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SOME ASPECTS OF COMPACT FUZZY SETS

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Abstract

The aim of the present is to study compact fuzzy set using the definition of C. L. Chang and obtain its several aspects .

Keywords and phrases : fuzzy set, compact fuzzy set, fuzzy topological spaces

বিমূর্ত সার *(Bengali version of the Abstract)*

এই পত্রের উদ্দেশ্য হচ্ছে সি - এল - চাঙ (C. L. Chang) - এর সংজ্ঞাকে ব্যবহার করে কম্‌পেক্ট ফাজি সেট্‌কে (compact fuzzy set) বিচার করা এবং ইহার বিভিন্ন অবয়বগুলি নির্ণয় করা ।

1. Introduction

The concept of fuzzy sets and fuzzy set operations was first introduced by L. A. Zadeh in his classical paper [9] in the year 1965 , describing fuzziness mathematically first time . C. L. Chang (1968) developed the theory of fuzzy topological spaces and fuzzy compactness was also studied in this paper. Compactness occupies a very important place in fuzzy topological spaces . The purpose of this paper is to study the concept due to C. L. Chang in more detail and to obtain several other aspects .

2. Preliminaries

We briefly touch upon the terminological concepts and some definitions , which are needed in the sequel . The following are essential in our study and can be found in the paper referred to .

2.1 Definition [9] **:** Let X be a non-empty set and I is the closed unit interval [0, 1]. A fuzzy set in X is a function u : X $\rightarrow$ I which assigns to every element x $\in$ X. u(x) denotes a degree or the grade of membership of x. The set of all fuzzy sets in X is denoted by I^X . A member of I^X may also be a called fuzzy subset of X .

2.2 Definition [7] **:** A fuzzy set is empty iff its grade of membership is identically zero . It is denoted by 0 or ϕ.

2.3 Definition [7] **:** A fuzzy set is whole iff its grade of membership is identically one in X . It is denoted by 1 or X .

2.4 Definition [2] **:** Let u and v be two fuzzy sets in X. Then we define

(i) u = v iff u(x) = v(x) for all x $\in$ X

(ii) u $\subseteq$ v iff u(x) $\leq$ v(x) for all x $\in$ X

(iii) λ = u $\cup$ v iff λ(x) = (u $\cup$ v) (x) = max [u(x) , v(x)] for all x $\in$ X

(iv) μ = u $\cap$ v iff μ(x) = (u $\cap$ v) (x) = min [u(x) , v(x)] for all x $\in$ X

(v) $\gamma = u^c$ iff γ(x) = 1 – u(x) for all x $\in$ X .

2.5 Definition [2] **:** Let f : X $\rightarrow$ Y be a mapping and u be a fuzzy set in X. Then the image of u , written $f(u)$, is a fuzzy set in Y whose membership function is given by

$$f(u)(y) = \begin{cases} \sup\{u(x) : x \in f^{-1}(y)\} & if \;\; f^{-1}(y) \neq \phi \\ 0 & if \;\; f^{-1}(y) = \phi \end{cases}$$

2.6 Definition [2] **:** Let f : X $\rightarrow$ Y be a mapping and v be a fuzzy set in Y. Then the inverse of v, written f^{-1}(v) , is a fuzzy set in X whose membership function is given by (f^{-1}(v)) (x) = v(f(x)) .

2.7 De-Morgan's laws [9] **:** De-Morgan's Laws valid for fuzzy sets in X i.e. if u and v are any fuzzy sets in X, then

(i) 1– (u $\cup$ v) = (1 – u) $\cap$ (1 – v)

(ii) 1 – (u $\cap$ v) = (1 – u) $\cup$ (1 – v)

For any fuzzy set in u in X , u$\cap$(1 – u) need not be zero and u$\cup$(1 – u)need not be one (Zadeh 1965).

2.8 Definition[2] **:** Let X be a non-empty set and t$\subseteq I^X$ i.e. t is a collection of fuzzy set in X. Then t is called a fuzzy topology on X if

(i) 0 , 1 $\in$t

(ii) $u_i \in$t for each i$\in$J , then $\bigcup_i u_i \in$t

(iii) u , v $\in$t , then u$\cap$v $\in$t

The pair (X , t) is called a fuzzy topological space and in short, fts. Every member of t is called a t-open fuzzy set. A fuzzy set is t-closed iff its complements is t-open. In the sequel, when no confusion is

likely to arise, we shall call a t-open (t-closed) fuzzy set simply an open (closed) fuzzy set .

2.9 Definition[7] **:** A fuzzy point λ in X is a fuzzy set in X which is zero everywhere except at one point , say x , where it takes value , say r with 0 < r < 1 . We denote it by x_r and we call the point x its support and r its value .

2.10 Definition[7] **:** Let x_r be a fuzzy point in an fts (X , t) . A fuzzy set λ in X is called a fuzzy neighborhood (in short nhd) of x_r iff there exist an open fuzzy set μ in X such that $x_r \in \mu \subseteq \lambda$.

2.11 Definition[2] **:** Let (X , t) and (Y , s) be two fuzzy topological spaces. A mapping f : $(X , t) \rightarrow (Y , s)$ is called an fuzzy continuous iff the inverse of each s-open fuzzy set is t-open.

2.12 Definition[7] **:** Let (X , t) be an fts and A $\subseteq$ X. Then the collection t_A = { u|A : u$\in$t } = {u$\cap$A : u$\in$t} is fuzzy topology on A, called the subspace fuzzy topology on A and the pair (A , t_A) is referred to as a fuzzy subspace of (X , t).

2.13 Definition [4] **:** An fts (X, t) is said to be fuzzy Hausdorfff iff for all x , y $\in$ X , x $\neq$ y, there exist u , v $\in$ t such that u(x) = 1 , v(y) = 1 and u $\cap$ v = 0 .

2.14 Distributive laws [9] **:** Distributive laws remain valid for fuzzy sets in X i.e. if u , v and w are fuzzy sets in X , then

(i) $u \cup (v \cap w) = (u \cup v) \cap (u \cup w)$

(ii) $u \cap (v \cup w) = (u \cap v) \cup (u \cap w)$.

2.15 Definition [3] **:** Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively and f is a mapping from (X, t) to (Y, s), then we say that f is a mapping from (A, t_A) to (B, s_B) if f(A) $\subseteq$ B.

2.16 Definition [3] **:** Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively. Then a mapping f : $(A, t_A) \to (B, s_B)$ is relatively fuzzy continuous iff for each v $\in s_B$, the intersection f^{-1}(v) $\cap$ A $\in t_A$.

2.17 Definition [7] **:** Let λ be a fuzzy set in an fts (X, t). Then the closure of λ is denoted by $\bar{\lambda}$ and defined by $\bar{\lambda} = \cap \{ \mu : \lambda \subseteq \mu$ and $\mu \in t^c \}$.

2.19 Definition [1] **:** An fts (X, t) is said to be fuzzy regular iff for each x $\in$ X and u $\in t^c$ with u(x) = 0 , there exist v , w $\in$ t such that u(x) = 1 , u $\subseteq$ w and v $\subseteq$ 1 – w .

2.20 Definition [1] **:** Let λ be a fuzzy set in X , then the set { x $\in$ X : λ(x) > 0 } is called the support of λ and is denoted by λ_0 or supp λ .

3. Characterizations of compact fuzzy sets .

Now we obtain some tangible features of compact fuzzy sets .

3.1 Definition [2] **:** Let (X, t) be an fts and λ be a fuzzy set in X . Let M = $\{ u_i$: i $\in$ J $\} \subseteq I^X$ be a family of fuzzy sets . Then M = $\{ u_i \}$ is called a cover of λ if $\lambda \subseteq \bigcup \{ u_i$: i $\in$ J } . If each u_i is open , then M = $\{ u_i \}$ is called an open cover of λ .

3.2 Definition : A fuzzy set λ in X is said to be compact iff every open cover of λ has a finite subcover i.e. there exist u_{i_1} , u_{i_2} , ….. , $u_{i_n} \in \{u_i\}$ such that $\lambda \subseteq u_{i_1} \cup u_{i_2} \cup$ ….. $\cup$ u_{i_n} . If $\mu \subset \lambda$ and $\mu \in I^X$, then μ is also compact . If λ(x) = 1 for all x $\in$ X , then this definition coincides an fts (X, t) with that of Chang .

3.4 Theorem : Let λ be a compact fuzzy set in an fts (X, t) and A $\subseteq$ X . Then the following are equivalent :

(i) λ is compact with respect to t .

(ii) λ is compact with respect to the subspace fuzzy topology t_A on A .

Proof : (i) $\Rightarrow$ (ii) : Let $\{u_i$: i$\in$ J } be a t_A– open cover of λ . Then by definition of subspace fuzzy

topology , there exists $v_i \in$ t such that u_i = A $\cap v_i \subseteq v_i$. Hence $\lambda \subseteq \bigcup\limits_{i \in J} u_i \subseteq \bigcup\limits_{i \in J} v_i$ and consequently $\{v_i$: i$\in$ J} is a t – open cover of λ . As λ is compact in (X, t) , then $\{v_i$: i$\in$ J} contains a finite subcover i.e. there exist v_{i_1} , v_{i_2} , ….. , $v_{i_n} \in \{v_i\}$ such that $\lambda \subseteq v_{i_1} \cup v_{i_2} \cup$ ….. $\cup$ v_{i_n} . But then $\lambda \subseteq$ A$\cap(v_{i_1} \cup v_{i_2} \cup$ ….. $\cup$ v_{i_n}) = (A$\cap v_{i_1}$)$\cup$(A $\cap v_{i_2}$)$\cup$….. $\cup$(A$\cap v_{i_n}$) = $u_{i_1} \cup u_{i_2} \cup$ ….. $\cup$ u_{i_n} . Thus $\{u_i$: i$\in$ J } contains a finite subcover of { u_1 , u_2 , …. , u_n } and hence (λ , t_A) is compact . (ii) $\Rightarrow$ (i) : Let $\{v_i$: i$\in$ J } be a t – open cover of λ . Set u_i = A $\cap v_i$, then $\lambda \subseteq \bigcup\limits_{i \in J} v_i$ implies that $\lambda \subseteq$A $\cap(\bigcup\limits_{i \in J} v_i) \subseteq \bigcup\limits_{i \in J}$(A $\cap v_i) \subseteq \bigcup\limits_{i \in J} u_i$. But $u_i \in t_A$, so $\{u_i$: i$\in$ J } is a t_A - open cover of λ . As λ is compact in (A , t_A) , then $\{u_i$: i$\in$ J } contains a finite subcover , say $\{u_{i_k}\}$ (k $\in$ J_n) . Accordingly , $\lambda \subseteq u_{i_1} \cup u_{i_2} \cup$…..$\cup u_{i_n} \subseteq$(A$\cap v_{i_1}$)$\cup$(A$\cap v_{i_2}$)$\cup$……$\cup$(A$\cap v_{i_n}$)

$\subseteq A \cap (v_{i_1} \cup v_{i_2} \cup \ldots\ldots \cup v_{i_n}) \subseteq v_{i_1} \cup v_{i_2} \cup \ldots\ldots \cup v_{i_n}$. Thus $\{ v_i : i \in J \}$ contains a finite subcover $\{ v_{i_k} \}$ (k $\in J_n$) and therefore λ is compact with respect to t .

3.5 Theorem : Let λ and μ be compact fuzzy sets in an fts (X, t). Then $\lambda \cup \mu$ is also compact .

Proof : Let M = $\{ u_i : i \in J \}$ be an open cover $\lambda \cup \mu$. Then M is any open cover of both λ and μ respectively . Since λ is compact in (X, t) , then each open cover of λ has a finite subcover i.e. there exist $v_{i_k} \in$ M (k $\in J_n$) such that $\lambda \subseteq \cup \{ v_{i_k} : k \in J_n \}$. Again

μ is compact in (X, t) , then each open cover of μ has a finite subcover i.e. there exist $w_{i_k} \in$ M (k $\in J_n$) such that $\lambda \subseteq \cup \{ w_{i_k} : k \in J_n \}$. Therefore $\{ v_{i_k} \} \cup \{ w_{i_k} \}$ is a finite subcover of M . Hence $\lambda \cup \mu$ is compact in (X, t) .

3.6 Theorem : Let (X, t) be an fts and λ be a fuzzy set in X . Then λ is compact iff for each $x_r \in \lambda$ with a fuzzy neighborhood v_r of x_r , there are finite number of fuzzy points $x_{r_1}, x_{r_2}, \ldots\ldots, x_{r_n}$ of λ such that $\lambda \subseteq \bigcup_{k=1}^{n} v_{r_k}$.

Proof : Suppose λ is compact and for each $x_r \in \lambda$, there is a fuzzy neighborhood v_r of x_r . Then by

definition of fuzzy neighborhood , there is an open fuzzy set u_r such that $x_r \in u_r \subseteq v_r$

and consequently

the family $\{ u_r : r \in J \}$ is an open cover of λ . As λ is compact , then there exist u_{r_1} ,

$u_{r_2}, \ldots\ldots, u_{r_n} \in$

$\{u_r\}$ such that $\lambda \subseteq u_{r_1} \cup u_{r_2} \cup \ldots\ldots \cup u_{r_n}$. But $u_{r_k} \subseteq v_{r_k}$ for each k , whence $\{ v_{r_1}$, v_{r_2} , ….. , $v_{r_n}\}$ is a cover of λ i.e. $\lambda \subseteq \bigcup_{k=1}^{n} v_{r_k}$. Hence we see that $x_{r_1} \in u_{r_1}$, $x_{r_2} \in u_{r_2}$, ….. , $x_{r_n} \in u_{r_n}$ and $\lambda \subseteq \bigcup_{k=1}^{n} u_{r_k}$.

Conversely , suppose that whenever , for each $x_r \in \lambda$, there is a fuzzy neighborhood v_r of x_r is given , there are finite number of fuzzy points x_{r_1} , x_{r_2} , ….. , x_{r_n} of λ such that $\lambda \subseteq \bigcup_{k=1}^{n} v_{r_k}$. As v_r is a fuzzy neighborhood of x_r , then there exist $u_r \in$ t such that $x_r \in u_r \subseteq v_r$. Then $\{u_r\}$ is an open cover of λ and implies that there is a finite subcover $\{u_{r_k}\}$ of $\{u_r\}$ with $x_{r_1} \in u_{r_1}$, $x_{r_2} \in u_{r_2}$, ….. , $x_{r_n} \in u_{r_n}$ and hence $\lambda \subseteq \bigcup_{k=1}^{n} u_{r_k}$. Therefore λ is compact .

3.7 Theorem : Let (A , t_A) and (B , s_B) are fuzzy subspaces of fts's (X , t) and (Y , s) respectively . Let λ be a compact fuzzy set in (A , t_A) and f : $(A , t_A) \to (B , s_B)$ be relatively fuzzy continuous , one – one and onto . Then f(λ) is also compact in (B , s_B).

Proof : Suppose λ is compact in (A , t_A) . Let M = { v_i : i$\in$ J } be an open cover of f(λ) in (B , s_B) i.e. f(λ)$\subseteq \bigcup_{i \in J}\{v_i\}$. Since $v_i \in s_B$, then there exist $u_i \in$ s such that $v_i = u_i \cap$ B . Hence f(λ)$\subseteq \bigcup_{i \in J}\{u_i \cap$ B} . As f is fuzzy relatively continuous , then $f^{-1}(v_i) \cap$ A $\in t_A$ and hence { $f^{-1}(v_i) \cap$ A : i$\in$ J } is an open cover of λ . Since λ is compact in (A , t_A) , then there exist $v_{i_k} \in \{v_i\}$ (k $\in J_n$) such that

$\lambda \subseteq \bigcup_{i \in J} \{ f^{-1}(v_{i_k}) \cap A \}$. Again , let v be any fuzzy set in B . Since f is onto , then for any y $\in$ B, we have f (f^{-1}(v)) (y) = sup { f^{-1}(v) (z) : z $\in$ f^{-1}(y) , f^{-1}(y) $\neq \phi$ } = sup {v (f (z)) : f (z) = y } = sup {v (y) } = v(y) i.e. f (f^{-1}(v)) = v . This is true for any fuzzy set in B . As f is one – one and onto , so f(1) = 1 . Therefore f(λ)$\subseteq$f ($\bigcup_{i \in J} \{ f^{-1}(v_{i_k}) \cap A \}$) , as f is one – one and onto . This implies that f(λ)$\subseteq \bigcup_{i \in J} f \{ f^{-1}(v_{i_k}) \cap A \}$ and hence f(λ)$\subseteq \bigcup_{i \in J} \{ v_{i_k} \cap$ f(A) } . Thus f(λ) is compact in (B , s_B) .

The following example will show that the compact fuzzy set in an fts need not be closed .

3.8 Example : Let X = { a , b } and I = [0 , 1] . Let u_1 , u_2 , u_3 , $u_4 \in I^X$ defined by u_1(a) = 0.4 , u_1(b) = 0.7 ; u_2(a) = 0.5 , u_2(b) = 0.3 ; u_3(a) = 0.5 , u_3(b) = 0.7 ; u_4(a) = 0.4 , u_4(b) = 0.3 . Now take t = { 0 , 1 , u_1 , u_2 , u_3 , u_4 } , then we see that (X , t) is an fts . Let $\lambda \in I^X$ defined by λ(a) = 0.5 , λ(b) = 0.4 . Clearly λ is compact . But λ is not closed , as its complement λ^c is not open in (X , t) .

The following example will show that the closure of a compact fuzzy set in an fts need not be compact .

3.9 Example : Let X = { a , b } and I = [0 , 1] . Let u_1 , u_2 , u_3 , $u_4 \in I^X$ defined by u_1(a) = 0.1 , u_1(b) = 0.3 ; u_2(a) = 0.4 , u_2(b) = 0.5 ; u_3(a) = 0.6 , u_3(b) = 0.7 ; u_4(a) = 0.8 , u_4(b) = 0.9 . Now take t = { 0 , 1 , u_1 , u_2 , u_3 , u_4 } , then we see that (X , t) is an fts . Let $\lambda \in I^X$ defined by λ(a) = 0.2 , λ(b) = 0.7 . Clearly λ is compact . Now , closed fuzzy sets are u_1^c(a) = 0.9 , u_1^c(b) = 0.7 ; u_2^c(a) = 0.6 , u_2^c(b) = 0.5 ; u_3^c(a) = 0.4 , u_3^c(b) = 0.3 ; u_4^c(a) = 0. 2 , u_4^c(b) = 0.1 . Therefore we see that $\bar{\lambda} = u_1^c$

i.e. $\bar{\lambda}$ (a) = 0.9 , $\bar{\lambda}$ (b) = 0.7 . Hence we observe that , there is no open cover of $\bar{\lambda}$. Thus $\bar{\lambda}$ is not compact .

3.10 Theorem : Let λ be a compact fuzzy set in a fuzzy Hausdorff space (X, t) with $\lambda_0 \subset$ X (proper subset). Suppose x $\notin \lambda_0$ (λ(x) = 0) , then there exist u , v $\in$ t such that u(x) = 1 , $\lambda_0 \subseteq v^{-1}(0, 1]$ and u $\cap$ v = 0 .

Proof : Let y $\in \lambda_0$. Then clearly x $\neq$ y . As (X, t) is fuzzy Hausdorff , then there exist u_y , $v_y \in$ t such that u_y(x) = 1 , v_y(y) = 1 and $u_y \cap v_y$ = 0 . Hence $\lambda \subseteq \cup$ { v_y : y $\in \lambda_0$ } i.e. { v_y : y $\in \lambda_0$ } is an open cover of λ . Since λ is compact in (X, t) , then there exist v_{y_1} , v_{y_2} , ……. , $v_{y_n} \in \{v_y\}$ such that $\lambda \subseteq v_{y_1} \cup v_{y_2} \cup$ ……. $\cup\, v_{y_n}$. Now, let v = $v_{y_1} \cup v_{y_2} \cup$ ….. $\cup\, v_{y_n}$ and u = $u_{y_1} \cap u_{y_2} \cap$ …… $\cap\, u_{y_n}$. Then we see that v and u are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. v , u $\in$ t . Furthermore , $\lambda_0 \subseteq v^{-1}(0, 1]$ and u(x) = 1 , as u_{y_i} (x) = 1 for each i.

Finally , we have to show that u $\cap$ v = 0 . As $u_{y_i} \cap v_{y_i}$ = 0 implies that u $\cap\, v_{y_i}$ = 0 , by distributive law , we see that u $\cap$ v = u $\cap$ ($v_{y_1} \cup v_{y_2} \cup$ ….. $\cup\, v_{y_n}$) = 0 .

3.11 Theorem : Let λ and μ are disjoint compact fuzzy sets in a fuzzy Hausdorff space (X, t) with λ_0 , $\mu_0 \subset$ X (proper subsets) . Then there exist u , v $\in$ t such that $\lambda_0 \subseteq u^{-1}(0, 1]$, $\mu_0 \subseteq v^{-1}(0, 1]$ and u $\cap$ v = 0 .

Proof : Let y $\in \lambda_0$. Then y $\notin \mu_0$, as λ and μ are disjoint . Since μ is compact in (X, t) , then by previous theorem , there exist u_y , $v_y \in$ t such that u_y(y) = 1 , $\mu_0 \subseteq v_y^{-1}(0, 1]$ and $u_y \cap v_y$ = 0 . As u_y(y) = 1 , then { u_y : y $\in \lambda_0$ } is an open cover of λ . Since λ is compact in (X, t) , then there exist u_{y_1} , u_{y_2} , ….. , $u_{y_n} \in \{u_y\}$ such that $\lambda \subseteq u_{y_1} \cup u_{y_2} \cup$ ……. $\cup u_{y_n}$. Furthermore , $\mu \subseteq v_{y_1} \cap v_{y_2} \cap$ ……. $\cap v_{y_n}$, as $\mu \subseteq v_{y_i}$ for each i . Now , let u = $u_{y_1} \cup u_{y_2} \cup$ …… $\cup u_{y_n}$ and v = $v_{y_1} \cap v_{y_2} \cap$ ….. $\cap v_{y_n}$. Thus we see that $\lambda_0 \subseteq u^{-1}(0, 1]$ and $\mu_0 \subseteq v^{-1}(0, 1]$. Hence u and v are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. u , v $\in$ t .

Lastly , we have to show that u $\cap$ v = 0 . First , we observe that $u_{y_i} \cap v_{y_i} = 0$ implies that $u_{y_i} \cap$ v = 0 , by distributive law , we see that u $\cap$ v = ($u_{y_1} \cup u_{y_2} \cup$ …… $\cup u_{y_n}$) $\cap$ v = 0 .

The following example will show that the compact fuzzy set in a fuzzy Hausdorff space need not be closed .

3.12 Example: Let X = { a , b } and I = [0 , 1] . Let u_1 , $u_2 \in I^X$ defined by u_1(a) = 1 , u_1(b) = 0 and u_2(a) = 0 , u_2(b) = 1 . Now , put t = { 0 , 1 , u_1 , u_2 } , then we see that (X, t) is a fuzzy Hausdorff space . Let $\lambda \in I^X$ defined by λ(a) = 0.3 , λ(b) = 0.7 . Hence by definition of compact fuzzy set , we observe that λ is compact . But λ is not closed , as its complement λ^c is not open in (X, t) .

3.13 Theorem : Let λ be a compact fuzzy set in a fuzzy regular space (X, t) with $\lambda_0 \subset$ X (proper subset) . For each x $\in$ X and u $\in t^c$ with u(x) = 0 , there exist v , w $\in$ t such that v(x) = 1 , u $\subseteq$ w ,

$\lambda_0 \subseteq v^{-1}(0, 1]$ and v $\subseteq$ 1 – w .

Proof : Suppose x $\in$ X , x $\in \lambda_0$ and u $\in$ t^c with u(x) = 0 . As (X, t) is fuzzy regular , then there exist

v_x , w_x $\in$ t such that v_x(x) = 1 , $u_x \subseteq$ w_x and v_x $\subseteq$ 1 – w_x . Hence $\lambda \subseteq \cup \{ v_x$: x $\in \lambda_0$ } i.e.

$\{ v_x$: x $\in \lambda_0$ } is an open cover of λ . Since λ is compact in (X, t) , then there exist v_{x_1} , v_{x_2} , ……. ,

$v_{x_n} \in \{ v_x \}$ such that $\lambda \subseteq v_{x_1} \cup v_{x_2} \cup$ ……. $\cup\, v_{x_n}$. Now , let v = $v_{x_n} \cup v_{x_2} \cup$ ….. $\cup\, v_{x_n}$ and w = w_{x_1} $\cap$ $w_{x_2} \cap$ …… $\cap\, w_{x_n}$. Then we see that v and w are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. v , w $\in$ t . Furthermore , $\lambda_0 \subseteq$ v^{-1}(0 , 1] , and v(x) = 1

and u $\subseteq$ w , as u $\subseteq w_{x_k}$ individually .

Lastly , we have to show that v $\subseteq$ 1 – w . As v_{x_k} $\subseteq$ 1– w_{x_k} implies that v_{x_k} $\subseteq$1 – w for each k and

hence it is clear that v $\subseteq$ 1 – w .

3.14 Theorem : Let λ and μ are disjoint compact fuzzy sets in a fuzzy regular space (X, t) with λ_0 , $\mu_0 \subset$ X (proper subsets) . Suppose for each x $\in$ X and u $\in$ t^c with u(x) = 0 , there exist v , w $\in$ t such that $\lambda_0 \subseteq v^{-1}(0, 1]$, $\mu_0 \subseteq w^{-1}(0, 1]$ and v $\subseteq$ 1 – w .

Proof : Suppose x $\in$ X , x $\in \lambda_0$ and u $\in$ t^c with u(x) = 0 . Then x $\notin$ μ_0 , as λ and μ are disjoint . As μ is compact in (X, t) , then by previous theorem , there exist v_x , w_x $\in$ t such that v_x(x) = 1 , $u_x \subseteq w_x$, $\mu_0 \subseteq w_x^{-1}(0, 1]$ and $v_x \subseteq 1 - w_x$. As v_x(x) = 1 , then { v_x : x $\in \lambda_0$ } is an open cover of λ .

Since λ is compact in (X, t) , then there exist v_{x_1} , v_{x_2} , ……. , v_{x_n} $\in$ { v_x } such that $\lambda \subseteq v_{x_1} \cup v_{x_2} \cup$ ……. $\cup\, v_{x_n}$. Furthermore , $\mu \subseteq w_{x_1} \cap w_{x_2} \cap$ …… $\cap\, w_{x_n}$, as $\mu \subseteq v_{y_k}$ individually . Now , let v = $v_{x_n} \cup v_{x_2} \cup$ ….. $\cup\, v_{x_n}$ and w = $w_{x_1} \cap w_{x_2} \cap$ …… $\cap\, w_{x_n}$. Thus we see that $\lambda_0 \subseteq v^{-1}(0, 1]$ and $\mu_0 \subseteq w^{-1}(0, 1]$. Hence v and w are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. v , w $\in$ t .

Lastly , we have to show that v $\subseteq$ 1 – w . As $v_{x_k} \subseteq 1 - w_{x_k}$ implies that $v_{x_k} \subseteq 1 - w$ for each k and

hence it is clear that v $\subseteq$ 1 – w .

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