

FAULT DETECTION TECHNIQUE OF ELECTRONIC GADGETS USING FUZZY PETRI NET ABDUCTION METHOD

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Abstract:

Fuzzy technique using Petri net is a formal tool for describing a Discrete event system model of an actual system. The advantage of this technique is that concurrent evolutions with various processes evolving simultaneously and partially independently can be easily represented and analyzed. In local control applications conditions /events are used to describe the control sequences of elementary devices. Petri nets are made up of places, transitions and tokens. A state is represented by distribution of tokens in places. Various approaches can be used to combine Petri nets and Fuzzy sets. In this paper the authors speak about the fault finding technique of electronic networks with different illustrations.

Keywords and phrases : Fuzzy sets, Petri nets, control sequences, technique of electronic networks

বিমূর্ত সার (Bengali version of the Abstract)

একটি প্রকৃত তন্ত্রের বিশিষ্ট ঘটনা তন্ত্রের মডেলকে বর্ণনা করার জন্য পেট্রিনেট (Petri net) ব্যবহার সমৃদ্ধ ফাজি কৃৎকৌশল (Fuzzy technique) হচ্ছে একটি বিধিবিৎ যন্ত্র। এই কৃৎকৌশলের সুবিধা হচ্ছে যে যুগপৎ এবং অংশত স্বাধীনভাবে উদ্ভূত বহুবিধ প্রক্রিয়া সহ সমকালীন অবঘাতনকে (concurrent evolutions) অত্যন্ত সহজভাবে প্রকাশ ও বিশ্লেষণ করা যায়। প্রাথমিক নকশার অনুক্রমিক নিয়ন্ত্রণকে বর্ণনা করার জন্য স্থানীয় নিয়ন্ত্রক প্রয়োগ শর্তে ঘটনাগুলির ব্যবহার করা হয়েছে। স্থান, অবস্থানান্তরতা (transitions) এবং টোকেনগুলি (tokens) নিয়ে তৈরী হয়েছে পেট্রিনেট। স্থানগুলিতে টোকেনের বন্টনের দ্বারা একটি অবস্থানকে

(state) উপস্থাপন করা হয়েছে । পেট্রিনেট এবং ফাজি সেটকে সংযুক্ত করতে বহুবিধ অভিগমনকে ব্যবহার করা যেতে পারে । এই পত্রে লেখকেরা বিভিন্ন উদাহরণের সাহায্যে ইলেকট্রনিক নেটওয়ার্কের ড্রাটি নির্ণায়ক কৃৎকৌশলের বিষয়ে বলেছেন ।

1. Introduction:

More recently, fuzzy logic has been successfully applied to a specialized structure of knowledge, called **Fuzzy Petri Nets** (FPN), [1],[3]-[9], [19], [23], [37] for handling one or more of the above problems. The concept of the management of imprecision of data with FPN was pioneered by Looney [24], who considered an acyclic model of FPN, for estimating the degree of truth of a proposition with a foreknowledge of its predecessors in the network. Chen et al. [7] presented an alternative model and an interactive algorithm for reasoning in the presence of both imprecision and uncertainty. Bugarin and Barro [1] refined the underlying concept of the model in [9] and extended it in the light of classical fuzzy logic [42]. The most challenging part of their work was reasoning under incomplete specification of knowledge. Yu improved the concept of structural mapping of knowledge onto FPN [40] and presented a new formalism [41] for reasoning with a knowledge base, comprising of fuzzy predicates [42], instead of fuzzy propositions [7]. Scarpelli et al. presented new algorithms for forward [36] and backward [35] reasoning on FPN which is of much interest. A completely different type of model of FPN using fuzzy t and s norms [13] was proposed by Pedrycz [29] for applications in supervised learning problems. There exists an extensive literature on FPN models [11], [1], [3], [8], which cannot be discussed here for lack of space. However, to the best of the author's knowledge, none of the existing models of FPN can handle the complexities in a reasoning system created by the coexistence of imprecision and inconsistency of data and uncertainty of knowledge. The complexity of the reasoning system is further complicated, when the knowledge base has an explicit self-reference to itself. The chapter presents

new models of FPN [20], pivoted around the work of Looney, for dealing with the above problems by a unified approach.

2. Imprecision Management in an Acyclic FPN

Before describing the technique for imprecision management in acyclic FPN, we present a few terminologies first.

6.2.2.1 Formal Definitions and the Proposed Model

Definition 1: An FPN can be defined as a 9-tuple:

$$FPN = \{P, Tr, D, I, O, cf, th, n, b\}$$

where

- $P = \{p_1, p_2, \dots, p_m\}$ is a finite set of places,
- $Tr = \{tr_1, tr_2, \dots, tr_n\}$ is a finite set of transitions.
- $D = \{d_1, d_2, \dots, d_m\}$ is a finite set of propositions,
- $P \cap Tr \cap D = \emptyset$ / $|P| = |D|$,
- $I : Tr \rightarrow P^\infty$ is the input function, representing a mapping from transitions to bags of (their input) places,
- $O : Tr \rightarrow P^\infty$ is the output function, representing a mapping from transitions to bags of (their output) places,
- $cf, th : Tr \rightarrow [0, 1]$ are association functions, representing a mapping from transitions to real values between 0 and 1
- $n : P \rightarrow [0, 1]$ is an association functions, representing a mapping from places to real values between 0 and 1
- $b : P \rightarrow D$ is an association function, representing a bijective mapping from places to propositions.

In realistic terminology, n_i represents the fuzzy beliefs of place p_i i.e., $n_i = n(p_i)$; $cf_j = cf(tr_j)$ and $th_j = th(tr_j)$ represent the CF and threshold of transition tr_j respectively. Further $d_i = b(p_i)$.

Definition 2: A transition tr_i is enabled if AND $\{n_i : p_i \in I(tr_j)\} > th_j$ where $n_i = n(p_i)$ and $th_j = th(tr_j)$. An enabled transition fires, resulting in a fuzzy truth token (FTT) at all its output arcs. The value of the FTT is a function of the CF of the transition and fuzzy beliefs of its input places.

The technique for computing the fuzzy beliefs at a place can be conveniently represented by a model. Based on Definitions 1 and 2 we propose a model, called the Belief Propagation Model.

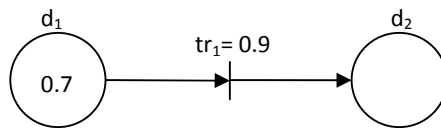
3. Proposed Model for Belief Propagation

Consider a place p_i which is one of the common output places of transitions tr_j , where $1 \leq j \leq m$. Now, for computing fuzzy beliefs n_i at place of p_i , first the condition for transition firing is checked for all tr_j . If a transition fires then the fuzzy beliefs of its input places are ANDed and then the result, called FTT is saved. If the transition does not fire, then the FTT corresponding to this transition is set to zero. The fuzzy belief of place p_i can now be computed by ORing the FTT associated with the tr_j for $1 \leq j \leq m$.

For the sake of brevity, the AND (Minimum of inputs) and OR (maximum of inputs) operators are represented by \boxtimes and \boxdot , respectively

3.1 Problem: Draw a Petri-net model for a given Production Rule. If (d_1) THEN (d_2), where d_1 = it is raining, d_2 = soil to be wetted. The transition value, tr_1 , that represents the trueness of this Production Rule also to be included. Determine the value of d_2 when d_1 and tr_1 are given as 0.7 and 0.9 respectively.

Solution: According to production rule the Petri-net model is given below.



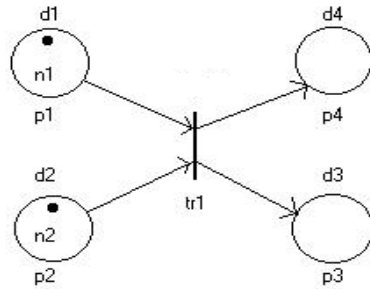
Here the circles are called as places or nodes and denoted by p_1, p_2, \dots etc. The bar is called transition and denoted as tr_1 . The value in the place are denoted as n_1, n_2, \dots etc. That is the value of $d_1 = n_1 = 0.7$ and the value of $d_2 = n_2$.

The value of d_2 can be obtained as $n_2 = n_1 \wedge tr_1 = 0.7 \wedge 0.9 = 0.7$.

3.2 Problem: Given a Production Rule : IF((d_1) AND (d_2)) THEN ((d_3) OR (d_4)). Here d_1 = it is hot, d_2 = the sky is cloudy, d_3 = it will rain, d_4 = humidity is high, P

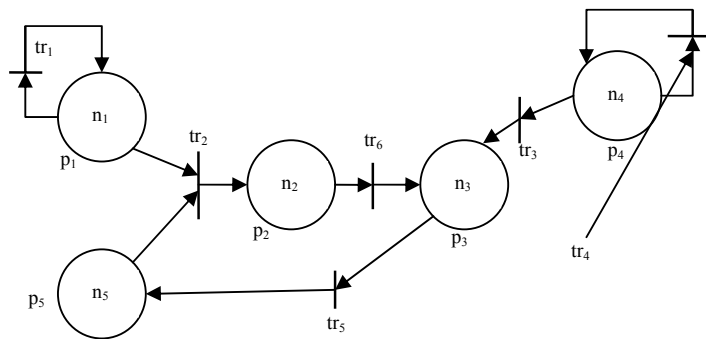
$= \{p_1, p_2, p_3, p_4\}$, $T = \{tr_1\}$, $D = \{d_1, d_2, d_3, d_4\}$, $tr_1 = 0.9$, $n_1 = 0.9$, $n_2 = 0.5$, and n_3 and n_4 are to be determined. Draw the Petri-net model for this production rule.

Solution: According to production rule the Petri-net model is given below.



$$n_3 = n_4 = [(n_1 \wedge tr_1) \wedge (n_2 \wedge tr_1)] = [(0.9 \wedge 0.9) \wedge (0.5 \wedge 0.9)] = 0.5.$$

3.3 Problem: Given a Petri-net model below, determine the P matrix and Q-matrix. Write the dynamic equation of Petri-net model.



Solution: The evaluation of P matrix and Q- matrix are given below.

From

To		tr_1	tr_2	tr_3	tr_4	tr_5	tr_6
P =	p_1	1	0	0	0	0	0
	p_2	0	1	0	0	0	0
	p_3	0	0	1	0	0	1
	p_4	0	0	0	1	0	0
	p_5	0	0	0	0	1	0

From

To						
		p ₁	p ₂	p ₃	p ₄	p ₅
Q =	tr ₁	1	0	0	0	0
	tr ₂	1	0	0	0	1
	tr ₃	0	0	0	1	0
	tr ₄	0	0	0	1	0
	tr ₅	0	0	1	0	0
	tr ₆	0	1	0	0	0

and $N(t) = [n_1 \ n_2 \ n_3 \ n_4 \ n_5]$.

The dynamic equation of Petri-net is written below.

$$N(t+1) = P \circ (Q \circ N^c(t))^c.$$

3.4 Problem: The P matrix and Q matrix of a Petri-net as well as the present state value ($N(t)$) of all the nodes of the petri-net are given below.

From

To		tr ₁	tr ₂	tr ₃	tr ₄	tr ₅
P =	p ₁	0	0	0	0	1
	p ₂	1	0	0	0	0
	p ₃	0	1	0	0	0
	p ₄	0	0	1	0	0
	p ₅	0	0	0	1	0

From

To		p ₁	p ₂	p ₃	p ₄	p ₅
Q =	tr ₁	1	0	0	0	0
	tr ₂	0	1	0	0	0
	tr ₃	0	1	1	0	0
	tr ₄	0	0	0	1	0

$$\begin{array}{cccccc} \text{tr}_5 & 1 & 0 & 0 & 0 & 0 \\ & n_1 & n_2 & n_3 & n_4 & n_5 \end{array}$$

and $N(t) = [0.9 \ 0.9 \ 0.6 \ 0.8 \ 0.7]$.

Determine the next nodes value, i.e., $N(t+1)$ and previous nodes values, i.e., $N(t-1)$ of this petri-net.

Solution: Using the equation of Petri-net's dynamic, we write

$$N(t+1) = P \circ (Q \circ N^c(t))^c$$

$$\begin{aligned} N^c(t) = \neg N(t) &= [(1-0.9) \ (1-0.9) \ (1-0.6) \ (1-0.8) \ (1-0.7)] \\ &= [0.1 \ 0.1 \ 0.4 \ 0.2 \ 0.3]. \end{aligned}$$

and $N(t) = [0.9 \ 0.9 \ 0.6 \ 0.8 \ 0.7]$.

Determine the next nodes value, i.e., $N(t+1)$ and previous nodes values, i.e., $N(t-1)$ of this petri-net.

Solution: Using the equation of Petri-net's dynamic, we write

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$$\begin{aligned} N^c(t) = \neg N(t) &= [(1-0.9) \ (1-0.9) \ (1-0.6) \ (1-0.8) \ (1-0.7)] \\ &= [0.1 \ 0.1 \ 0.4 \ 0.2 \ 0.3]. \end{aligned}$$

$$\begin{aligned} N(t+1) &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \circ \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0.1 \\ 0.1 \\ 0.4 \\ 0.2 \\ 0.3 \end{bmatrix} \right)^c \\ &= [0.9 \ 0.9 \ 0.9 \ 0.6 \ 0.8] \end{aligned}$$

For $N(t-1)$, we rewrite Petri-net's dynamic as

$$N(t) = P \circ (Q \circ N^c(t-1))^c$$

$$P^{-1} \circ N(t) = (Q \circ N^c(t-1))^c \quad (\text{Since, } P^{-1} \circ P = I)$$

$$(P^{-1} \circ N(t))^c = Q \circ N^c(t-1)$$

$$Q^{-1} \circ (P^{-1} \circ N(t))^c = N^c(t-1) \quad (\text{Since, } Q^{-1} \circ Q = I)$$

$$N^c(t-1) = Q^{-1} \circ (P^{-1} \circ N(t))^c$$

$$(N(t-1))^c = \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \right)^{-1}$$

$$\circ \left(\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0.9 \\ 0.9 \\ 0.6 \\ 0.8 \\ 0.7 \end{bmatrix} \right)^c$$

Using Algorithm I, P^{-1} and Q^{-1} are evaluated below.

$$(N(t-1))^c = \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right)$$

$$\circ \left(\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 0.9 \\ 0.6 \\ 0.8 \\ 0.7 \end{bmatrix} \right)^c$$

$$(N(t-1))^c = [0.1 \quad 0.4 \quad 0.2 \quad 0.2 \quad 0.1]$$

$$N(t-1) = [0.9 \quad 0.6 \quad 0.8 \quad 0.8 \quad 0.9].$$

Fuzzy Abduction Technique to Diagnosis an Electronics Circuit:

We have taken a simple example to diagnosis an electronics circuit by abductive or backward reasoning. This electronic circuit comprises backward

reasoning. This electronic circuit comprises with two diodes and three $1\text{ k}\Omega$ resistances as depicted bellow. In this circuit, we want to diagnosis whether the diodes D_1 and D_2 are working properly. And this diagnosis will be done by applying D_2 are working properly. And this diagnosis will be done.

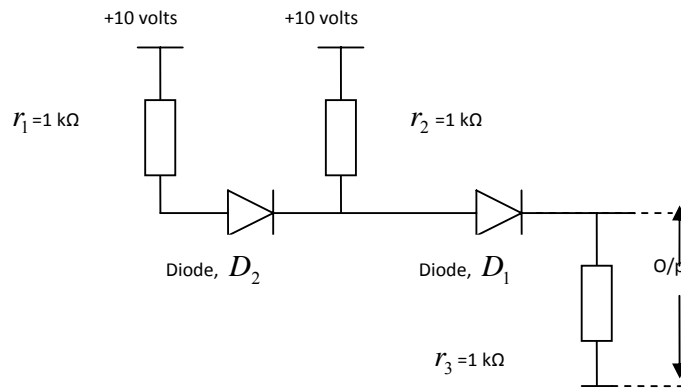


Fig:1 Schematic diagram of OR gate circuit where diodes are shown
 abductive reasoning with the help of output voltage i.e., voltage across r_3
 resistance. It is obvious, if diode D_1 is defective, then output voltage will be 0 volts. If diode D_2 is defective and diode D_1 is not defective, then output voltage is 5 volts (voltage across $r_3 = \frac{1}{1+1} \times 10\text{volts} = 5\text{ volts}$). And if the two diodes D_1 and D_2 are not defective, then applying voltage divider rule, we have output voltage as 6.66 volts (voltage across $r_3 = \frac{1}{1+0.5} \times 10\text{volts} = 6.66\text{ volts}$). Thus we have three membership curve of output voltage as LOW, MEDIUM and HIGH for 0 volts, 5 volts and 7 volts respectively.

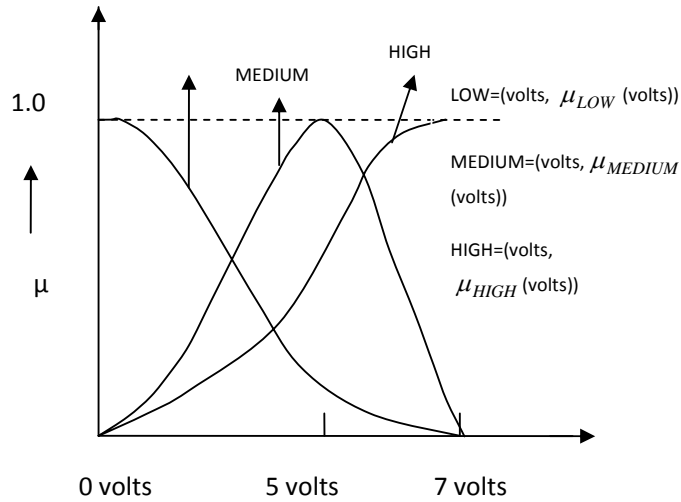


Fig2: LOW, MEDIUM and HIGH membership curve of output voltage.

In the other hand the defectiveness of diodes (D_1 and D_2 both) will be measured by forward junction voltage of diode. If the forward junction voltage of diode is greater than 0.7 V, then the diode is defective and otherwise it is proper. Thus for both of the diodes, we have two membership curve as LOW and HIGH forward junction voltage (fig. 13).

Construction of Rules:

Let us consider three rules:

Rule1: If the forward junction voltage of diode D_1 is HIGH, then output voltage is LOW.

Rule2: If the forward junction voltage of diode D_1 is LOW and D_2 is HIGH, then output voltage is MEDIUM.

Rule3: If the forward junction voltage of diode D_1 is LOW and D_2 is LOW, then output voltage is HIGH.

Figure 4: Inferential Graph representing the relationship between antecedent and consequent clause.

Abductive Reasoning: Now, we assume the primary membership distribution of “output voltage as *MEDIUM*” is: Source 1: (0.2 0.9 0.1), source 2: (0.1 1.0 0.2) and source 3: (0.1 1.0 0.1). Furthermore, the secondary membership distribution of “output voltage as *MEDIUM*” is: $Q' = [1.0 \ 0.7 \ 0.7 \ 0.6 \ 0.9 \ 0.8 \ 0.9 \ 0.5 \ 0.8]$. Thus, the primary membership distribution for output voltage as ‘*MEDIUM*’ is obtained using the above primary distributions from all sources corresponding to best secondary membership values as, ${}^{best}\mu_{Output_Volt_MEDIUM'}(y) = {}^{best}\mu_{B'}(x_1) = [0.2 \ 1.0 \ 0.1]$. Now, we have the primary distribution for diode D_1 as ${}^{best}\mu_{A'}(x_1) = {}^{best}\mu_{B'}(x_1) \circ ({}^1R_1)^T = [0.2 \ 1.0 \ 0.1] \circ$

$\begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.9 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} = [0.9 \ 0.3 \ 0.1]$. Similarly, we obtain the primary distribution for diode D_2 as ${}^{best}\mu_{c'}(x_1)$

$[1.0 \ 0.1]$. Now, we have the primary distribution for diode D_1 as ${}^{best}\mu_{A'}(x_1) = {}^{best}\mu_{B'}(x_1) \circ ({}^1R_1)^T = [0.2 \ 1.0 \ 0.1] \circ$

$\begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.9 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} = [0.9 \ 0.3 \ 0.1]$. Similarly, we obtain the primary distribution for diode D_2 as ${}^{best}\mu_{c'}(x_1)$

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