}_{x\,j-1}}{(\Delta Y)^2}$$

Finite Difference Energy Equation

$$\frac{\bar{T}'_j - \bar{T}_j}{\Delta\tau} - \frac{\bar{T}_{j+1} - \bar{T}_j}{\Delta Y} = \frac{1}{P_r}\frac{\bar{T}_{j+1} - 2\bar{T}_j + \bar{T}_{j-1}}{(\Delta Y)^2} + \frac{E_c}{P_m}\left(\frac{\bar{H}_{x\,j+1} - \bar{H}_{x\,j}}{\Delta Y}\right)^2 + E_c\left(\frac{U_{j+1} - U_j}{\Delta Y}\right)^2$$

and the initial and boundary conditions with the finite difference scheme are,

$$\left.\begin{array}{lll} U_j^0 = 0, & \bar{H}_{x\,j}^0 = 0, & \bar{T}_j^0 = 0 \\ U_j^n = 0, & \bar{H}_{x\,j}^n = 1, & \bar{T}_j^n = 1 \\ U_L^n = 0, & \bar{H}_{xL}^n = 0, & \bar{T}_L^n = 0 \end{array}\right\}, \quad \text{where} \quad L \to \infty.$$

Here the subscripts j designate the grid points in Y direction and the superscript n represents a value of time, $\tau = n\Delta\tau$ where $n = 0, 1, 2, \ldots$. From the initial condition, we have the values of U, $\bar{H}_x$, $\bar{T}$ are zero. During any one time-step, the coefficient U_j appearing in equations are treated as constant. Hence at the end of any time-step $\Delta\tau$, the new temperature $\bar{T}'$, the new velocity U', the new induced magnetic field $\bar{H}_x'$ at all interior nodal points may be obtained by successive applications of energy, momentum and induction equations respectively. This process is repeated in time and it is provided the time-step is sufficiently small, hence U, $\bar{H}_x$, $\bar{T}$ should eventually converge to values which approximate the steady-state solution of the problem.
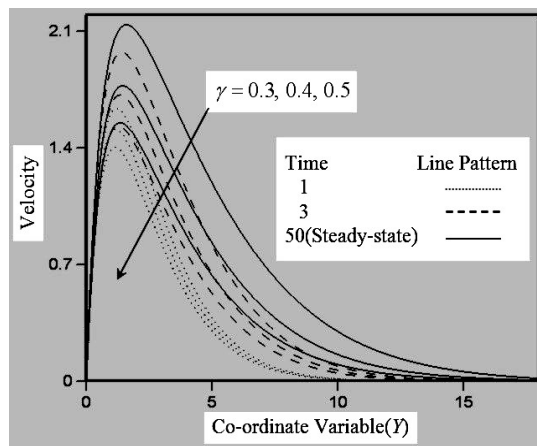
4. Results and Discussion

In order to discuss the result of the problem, the steady state solutions are obtained by using the finite difference method. To discuss the physical situation of the model, we have computed the numerical values of the non-dimensional velocity, induced magnetic field and temperature within the boundary layer for different

values of magnetic parameter (M), magnetic diffusivity number (P_m), Grashof number (G_r), Prandtl number (P_r), permeability number (γ) and Eckert number (E_c). It is observed that the results of the computations for flow variables show little changes after the time $\tau = 20$. Hence the solution at $\tau = 50$ are essentially steady state solutions. In this section, the fluid velocity, induced magnetic field and temperature versus the co-ordinate variable Y are illustrated in Figs. 1-12 for the time $\tau = 1, 2, 50$.

The transient velocity profiles have been drawn in Figs. 1-4. The Fig. 1 represents, the fluid velocity gradually increases with the increasing value of Grashof number or time. In Fig. 2, we see that the velocity rapidly decreases with the increases of permeability number while it rises with the increase of time. It is observed from Fig. 3 that the velocity increases in case of strong magnetic diffusivity number. A strong decreasing effect of Prandtl number on velocity profiles is observed from Fig. 4.
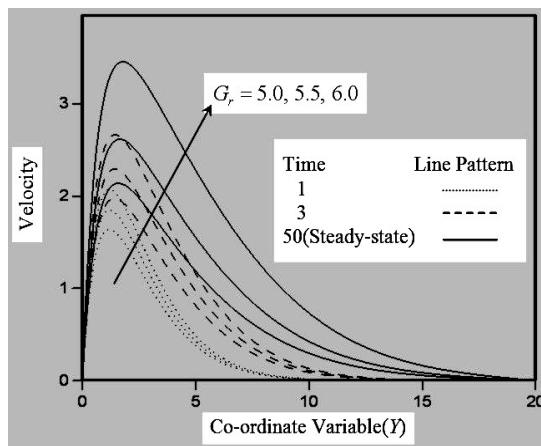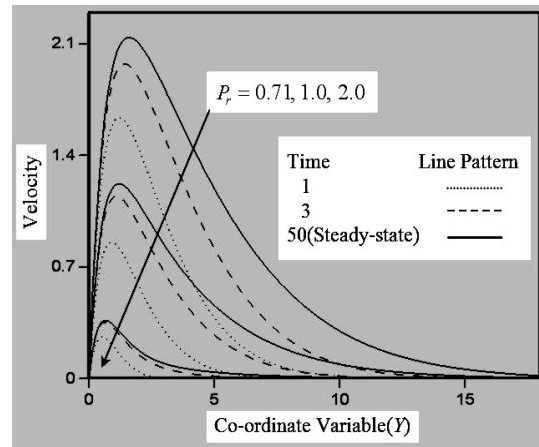

**Fig**. 1. Velocity Profiles for G_r **Fig**. 2. Velocity Profiles for γ
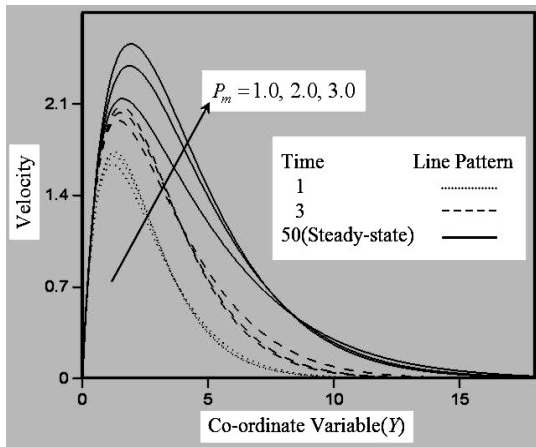
Fig. 3. Velocity Profiles for P_m


Fig. 4. Velocity Profiles for P_r

The transient profiles of induced magnetic field are displayed in Figs. 5-8 for an externally cooled plate $\left(G_r > 0\right)$. The effect of Grashof number on the induced magnetic field is observed from the Fig. 5. It is shown that the induced magnetic field strongly decreases for the increase of G_r or the time. Decreasing effect of magnetic parameter on the induced magnetic field is observed from Fig. 6. The Fig. 7 shows that $\bar{H}_x$ decreases near the plate but increases far away from the plate with the rise of P_m. It is observed from the Fig. 8 that the induced magnetic field is increasingly affected by the Prandtl number.
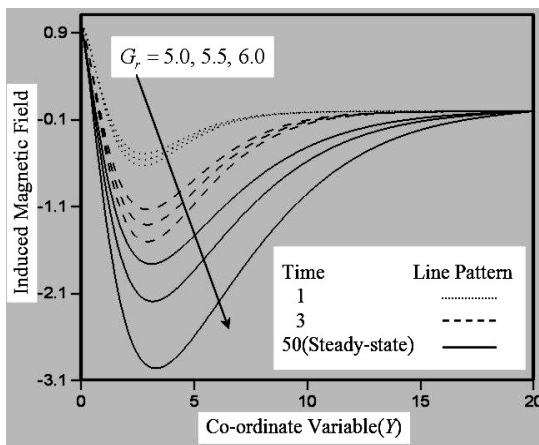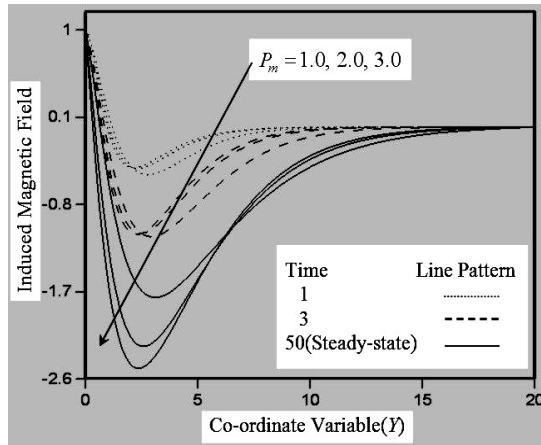

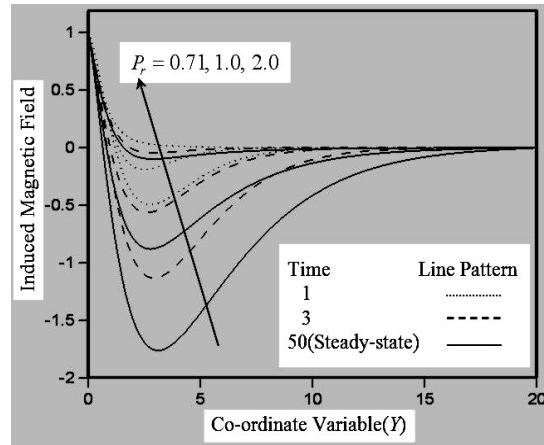**Fig**. 5. Induced Magnetic Field for G_r
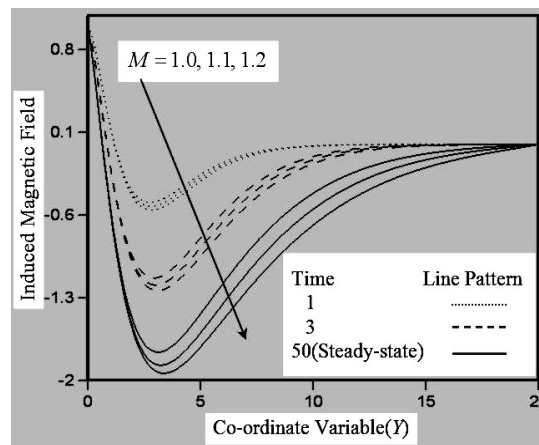
**Fig**. 6. Induced Magnetic Field for M

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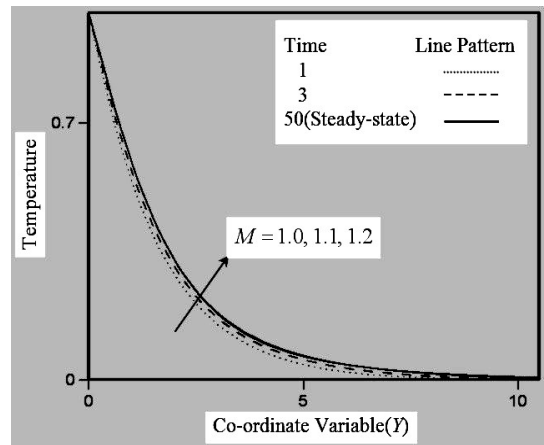
Fig. 7. Induced Magnetic Field for G_r **Fig**. 8. Induced Magnetic Field for M

The transient temperature distributions are shown in Figs. 9-12. A minor increasing effect of Grashof number on the temperature of fluid is observed from Fig. 9. In Fig. 10, we see a negligible effect of M on the profiles of temperature. It is observed from Fig. 11 that the temperature increases in case of strong Eckert number or time. The Fig. 12 shows that the fluid temperature gradually decreases with the rise of Prandtl number P_r while it increases with the increasing values of time.
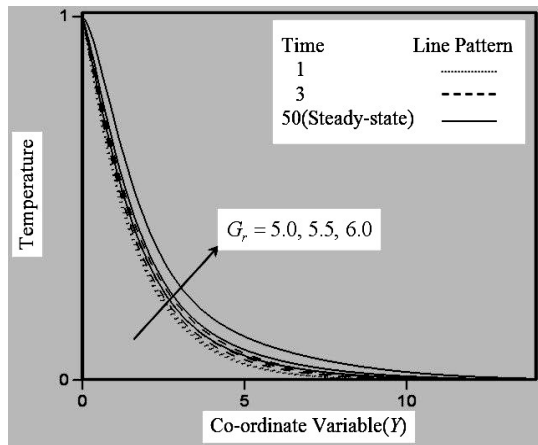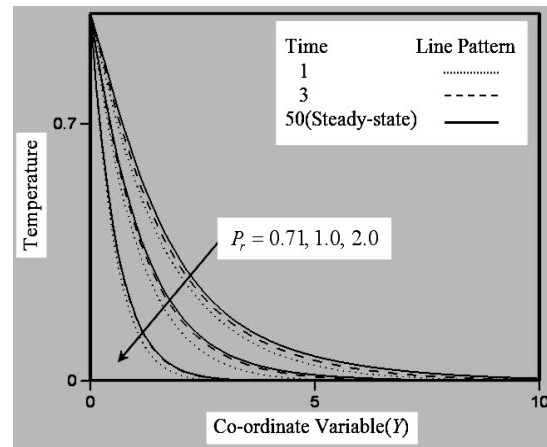

Fig. 9. Temperature Profiles for G_r **Fig**. 10. Temperature Profiles for M
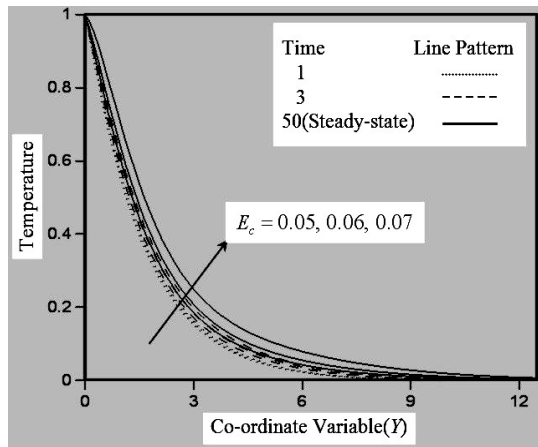
Fig. 11. Temperature Profiles for E_c **Fig**. 12. Temperature Profiles for P_r

5. Conclusions

A numerical investigation on a natural convective heat transfer unsteady flow through a porous medium is completed in the presence of an induced magnetic field. From the graphical representation of the results, some important findings of the present problem are listed below,

1. The transient velocity increases with the rise of G_r or P_m while it decreases in case of strong γ or P_r.

2. The transitory induced magnetic field decreases with the increase of G_r, M or P_m while it rises with the increase of P_r.

3. The transient fluid temperature increases in case of strong G_r, M or E_c while it decreases for the increasing value of Prandtl number.

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