

STATE SPACE ANALYSIS OF A SOLAR POWER ARRAY TAKING A HIGHER DEGREE OF NON-LINEARITY INTO ACCOUNT

By

¹Adhir Baran Chattopadhyay, ²Sunil Thomas, ³Aliakbar Eski, ⁴Ruchira Chatterjee

^{1,2}Department of Electrical and Electronics Engineering, Birla Institute of Science and Technology, Pilani – Dubai Campus

^{3,4}Department of Electronics and Instrumentation Engineering, Birla Institute of Science and Technology, Pilani – Dubai Campus

Abstract:

This paper develops a mathematical technique for the solution of a non linear state variable model of a solar array power system powering a non linear load. The significance of the technique lies in the fact that experimental complexities can be avoided to reach a desired conclusion regarding the design of the controller associated with a solar power array system. An iterative method has been used in which the initiating assumption has been made to consider the system to depend entirely upon its initial values at the instant $t = 0$ and taking the forcing function to be zero at that instant. In the next step we use the solution at $t = 0$ and plug it into the equation iteratively while having a non zero value of the forcing equation during the second iteration. The non linearity lies in the fact that the forcing function is a function of the state variable itself. We have applied the Maclaurin series to find the laplace transform of certain mathematical functions containing a singularity at the zero time instant. The time response is obtained and then it is plotted by using MATLAB and various graphs have been obtained.

Keywords and phrases : solar array power system, non linear state variable model, forcing function, laplace transform, time response,

বিমূর্ত সার (Bengali version of the Abstract)

একটি অরৈখিক ভারের শক্তি সম্পন্নকারী সৌর সজ্জা শক্তি তন্ত্রের (solar array power system) একটি অরৈখিক অবস্থা চল মডেলের (non linear state variable model) সমাধানের জন্য এই পত্রে একটি গাণিতিক কুংকৌশল উদ্ভাবন করা হয়েছে। এই কুংকৌশলের গুরুত্ব এইখানে যে পরীক্ষণজনিত জটিলতাকে এড়িয়ে সৌরশক্তি সজ্জা তন্ত্রের সঙ্গে যুক্ত নিয়ন্ত্রকের

নকশা প্রসঙ্গে কাম্য সিদ্ধান্তে পৌছানো যায়। একটি পুনরাবৃত্তিমূলক পদ্ধতিকে (iterative method) ব্যবহার করা হয়েছে এই প্রারম্ভিক ধারণাকে ধরে নিয়ে যে তত্ত্বটি সম্পূর্ণভাবে নির্ভরশীল ইহার প্রারম্ভিক মানের তাৎক্ষণিক সময় $t = 0$ এবং এই তাৎক্ষণিক সময়ে বল প্রয়োগকারী অপেক্ষকের (forcing function) মান শূন্য ধরা হয়েছে। পরবর্তী ধাপে $t = 0$ তে প্রাপ্ত সমাধানকে ব্যবহার করা হয়েছে এবং ইহাকে সমীকরণে পুনঃ পুনঃ যুক্ত করা হয়েছে পুনরাবৃত্তিমূলক ভাবে যখন দ্বিতীয় পুনরাবৃত্তিকরণের সময় বল প্রয়োগকারী সমীকরণের মান শূন্য নয়। অরৈখিকতার সত্যাসত্য দাড়িয়ে আছে যখন বল প্রদানকারী অপেক্ষকটি অবস্থা - চলের (state variable) নিজেরই অপেক্ষক। তাৎক্ষণিক সময় মান শূন্যতে একটি বিশিষ্টতা (singularity) ধারণকারী গাণিতিক অপেক্ষকের ল্যাপলাস রূপান্তরকে (Laplace transform) নির্ণয় করার জন্য আমরা মেক্লেইরিন শ্রেণীকে (Maclaurin series) ব্যবহার করেছি। সময়ের প্রতিক্রিয়া নির্ণয় করা হয়েছে এবং ম্যাটল্যাবের (MATLAB) সাহায্যে ইহাকে প্লট করা হয়েছে এবং বহুবিধ লেখচিত্র নির্ণয় করা হয়েছে।

1. Introduction

Nowadays renewable sources of energy are attracting major attention from researchers. This is mainly due to the fact that the energy demands are increasing and the common supply i.e. fossil fuel is slowly getting depleted. Solar Arrays were initially employed to power satellites and spacecraft (Bae et al., 2008) (Cho et al., 1990) (Paulkovich 1967), but over the years their application scope has broadened and they are now also being used in terrestrial applications. In light of these recent developments, they are bound to play a promising role in future.

The research papers (Paulkovich 1967), (Mourra et al., 2010), (Cho and Cho 2001), (Jensen et al., 2010), (Wang et al., 2010a,b), (Ramaprabha et al., 2009) and (Bouchafaa et al., 2010) indicate current trends in the solar power research area.

Solar Power upto now also has been expensive and significant work has been done on reducing the cost of not only the solar device but also of its associated power electronics and converter by reducing the hardware and stresses of the operating conditions.

It sometimes is required that the Solar Power be available in small devices in which the power source and the regulator have to be inside the device and also at times integrated into the circuit. This leads to the need for power converters at the micro level that can convert efficiently and cheaply.

Even though some analysis has been covered in literature, the authors of the present paper feel that the state variable model containing some non-linearity can be dealt analytically using some special mathematical technique to have a better prediction on the current through the inductor and voltage across the capacitor in connection with the controller circuit of the solar array power system. Such work is based on the strategy stated in research paper (Bae et al., 2008), but the exact way of tackling the nonlinearity mathematically has not been presented in that paper. Such detailed aspects have been preliminaries included in the analysis presented in the paper by (Chattopadhyay *et. al*, 2011). In this paper, the non-linearity lied in the fact that the control input u_2 , the current through the capacitor, was in fact an inverse function of the state variable x_2 which was the voltage across the capacitor. Such a condition generally leads to a constant power load. The authors of the present paper have now taken a higher degree of non-linearity into account, specifically; the control variable u_2 now is an inverse square function of the state variable x_2 . Such a condition leads to special kinds of load such as UJT, tetrode Valves, Tunnel and gun diodes which are predominantly in use in the field of high frequency generation.

2. Problem formulation

The Analysis of this paper is based on the configuration of the solar power array system presented in fig 1. (Bae. Et al 2008). To start the analysis we define the kind of load, which in this case is a load with negative resistance, to be specific, an N – shaped Negative Differential Resistance, such as a tunneling diode. The I-V characteristics of an Tunneling diode looks like shown in Fig 1.

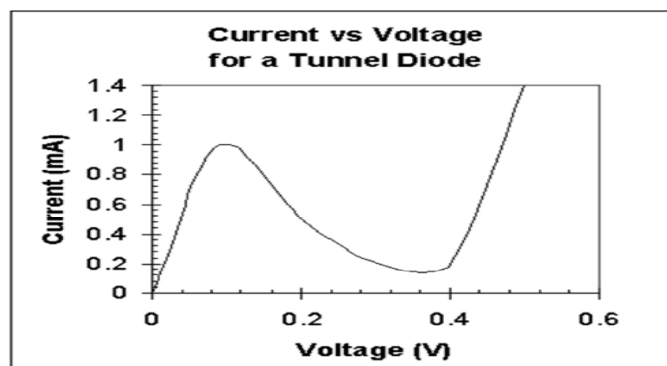


Fig 1. The I-V characteristics of a Tunneling diode

As can be seen there is a portion the curve that has a negative slope which corresponds to the $-dI/dV$ negative differential conductance (in this case)

characteristics of the device. Looking at the negative resistance portion of the curve, it is observed that the curve can be approximated by the equation $I = k/V^2$. This is precisely the kind of device that forms the load of the solar power array in our analysis. The equivalent circuit diagram for the system is shown in fig.3

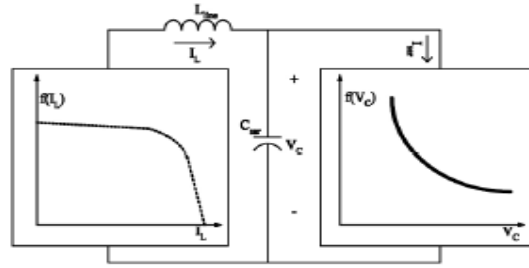


Fig.3 Equivalent circuit diagram for the system

The Mathematical technique has been summarized in fig. 3 and can be treated as a generalized approach to non-linear state variable analysis. The state equations are given as follows:

$$\frac{di_L}{dt} = \frac{f(i_L)}{L} - \frac{V_C}{L}$$

$$\frac{dV_C}{dt} = \frac{i_L}{C} - \frac{f(V_C)}{C}$$

We use these state equations and develop an iterative technique which we will apply on the state variable model that we develop in the next section.

Our function, $f(V_C)$ is defined by:

$$f(V_C) = \frac{C_1}{x_2(t)^2}$$

Here $f(V_C)$ corresponds to the current through the device and $x_2(t)$ is the voltage across it. This characteristic of the device is further established by looking at its (Power x Voltage) values for different values of Voltage. Theoretically,

$$I = \frac{C_1}{V^2}$$

$$(V \cdot I) \cdot V = C_1$$

$$P \cdot V = C_1 = \text{Constant}$$

Experimentally, from the graph:

Table 1 : Power-voltage product of a Tunnel Diode showing nonlinearity and negative resistance

V(Volts)	I(Amperes)	P(Watts)	PV(Watts-volts)	Error	Remarks
0.16	0.7	0.112	0.01792	0.00167	It's observed that the value of the Power and voltage product remains fairly constant with a mean error of 9.78×10^{-4} and a maximum error of 1.67×10^{-3} .
0.2	0.5	0.1	0.02	0.00041	
0.24	0.36	0.0864	0.020736	0.001146	
0.28	0.24	0.0672	0.018816	0.000774	
0.32	0.2	0.064	0.02048	0.00089	
		Avg:	0.0195904	0.000978	

3. Development of a nonlinear state variable model and method of solution

The analysis is based on the configuration of the solar array power system presented in Fig 2. ^[3]

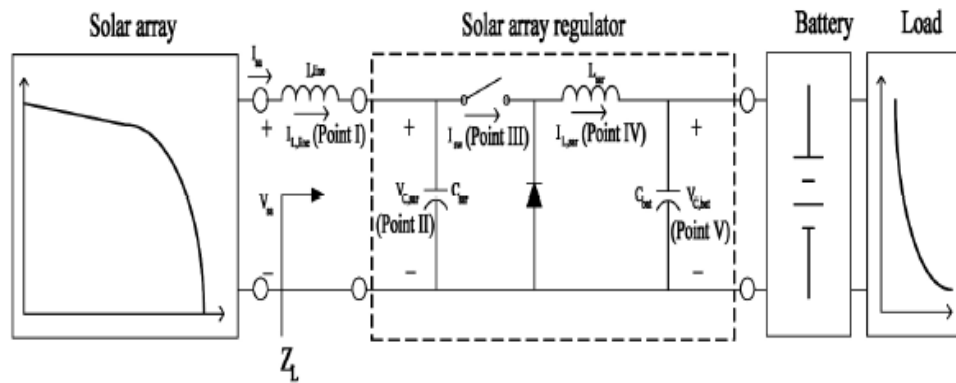


Fig 2.Solar array power system

Let the state variable function for the formulation of the state variable matrix

$$\frac{dx}{dt} = Ax + Bu \quad (1)$$

be:

where 'x' is the state variable and 'u' is the forcing function. For the case of a solar array power system,

$$u = f(x) \Rightarrow \frac{dx}{dt} = Ax + Bf(x) \quad (2)$$

Equation 2 clearly indicates that the control input is not independent rather it depends on the state variable itself. Hence the traditional mathematical technique

of solving a linear state variable model will not hold well in the present case. A flow chart for the proposed iterative method for solving the non-linear system is shown in Figure 4 ^[3]

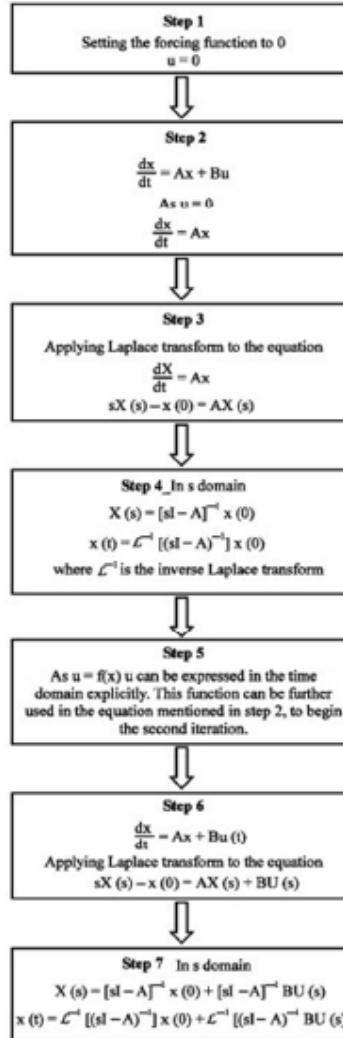


Fig.4. Flow chart for iteration algorithm

The state equations are given as follows:

$$\frac{di_L}{dt} = \frac{1}{L} f(i_L) - \frac{v_c}{L} \quad (3)$$

$$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{1}{C} f(v_c) \quad (4)$$

Let $i_L = x_1$ and $V_c = x_2$

With the help of these state equations, the iterative technique has been applied on the state variable model developed in the next section.

Therefore,

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & -1/C \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix} \quad (5)$$

Let $f(x_1) = u_1$ and $f(x_2) = u_2$

For the first iteration, $u_1 = u_2 = 0$ is considered. The system is treated as autonomous and as depending only on the initial value. Therefore,

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (6)$$

The above matrix corresponds to:

$$\frac{dx}{dt} = Ax$$

From the algorithm,

$$X(s) = [sI - A]^{-1} x(0) \quad (7)$$

After necessary mathematical manipulation and applying inverse Laplace transform formulae, the state solution appears as:-

$$x_1(t) = x_1(0) \cos(\omega_0 t) - \frac{x_2(0)}{\omega_0 L} \sin(\omega_0 t) \quad (8)$$

$$x_2(t) = \frac{x_1(0)}{\omega_0 C} \sin(\omega_0 t) + x_2(0) \cos(\omega_0 t) \quad (9)$$

The functions $x_1(t)$ and $x_2(t)$ are obtained at the force free condition of the present system. Here it is required to find the actual state variable response of the system due to the actual control inputs u_1 and u_2 . It was assumed that $u_1 = f(x_1)$ and $u_2 = f(x_2)$ but these assumptions will not serve the actual purpose.

For simplicity of calculations, the output of the solar array is considered to be a constant

Defining the following functions for u_1 and u_2 as given In Fig 5 and Fig 6 ^[3]

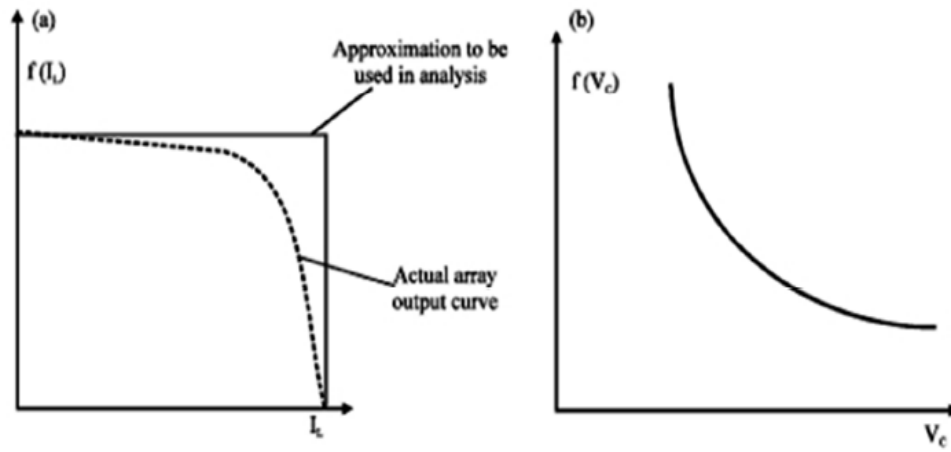


Fig 5. Defining the function for u_1 .

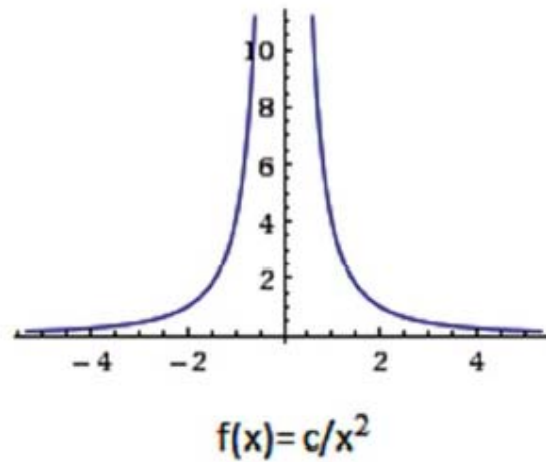


Fig6. Defining the function for u_2

$$u_1 = f(x_1) = \text{constant} = k$$

$$u_2 = \frac{c_1}{x(t)^2} \quad (10)$$

$$= \frac{C_1}{\left[\frac{x_1(0)}{\omega_0 C} \sin(\omega_0 t) + x_2(0) \cos(\omega_0 t) \right]^2} \quad (11)$$

$$= \frac{C_1}{[P \sin(\omega_0 t) + Q \cos(\omega_0 t)]^2} \quad (12)$$

Where: $P = \frac{x_1(0)}{\omega_0 C}, Q = x_2(0)$ and
 $r = \sqrt{P^2 + Q^2}$

Therefore

$$\begin{aligned} (x_2)^2 &= [P \sin(\omega_0 t) + Q \cos(\omega_0 t)]^2 \\ &= P^2 \sin^2(\omega_0 t) + Q^2 \cos^2(\omega_0 t) + 2PQ \sin(2\omega_0 t) \end{aligned} \quad (13)$$

$$= P^2 \frac{1}{2} (1 - \cos(2\omega_0 t)) + Q^2 \frac{1}{2} (1 + \cos(2\omega_0 t)) + PQ \sin(2\omega_0 t)$$

$$= \cos(2\omega_0 t) \left(\frac{Q^2}{2} - \frac{P^2}{2} \right) + PQ \sin(2\omega_0 t) + \left(\frac{P^2 + Q^2}{2} \right) \quad (14)$$

$$\text{Let } a = \left(\frac{Q^2}{2} - \frac{P^2}{2} \right)$$

$$b = PQ$$

$$d = \left(\frac{P^2 + Q^2}{2} \right)$$

$$a = r \sin \theta \quad b = r \cos \theta$$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \frac{a}{b}$$

Therefore

$$x_2 = r \sin(2\omega_0 t + \theta) + d \quad (15)$$

And

$$u_2 = \left[\frac{c_1}{r \sin(2\omega_0 t + \theta) + d} \right] \quad (16)$$

Incorporating the values of u_1 and u_2 in the following equation:

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & -1/C \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix} \quad (17)$$

Applying The Laplace Transform to the above equations:

$$\begin{bmatrix} sX_1(s) - x_1(0) \\ sX_2(s) - x_2(0) \end{bmatrix} = A \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + B \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \quad (18)$$

$$U_1(s) = \frac{k}{s} \quad (19)$$

$$U_2 = L \left[\frac{c_1}{r \sin(2\omega_0 t + \theta) + d} \right] \quad (20)$$

$$= L \left[C_1 \left\{ 1 + \frac{r}{d} \sin(2\omega_0 t + \theta) \right\}^{-1} \right] \quad (21)$$

Using Negative Binomial Series

$$(x + 1)^{-n} = 1 - nx + \frac{1}{2}n(n+1)x^2 - \frac{1}{6}n(n+1)(n+2)x^3 + \dots$$

$$= L \left[C_1 d \left\{ 1 - \frac{r}{d} \sin(2\omega_0 t + \theta) \right\} \right]$$

$$= L [C_1 d - C_1 r \sin(2\omega_0 t + \theta)]$$

$$= \frac{C_1}{s} - C_1 r \left(\frac{s \sin \theta + 2\omega_0 \cos \theta}{s^2 + 4\omega_0^2} \right) \quad (22)$$

For the second iteration, the function for $X(s)$ is as follows:

$$X(s) = [sI - A]^{-1} x(0) + [sI - A]^{-1} B U(s) \quad (23)$$

After applying the traditional formulae, the state space variable solutions are obtained and presented as follows:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = L^{-1} \left\{ \frac{1}{s^2 + 1/LC} \left[\begin{bmatrix} x_1(0)s - \frac{x_2(0)}{L} + \frac{k}{L} + \frac{C_1 d}{LCs} - \frac{C_1 r}{LC} \left(\frac{s \sin \theta + 2\omega_0 \cos \theta}{s^2 + 4\omega_0^2} \right) \right] \right. \right. \\ \left. \left. + \begin{bmatrix} \frac{x_1(0)}{C} + x_2(0)s + \frac{k}{sLC} - \frac{C_1 d}{C} + \frac{sC_1 r}{C} \left(\frac{s \sin \theta + 2\omega_0 \cos \theta}{s^2 + 4\omega_0^2} \right) \right] \right\} \quad (24)$$

Applying the initial conditions as $x_1(0) = 0$, $x_2(0) = 1$ (based on practical established results of above equation (24) individually and letting $\frac{1}{\sqrt{LC}} = \omega_0$

Hence,

$$\begin{aligned} x_1(t) = & \frac{k}{\omega_0 L} \sin \omega_0 t - \frac{x_2(0)}{\omega_0 L} \sin \omega_0 t + C_1 d (1 - \cos \omega_0 t) + \\ & C_1 r \sin \theta \left[\frac{+1}{3\omega_0^2} \cos \omega_0 t - \frac{1}{3\omega_0^2} \cos 2\omega_0 t \right] + C_1 r 2 \cos \theta \left[\frac{1}{3} \sin \omega_0 t - \right. \\ & \left. \frac{1}{6} \sin 2\omega_0 t \right] \end{aligned} \quad (25)$$

and

$$\begin{aligned} x_2(t) = & k(1 - \cos \omega_0 t) + \frac{-C_1 d}{C \omega_0} \cdot \sin \omega_0 t + \frac{C_1 r}{C} \sin \theta \frac{2}{3\omega_0} (2 \sin 2\omega_0 t - \sin \omega_0 t) + \\ & \frac{C_1 r}{C} \cos \theta \frac{2}{3\omega_0} (\cos 2\omega_0 t - \cos \omega_0 t) + \cos(\omega_0 t) \end{aligned} \quad (26)$$

The following values have been chosen after several numerical approximations (Chattopadhyay et.al, 2011),

$$k = 4, C_1 = 4, L = 100 \mu H, C = 200 \mu F, \theta = \frac{3\pi}{2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 7071.067812 \text{ Hz}$$

$$\theta = \tan^{-1} \frac{Q}{P} = \frac{3\pi}{2}$$

$$\frac{Q}{P} = \tan \frac{3\pi}{2} = \infty$$

$$\text{As } Q = x_2(0) \neq \infty$$

Therefore

$$P = \frac{x_1(0)}{\omega_0 C} = 0$$

Therefore, $x_1(0) = 0$ and assuming $Q = x_2(0) = 1 \text{ V}$,

$$\text{Let } a = \left(\frac{Q^2}{2} - \frac{P^2}{2} \right)$$

$$b = PQ$$

$$d = \left(\frac{P^2 + Q^2}{2} \right)$$

$$a = r \sin \theta \quad b = r \cos \theta$$

$$d = 0.5$$

$$a = 0.5$$

$$b = 0$$

$$r = 0.5$$

Equation 25 and 26 can be reduced to the following:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 5.66 \sin \omega_0 t - 1.4142 \sin \omega_0 t + 2 - 2 \cos \omega_0 t + 0.6667 (\cos \omega_0 t - \cos 2\omega_0 t) \\ 4 - 4 \cos \omega_0 t - 1.4142 \sin \omega_0 t - 0.9428 (2 \sin 2\omega_0 t - \sin \omega_0 t) + \cos \omega_0 t \end{bmatrix}$$

(27)

The expression for capacitor current is

$$i_c(t) = C \left(\frac{dx_2(t)}{dt} \right)$$

$$= 200 * 10^{-6} (4\omega_0 \sin \omega_0 t - 1.4140\omega_0 \cos \omega_0 t - 0.9428\omega_0 (4\cos 2\omega_0 t - \cos \omega_0 t) - \omega_0 \sin \omega_0 t)$$

(28)

And expression for inductor voltage is

$$v_L(t) = L \left(\frac{dx_1(t)}{dt} \right)$$

$$= 100 * 10^{-6} (5.66\omega_0 \cos \omega_0 t - 1.4142\omega_0 \cos \omega_0 t + 2\omega_0 \sin \omega_0 t - 0.6667\omega_0 (\sin \omega_0 t - 2\sin \omega_0 t))$$

(29)

4. Results

The above equations were plotted in MATLAB and obtained the plots for the various state variables.

State Variable x1(t)

The plot for the state variable x1(t) which is the current through the inductor, is as shown in Fig7.

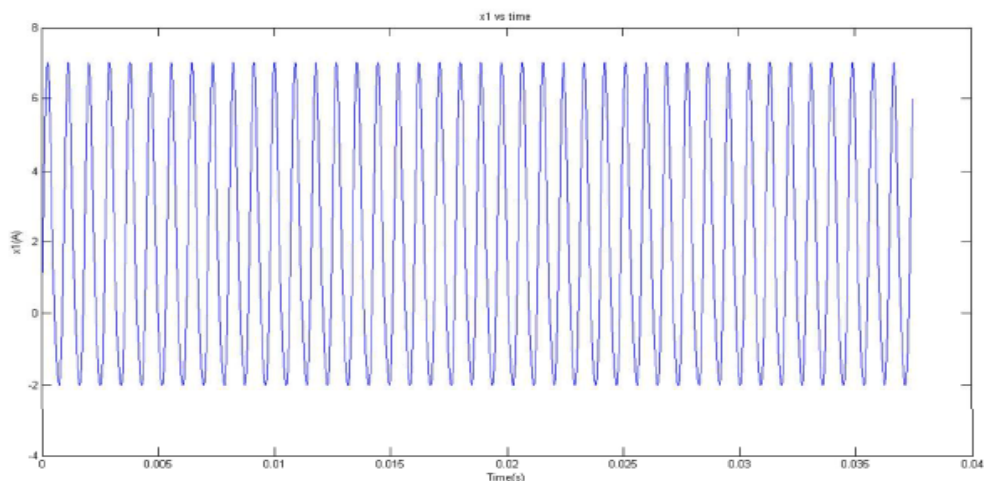


Fig. 7 The plot for the state variable $x_1(t)$ v/s time

From the plot it is evident that the current oscillates about a certain value or an average DC value.

We find this by taking an average of the function simply by integrating the wave form to get the DC value which turns out to be 2 A.

State Variable $x_2(t)$

The plot for the state variable $x_2(t)$, which is the voltage across the capacitor, is as shown in Fig 8.

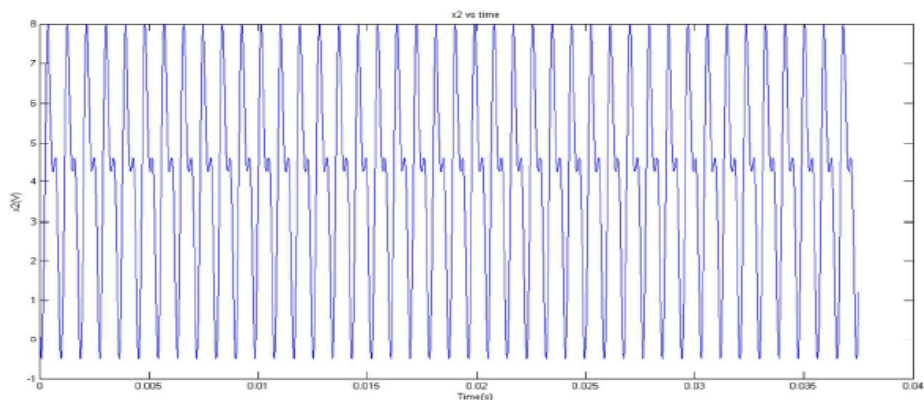


Fig.8. The plot for the state variable $x_2(t)$ v/s time

From the plot it is observable that the waveform is not a pure sinusoid but has a visible 'wobble' or a transient in its waveform. This is because of the second degree non-linearity presented by the load which is injecting a second harmonic

which in turn is causing this ‘wobble’ in the waveform. As with the state variable $x_1(t)$, the waveform oscillates about a average DC value which turns out to be $3.995 \approx 4$ V

Current through Capacitor

The plot for the current through capacitor is obtained as shown in Fig 9.

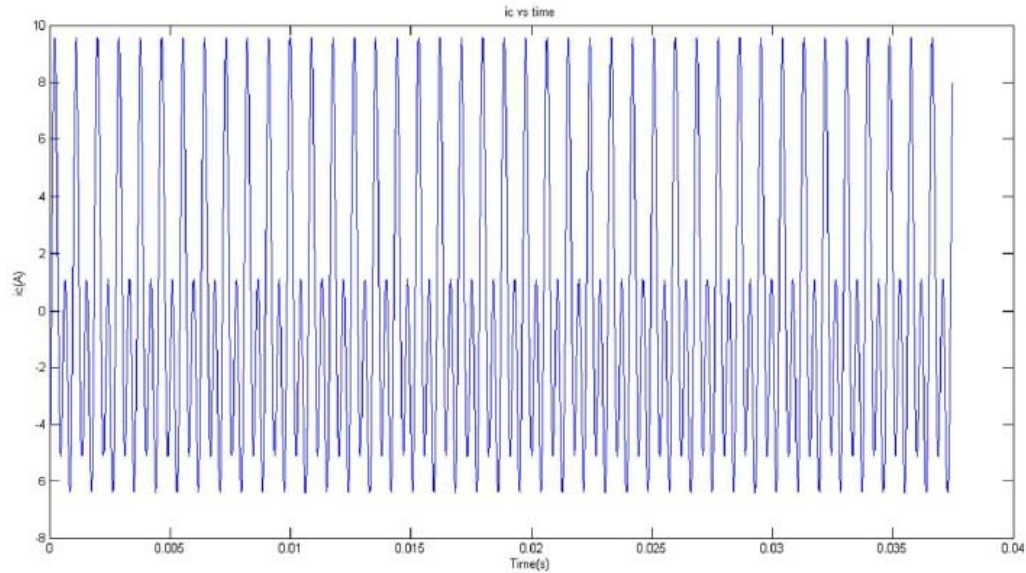


Fig 9. Plot of $I_c(t)$ v/s time

From the plot it is clearly seen that there is a noticeable amount of transient or ‘wobble’ is much higher than that in the $x_2(t)$ plot. The ‘wobble’ as the current is the derivative of the voltage across the capacitor. The current through the capacitor, as expected, has a very small average value of 0.0014 A. The small value is expected since the capacitor has a constant stable DC average value.

Voltage across the inductor

The plot for the voltage across the inductor is obtained as shown in Fig 10.

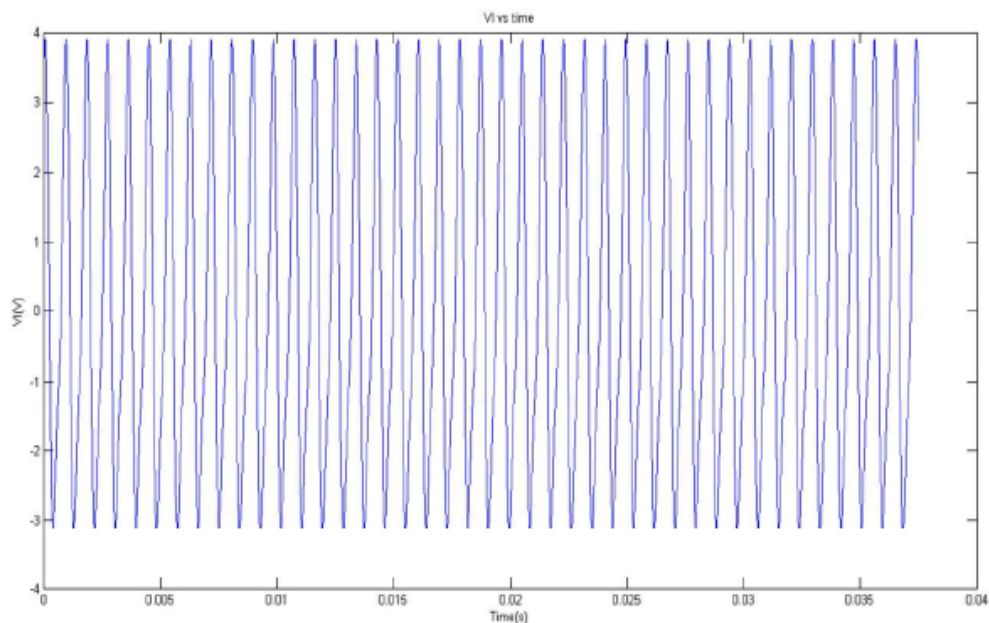


Fig 10. The plot for the voltage across the inductor v/s time

We can clearly see that the voltage just as the current through it oscillates about a certain point, which in this case as expected is $5\text{E-}4 \approx 0$ V. This is expected as the inductor too, like the capacitor has an average DC value.

An interesting point to note would be that although the non-linear load's second degree of non-linearity is injecting a second order harmonic into both, the voltage across the capacitor and the current through the inductor, it is only the voltage across the capacitor which actually indicates any transient or 'wobble'. This is solely because in the expression for the inductor it can be shown that the ratio of the first harmonic to the second harmonic is 6.7:1, while in the expression for the capacitor voltage the same ratio is just 1.6:1 when plotted with such ratios, the harmonic in the inductor is not that easily visible than in the capacitor.

Power supplied to the load

The power supplied to the load is simply calculated by the voltage across it x the current through it,

$$P = x_2 \cdot (x_1 - I_c).$$

The Power is plotted and shown in Fig 11.

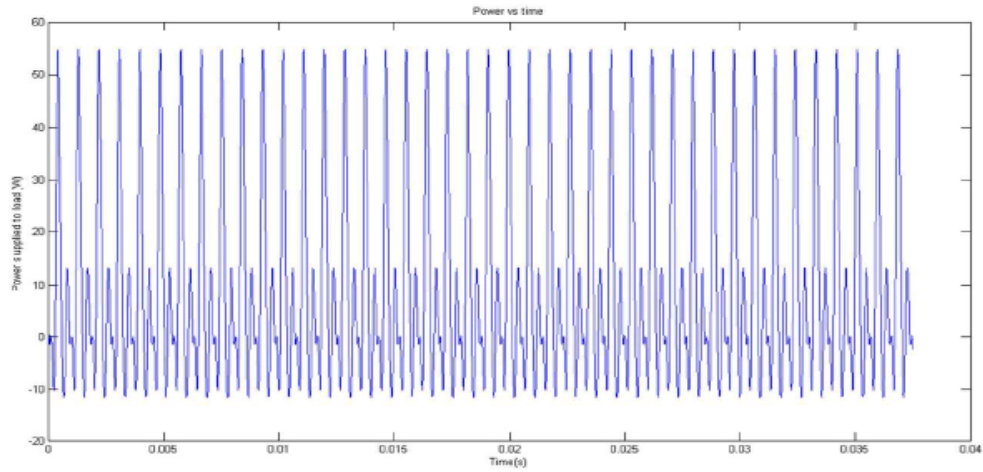


Fig 11. Plot of power v/s time

A noticeable characteristic of the plot is the double 'wobble' or the transient which arises due to the product of the voltage and current, which gives rise to even higher harmonics hence creating the 'wobble'. The average value of this power plot is given to be 9.58 W.

Time(s)	x1(A)	x2(V)	Ic(A)	VI(V)
0	0	1	0.256224	0.345952
0.002	6.919037	3.562238	0.340484	-0.21473
0.004	2.624488	6.967074	-0.24842	-0.21116
0.006	-1.59879	4.482833	-0.35665	0.084197
0.008	0.08514	0.916319	0.278054	0.342917
0.01	6.943478	3.696864	0.320353	-0.22014
0.012	2.540676	6.90039	-0.24025	-0.20826
0.014	-1.62449	4.497845	-0.36639	0.089816
0.016	0.171826	0.833945	0.299755	0.339668
0.018	6.965179	3.831498	0.30012	-0.22539
0.02	2.457438	6.832695	-0.23202	-0.20535
0.022	-1.64955	4.512505	-0.37595	0.095479
0.024	0.260019	0.752997	0.3213	0.336206
0.026	6.984133	3.966	0.27981	-0.23047
0.028	2.374805	6.764103	-0.22374	-0.20244
0.03	-1.67396	4.526719	-0.38531	0.101182
0.032	0.349683	0.673597	0.342663	0.332533
0.034	7.000338	4.100227	0.25945	-0.23538
0.036	2.292806	6.69473	-0.21543	-0.19952
0.038	-1.69769	4.540394	-0.39445	0.106923
0.04	0.440779	0.595863	N/A due to differentiation	

Table 2: Values of the functions at various times

5. Conclusions

The state variable model of a solar array power system taking a higher degree of non-linearity into account has been developed and the concerned solution has been achieved using an iterative mathematical technique. Using this technique, the time response expressions for the state variables have been obtained in explicit form. With the help of these time response expressions, simulation has been performed in MATLAB to assess the power transferred to the load. The power found from the current through inductor and the voltage across capacitor is found to be just 8 W which is violating the conservation of energy. However this error can be attributed to the approximation error due to the iterative method and the behavior of the non-linear load since it hasn't been show to be piecewise linear. The time response expressions developed can also be used to analyze the stability of the system. In general, the method used in this paper can be used for the analysis of any non-linear state model. Such analysis will help a designer to design the values of the controller elements, especially in the necessity of driving a typical non-linear semiconductor element like a Uni-junction transistor (UJT), Tunnel and Gunn diodes. In this context it is relevant to point out that UJT and tunnel diode are

applied in connection with Thyristor triggering and generation of high frequency signals, respectively.

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