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FREE CONVECTIVE MASS TRANSFER FLOW THROUGH A POROUS MEDIUM IN A ROTATING SYSTEM

By

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Abstract

An analytical investigation on a free convective mass transfer steady flow along a semi-infinite vertical plate bounded by a porous medium with large suction is completed in a rotating system. A mathematical model related to the problem is developed from the basis of studying Fluid Dynamics(FD). Non-dimensional system of equations is obtained by the usual similarity transformation with the help of similar variables. The perturbation technique is used to solve the momentum wiith concentration equations. The chief physical interest of the problem as shear stress and Sherwood number are also calculated here. The numerical values of velocities, concentration, shear stress and Sherwood number are plotted in figures. In order to observe the effects of various parameters on the flow variables, the results are discussed in detailed with the help of graphs.

Last of all, some important findings of the problem are concluded in this study.

Keywords and phrases : convective mass transfer, steady flow, shear stress, Sherwood number

বিমূর্ত সার (Bengali version of the Abstract)

একটি অর্ধ অসীম উল্লম্ব প্লেটের দৈর্ঘ্য বরাবর বৃহৎ শোষণ সহ সচ্ছিদ্র মাধ্যমের দ্বারা আবদ্ধ মুক্ত পরিচালন ভর স্থানান্তরণের স্থায়ী প্রবাহের উপর একটি বৈশ্লেষিক অনুসন্ধানকে একটি আবর্তন তন্ত্রে সম্পূর্ণ করা হয়েছে । ফ্লুয়িড গতি বিদ্যার (FLUID DYNAMICS) অনুসন্ধানের ভিত্তি থেকে

উক্ত সমস্যা সম্পর্কিত গাণিতিক মডেলের উদ্ভাবন করা হয়েছে । প্রথানুসারে সদৃশ স্থানান্তরণের মাধ্যমে সদৃশ তলের সাহায্যে অ - মাত্রিক তন্ত্রের সমীকরণটি নির্ণয় করা হয়েছে । এই সমস্যার মুখ্য ভৌত আকর্ষণ হিসাবে কৃন্তন পীড়ণ এবং শেরউড নাম্বার (SHERWOOD NUMBER) গণনা করা হয়েছে । গতিবেগ, গা ঢ়তা, কৃন্তন পীড়ন এবং শেরউড নাম্বারের সাংখ্যমানগুলিকে সংখ্যায় প্লট করা হয়েছে । প্রবাহ চলের উপর বিবিধ প্রচলের প্রভাবকে বুঝবার জন্য ইহাদের ফলাফলগুলি বিশদভাবে আলোচনা করা হয়েছে লেখচিত্রের সাহায্যে ।

অবশেষে, এই অনুসন্ধানে সমস্যার কিছু গুরুত্বপূর্ণ সিদ্ধান্তকে চূড়ান্ত করা হয়েছে ।

1. Introduction

The processes of mass transfer play an important role in the production of materials in order to obtain the desired properties of a substance. Separation processes in chemical engineering such as the drying of solid materials, distillation, extraction and absorption are all affected by the process of mass transfer. Chemical reactions including combustion processes are often decisively determined by the mass transfer. *Callahan* and *Marner*(1976) studied a free convective unsteady flow with mass transfer past a semi-infinite plate. An investigation on free convective unsteady flow with mass transfer past an infinite vertical porous plate with constant suction has been completed by *Soundalgekar* and *Wavre*(1977). Transient free convection flow on a semi-infinite plate with mass transfer has been observed by *Soundalgekar* and *Ganesan*(1980).

Since porous media are very widely used to insulate a heated body to maintain its temperature so to make the heat insulation of the surface more effective, it is necessary to study the flow through a porous medium. *Raptis et al.*(1981) have observed the steady free convective flow through a porous medium bounded by an infinite surface by use of the model of *Yamamoto* and *Iwamura*(1976) for the flow near the surface. A natural convective flow about a vertical plate embedded in porous medium have been analyzed by *Kim* and *Vafai*(1989). Recently, *Magyari et al*.(2004) have studied a free convective unsteady flow in a porous medium.

The flow problems in a rotating system are very important in rocket propulsion control, crystal growth technology, astrophysical plasma fluid dynamics and tribological regulation in moving machine parts. The Ekman boundary layers of an incompressible fluid have been investigated as basic boundary layers in a rotating environment appearing in the oceanic, atmospheric, cosmic fluid dynamics and solar physics or geophysical problems. *Greenspan*(1968) was the first author to recognize the Ekman boundary layer near a flat plate in a rotating fluid and find out the viscous and the Coriolis forces are of same order of magnitude. A free convective mass transfer flow in a rotating fluid past an infinite porous plate has been studied by *Raptis* and *Perdikis*(1982). Hence, our main aim is to investigate a free convective mass transfer Ekman boundary layer flow along a semi-infinite vertical plate surrounded by a porous medium with large suction. These types of problems play a decisive role in a number of industrial applications as chemical engineering, fiber and granular insulation, crystal growth technology, rocket propulsion control etc.

2. Mathematical Model of Flow

A free convective mass transfer steady flow of an electrically conducting incompressible viscous fluid past an electrically non-conducting semi-infinite vertical plate bounded by a porous medium is considered in a rotating system. The flow is assumed to be in the x-direction which is chosen along the plate in upward direction and y-axis is normal to it. Initially, we consider that the plate as well as the fluid particles is at rest at the same species concentration level $C\left(=C_\infty\right)$ at all points, where C_∞ denotes the uniform concentration of fluid. It is also assumed that the plate be at rest after that the system is allowed to rotate with a constant angular velocity $\mathbf{\Omega}$ about the y-axis. Hence the angular velocity vector is of the form $\mathbf{\Omega}=\left(0,-\Omega,0\right)$.

Within the framework of the above stated assumptions, we have the following system of coupled non-linear partial differential equations in accordance with the Boussinesq's approximation,

Continuity Equation $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Momentum Equations $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(C-C_\infty)+\upsilon\frac{\partial^2 u}{\partial y^2}-2w\Omega-\frac{\upsilon}{K'}u$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\upsilon\frac{\partial^2 w}{\partial y^2}-2u\Omega-\frac{\upsilon}{K'}w$$

Concentration Equation $$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial x^2}$$

under also the Ekman bounder layer phenomena, the appropriate boundary conditions of the problem are as given below,

$$u=0,\quad v=V(x),\quad w=0,\quad C=C_w,\quad \text{at } y\to 0$$
$$u=0,\quad v=0,\quad w=0,\quad C=C_\infty,\quad \text{as } y\to\infty$$

where x & y denote the cartesian coordinates in two directions, u, v & w are velocity components of flow, g is the local acceleration due to gravity, β is the thermal expansion coefficient, υ is the kinematic viscosity, K' is the permeability of porous medium, D_m is coefficient of mass diffusivity and C_w represents the species concentration near at the plate.

In order to obtain the non-dimensional system of equations, it is required to introduce the following similar variables,

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}},\qquad \psi=\sqrt{2\upsilon xU_0}\,f(\eta)\quad\text{and}\quad \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.$$

Introducing the above stated variables, we have the followings,

$$u=U_0 f'(\eta),\qquad v=\sqrt{\frac{\upsilon U_0}{2x}}\left[\eta f'(\eta)-f(\eta)\right]\quad\text{and}\quad w=U_0 g.$$

Using the above relations, we have the following dimensionless equations,

$$f'''(\eta)+f(\eta)f''(\eta)-Kf'(\eta)+G_m\phi(\eta)+2Eg(\eta)=0$$
$$g''(\eta)+g'(\eta)f(\eta)-2Ef'(\eta)-Kg(\eta)=0$$
$$\varphi''(\eta)+S_c f(\eta)\varphi'(\eta)=0$$

where, $K = \dfrac{2x\nu}{K' U_0}$ (**Permeability Parameter**), $E = \dfrac{2\Omega x}{U_0}$ (**Ekman Number**),

$G_m = \dfrac{2g\beta x}{U_0^2}(C_w - C_\infty)$ (**Modified Grashof Number**), $S_c = \dfrac{\nu}{D_m}$ (**Schmidt Number**)

and $f'(\eta)$, $g(\eta)$ and $\varphi(\eta)$ represent the non-dimensional primary velocity, secondary velocity of fluid and species concentretion respectively.

Also the dimensionless boundary conditions,

$$f = f_w, \quad f' = 0, \quad g = 0, \quad \varphi = 1, \quad \text{at} \quad \eta = 0$$

$$f' = 0, \quad g = 0, \quad \varphi = 0, \quad \text{as} \quad \eta \to \infty$$

where, $f_w = -V(x)\sqrt{\dfrac{2x}{U_0 \nu}}$ is the transpiration parameter. Here $f_w > 0$ indicates the suction and $f_w < 0$ indicates the injection.

3. Analytical Solutions

Since the solution is sought for the large suction, so we take the followings,

$$\xi = \eta f_w, \quad f(\eta) = f_w F(\xi), \quad \varphi(\eta) = f_w^2 \phi(\xi), \quad g(\eta) = f_w^2 G(\xi).$$

Using the above quantities, we have the following system of equations,

$$F''' + FF'' = \varepsilon(KF' - G_m \phi - 2EG), \quad G'' + FG' = \varepsilon(KG + 2EF'), \quad \phi'' + F\phi' S_c = 0$$

with boundary conditions,

$$F = 1, \quad F' = 0, \quad G = 0, \quad \phi = \varepsilon \quad \text{at} \ \xi = 0$$

$$F' = 0, \quad G = 0, \quad \phi = 0 \quad \text{as} \ \xi \to \infty \quad \text{where, } \varepsilon = \frac{1}{f_w^2}.$$

Now for the large suction $f_w > 1$, ε will be very small. Hence F, G and ϕ can be expended in terms of the small perturbation quantity ε as,

$$F(\xi) = 1 + \varepsilon F_1(\xi) + \varepsilon^2 F_2(\xi) + \varepsilon^3 F_3(\xi) + \cdots\cdots\cdots\cdots$$

$$G(\xi) = \varepsilon G_1(\xi) + \varepsilon^2 G_2(\xi) + \varepsilon^3 G_3(\xi) + \cdots\cdots\cdots\cdots$$

$$\phi(\xi) = \varepsilon \phi_1(\xi) + \varepsilon^2 \phi_2(\xi) + \varepsilon^3 \phi_3(\xi) + \cdots\cdots\cdots\cdots$$

Introducing $F(\xi)$, $G(\xi)$ and $\phi(\xi)$ in the above system of equations, we get the first order equations, $F_1''' + F_1'' = 0$, $G_1'' + G_1' = 0$ and $\phi_1'' + \phi_1' S_c = 0$

with first order boundary conditions,

$$F_1 = 0\,, \quad F_1' = 0\,, \quad G_1 = 0\,, \quad \phi_1 = 1 \quad \text{at } \xi = 0$$
$$F_1' = 0\,, \quad G_1 = 0\,, \quad \phi_1 = 0 \quad \text{as } \xi \to \infty\,.$$

Also we have the second order equations,

$$F_2''' + F_2'' + F_1 F_1'' = KF_1 - G_m \phi_1 - 2EG_1$$
$$G_2'' + G_2' + F_1 G_1' = KG_1 + 2EF_1' \quad \text{and} \quad \phi_2'' + S_c\,\phi_2' + F_1\,\phi_1' S_c = 0$$

with second order boundary conditions,

$$F_2 = 0\,, \quad F_2' = 0\,, \quad G_2 = 0\,, \quad \phi_2 = 0 \quad \text{at } \xi = 0$$
$$F_2' = 0\,, \quad G_2 = 0\,, \quad \phi_2 = 0 \quad \text{as } \xi \to \infty\,.$$

And we obtain the third order equations,

$$F_3''' + F_2 F_1'' + F_1 F_2'' + F_3'' = KF_2' - G_m \phi_2 - 2EG_2$$
$$G_3'' + F_2 G_1' + F_1 G_2' + G_3' = KG_2 + 2EF_2' \quad \text{and} \quad \phi_3'' + S_c\,\phi_3 + F_1\,\phi_2\,S_c + F_2\,\phi_1\,S_c = 0$$

with the third order boundary conditions,

$$F_3 = 0\,, \quad F_3' = 0\,, \quad G_3 = 0\,, \quad \phi_3 = 0 \quad \text{at } \xi = 0$$
$$F_3' = 0\,, \quad G_3 = 0\,, \quad \phi_3 = 0 \quad \text{as } \xi \to \infty\,.$$

Using the prescribed boundary conditions upon simplification, we obtain the

First order solution, $F_1 = 0$, $G_1 = 0$ and $\phi_1 = e^{-S_c \xi}$

Second order solution, $F_2 = A_{20}\, e^{-S_c \xi}$, $G_2 = 0$ and $\phi_2 = 0$

and Third order solution, $F_3 = A_{21}\, e^{-S_c \xi}$, $G_3 = A_{22}\, e^{-S_c \xi}$ and $\phi_3 = A_{23}\, e^{-2S_c \xi}$.

From the first, second and third order solutions, the velocities and concentration functions are obtained as follows,

Primary velocity, $f'(\eta) = -\varepsilon\, S_c\, A_{20}\, e^{-S_c\, \eta\, f_w} - \varepsilon^2\, S_c\, A_{21}\, e^{-S_c\, \eta\, f_w}$

Secondary velocity, $g(\eta) = \varepsilon^2 A_{22}\, e^{-S_c\, \eta\, f_w}$

Concentration, $\varphi(\eta) = e^{-S_c\, \eta\, f_w} + \varepsilon^2 A_{23}\, e^{-2S_c\, \eta\, f_w}$.

4. Shear Stress and Sherwood Number

Since the quantities of chief physical interest are shear stress and Sherwood number, so the primary shear stress near at the plate is defined as, $\tau_x = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ which implies that, $\tau_x \,\alpha \left(\varepsilon S_c^2 A_{20} + \varepsilon^2 S_c^2 A_{21}\right)$; the secondary shear stress near at plate is also defined as, $\tau_z = \mu\left(\frac{\partial w}{\partial y}\right)_{y=0}$ which implies that, $\tau_z \,\alpha \left(-S_c \,\varepsilon^2 A_{22}\right)$ and the Sherwood number is defined as, $S_h = \mu\left(-\frac{\partial C}{\partial y}\right)_{y=0}$ which implies that, $S_h \,\alpha \left(-S_c - 2S_c \,\varepsilon^2 A_{23}\right)$.

5. Results and Discussion

In order to discuss the results of the problem, analytical solutions of the system of coupled non-linear partial differential equations are obtained by using the perturbation technique. To analyze the physical situation of the model, we have computed the numerical values of the flow variables for different values of suction parameter(f_w), modified Grashof number(G_m), Schmidt number(S_c), Ekman number (E) and the permeability parameter(K). The fluid velocities and species concentration versus the non-dimensional length scale η are plotted in Figs. 1-5.

The primary velocity profiles for different values of K, G_m, S_c and f_w are displayed in Fig. 1 and Fig. 2. It is observed from Fig. 1 that the primary velocity gradually decreases with the rise of K but it rapidly increases in case of strong G_m. Fig. 2 declare that primary velocity swiftly decreases with the increase of S_c or f_w.

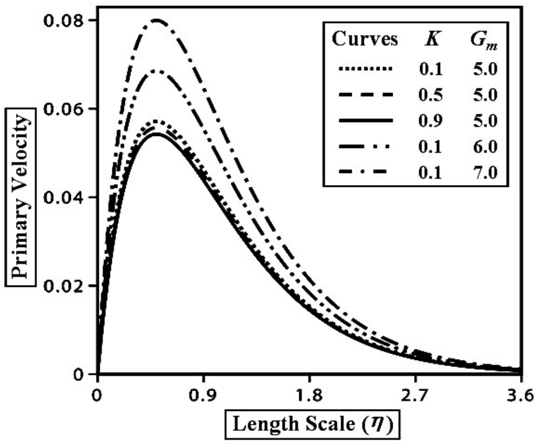

Fig.1. Primary Velocity Profiles for K & G_m

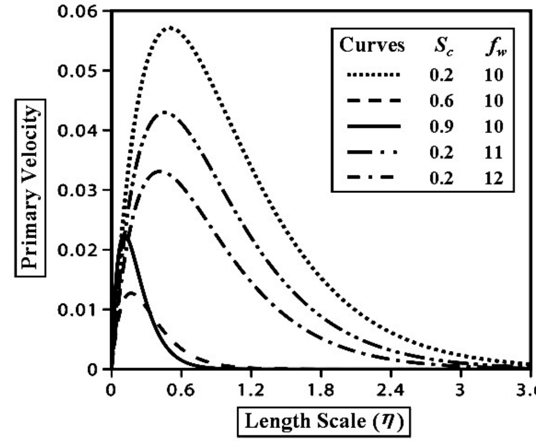

Fig.2. Primary Velocity Profiles for S_c & f_w

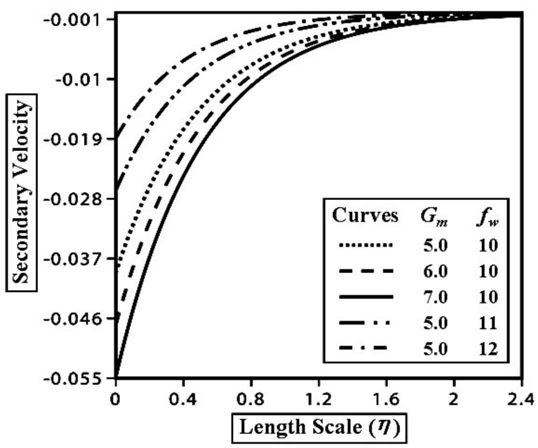

Fig.3. Secondary Velocity Profiles for G_m & f_w

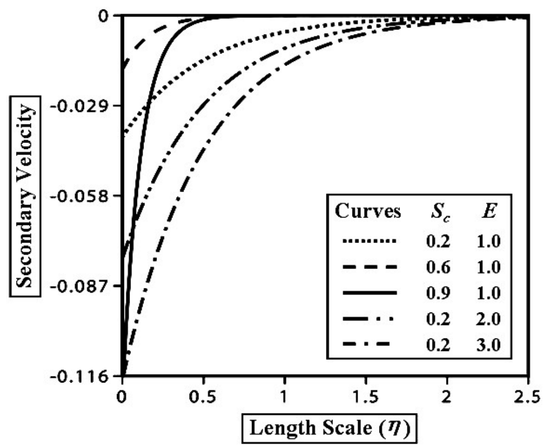

Fig.4. Secondary Velocity Profiles for S_c & E

The secondry velocity profiles for different values of G_m, f_w, S_c and E are shown in Fig. 3 and Fig. 4. We see in Fig. 3, the secondry velocity decreases for the increasing value of G_m while it increases with the rise of f_w. It is observed from Fig. 4, the secondry velocity is increasingly affected by S_c but decreasingly affected by E. Decreasing effect of f_w or S_c on species concentration is observed from Fig. 5.

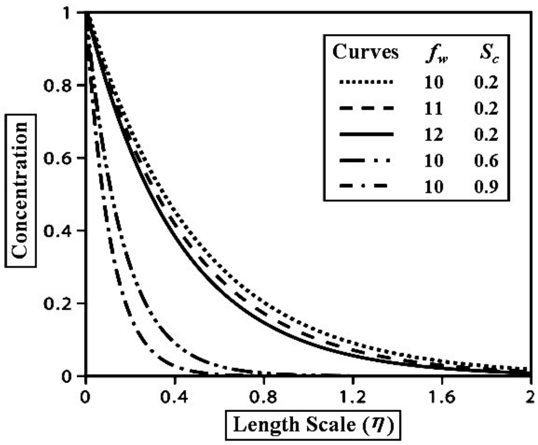

Fig.5. Concentration Profiles for f_w & S_c

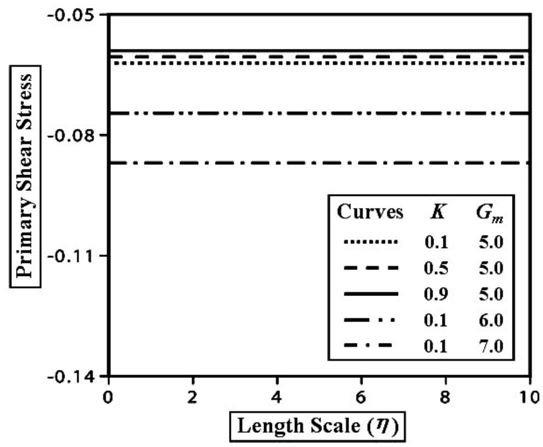

Fig.6. Primary Shear Stress for K & G_m

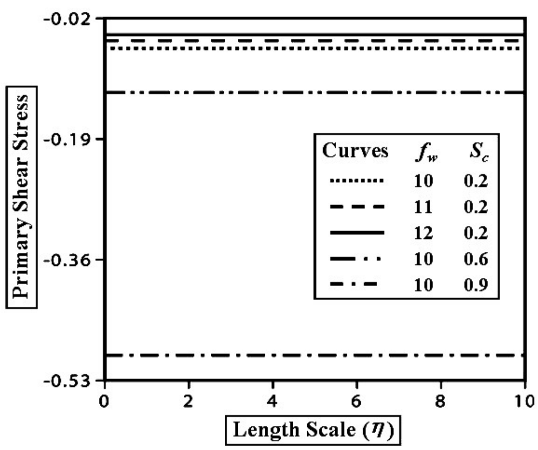

Fig.7. Primary Shear Stress for f_w & S_c

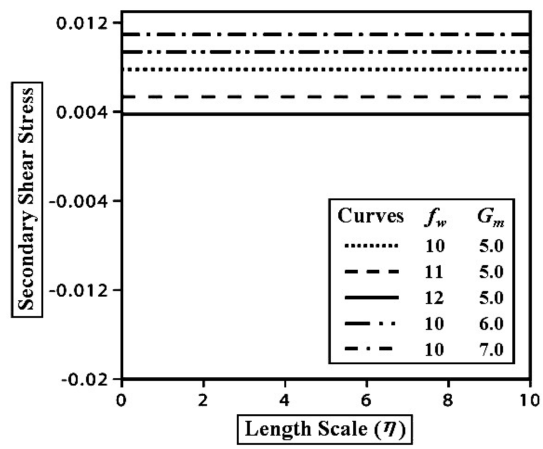

Fig.8. Secondary Shear Stress for f_w & G_m

The shear stress near at plate and Sherwood number versus the dimensionless length scale η are illustrated in Figs. 6-10. The curves of primary shear stress are shown in Figs. 6-7. It is found from Fig. 6 that the primary shear stress increases in case of strong K while it decreases with the increase of G_m. The Fig. 7 shows that the primary shear stress rises for the increasing value of f_w but it decreases with the increase of S_c. We see in Fig. 8, the secondary shear stress fall for the increasing values of f_w but it rises in cse of strong G_m. It is studied from Fig. 9 that secondary shear stress gradually increases with the rise of E or S_c. A negligible effect of f_w

and a strong decreasing effect of S_c on Sherwood number are observed from Fig.10.

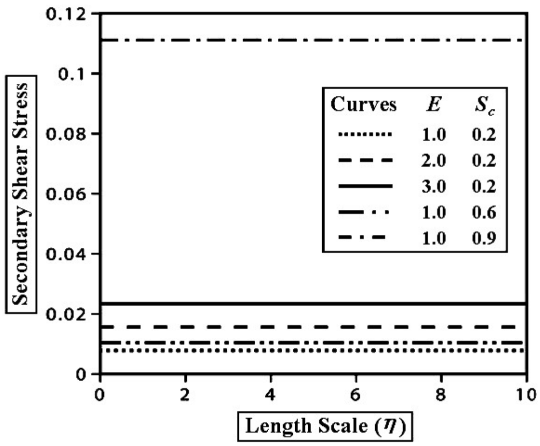

Fig.9. Secondary Shear Stress for E & S_c

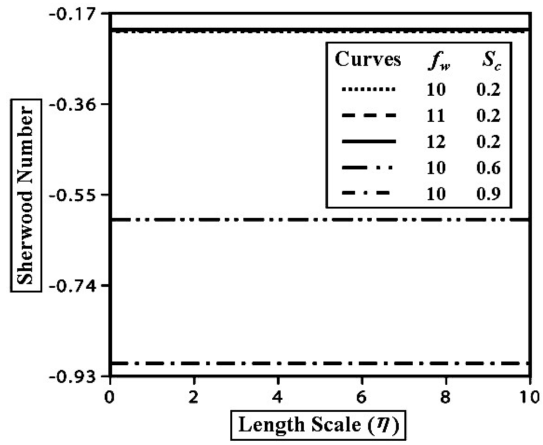

Fig.10. Sherwood Number for f_w & S_c

6. Conclusions

Some of the important findings of the present problem obtained from the graphical representation of the results are listed below,

1. The primary velocity of fluid particles increase with the rise of G_m while decrease for the increase of F_w, S_c or K.
2. The secondary fluid velocity increases in case of strong F_w or S_c but it decreases with the rise of G_m or E.
3. The species concentration decreases with the increases of F_w or S_c.
4. The primary shear stress near at the plate increases in case of strong F_w or K while it decreases with the increase of G_m or S_c.
5. The secondary shear stress rises with the increase of S_c, G_m or E while it decreases for the increase of F_w.
6. The Sherwood number decreases with the increases of S_c.

Appendix

$$A_{20} = \frac{G_m}{S_c^2(S_c-1)}, \quad A_{21} = \frac{K\,A_{20}}{S_c(S_c-1)}, \quad A_{22} = \frac{-E\,A_{20}}{(S_c-1)} \quad \text{and} \quad A_{23} = \frac{-A_{20}}{4S_c}.$$

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