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FAULT DETECTION IN ENGINEERING APPLICATION USING FUZZY PETRI NET AND ABDUCTION TECHNIQUE

By

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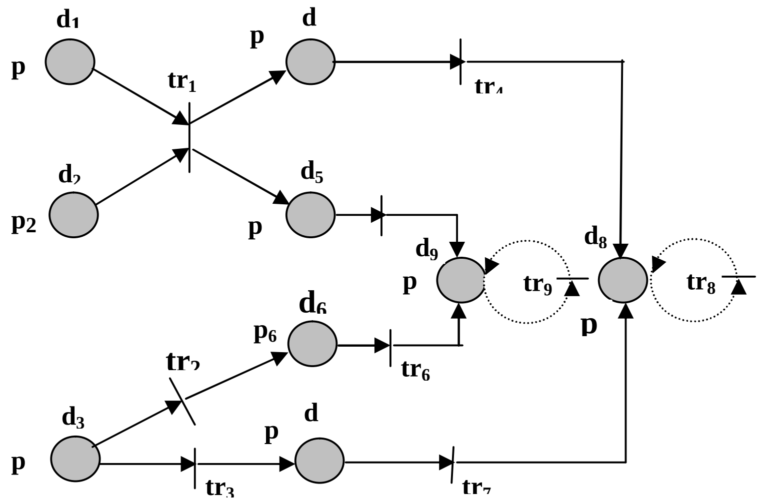
Abstract:

This paper addresses on engineering application using fuzzy abduction and Petri net technique. The problems are introduced informally about the fault finding technique of electronic networks with different illustrations, so that anyone without any background in the specific domain easily understands them. and easily find out the fault of the complicated electronic circuit. The problems require either a mathematical formulation or a computer simulation for their solutions. The detail outline of the solution of the engineering problem is illustrated here.

Keywords and Phrases : *Fuzzy abduction , Petri net, Relational matrix, Abductive Reasoning*

1. Introduction.

Consider the problem of diagnosis of a 2-diode full wave rectifier circuit. The expected rectifier output voltage is 12 volts, when the system operates properly. Under defective condition, the output could be close to 0 volts or 10 volts depending on the number of defective diodes. The knowledge base of the diagnosis problem is extended into a FPN (vide fig. 1). The task here is to identify the possible defects: defective (transformer) or defective (rectifier). Given the belief distribution of the predicate close-to (rectifier-out, 0 v) and more-or-less (rectifier-out, 10 v) (vide fig. 9 and 10), one has to estimate the belief distribution of the predicate: defective (transformer) and defective (rectifier). However, for this estimation, one should have knowledge of the relational matrices corresponding to input-output place pairs of each transition and the thresholds. Let us assume for the sake of simplicity that the thresholds are zero and the relational matrices for each input-output pair of transition tr_1 through tr_7 are equal. So, for $1 \le i \le 7$ let $\mathbf{R_i}$ = $\begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.4 & 0.6 & 0.9 \\ 0.8 & 0.7 & 0.2 \end{bmatrix}$.



d_1= defective (transformer), d_2 =close-to (primary, 230), d_3=defective (rectifier), d_4= Close-to (trans-out, 0V), d_5 = Open (one half-of-secondary-coil), d_6 = Defective (one-diode), d_7 = Defective (two-diodes), d_8 =Close-tp (rectifier-out, 0V), d_9 =More-or-less (rectifier-out, 0V).

Fig. 1 A FPN representing diagnostic knowledge of a 2- diode full wave rectifier.

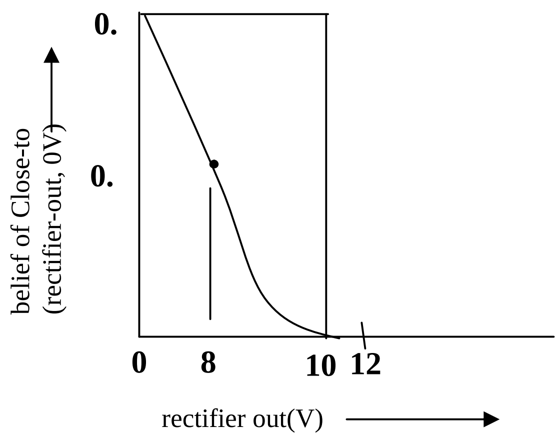


Fig. 2: Belief distribution of Close-to (rectifier-out, 0V).

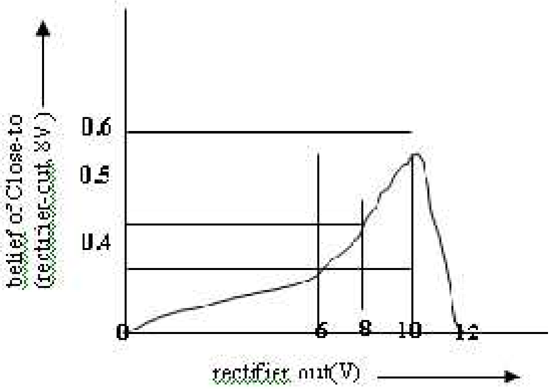


Fig. 3: Belief distribution of More-or-Less (rectifier-out, 10V).

2. Outlines Based Approach:

***Step* 1**: Given, the relational matrices for each input-output pair of transition tr_1 through tr_7 are equal. And these are, $\mathbf{R_i} = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.4 & 0.6 & 0.9 \\ 0.8 & 0.7 & 0.2 \end{bmatrix}$ for $1 \le i \le 7$.

The $\mathbf{R_i}^{-1}$ = Pre-invrrse of ($\mathbf{R_i}$) = $\begin{bmatrix} 0.2 & 0.2 & 0.8 \\ 0.2 & 0.2 & 0.7 \\ 0.4 & 0.9 & 0.4 \end{bmatrix}$, is determined by the Algorithm is given below.

At i =1

Evaluate_ q_{ik} (i)
With $q_{11} = r_{11}$ =0.3; $h_1(q_{11}) = q_{11} - \{(q_{11} \wedge r_{12}) + (q_{11} \wedge r_{13})\}/2 = 0.0$ With $q_{12} = r_{21}$ =0.4; $h_1(q_{12}) = q_{12} - \{(q_{12} \wedge r_{22}) + (q_{12} \wedge r_{23})\}/2 = 0.0$ With $q_{13} = r_{31}$ =0.8; $h_1(q_{13}) = q_{13} - \{(q_{13} \wedge r_{32}) + (q_{13} \wedge r_{33})\}/2 = 3.5$
Max{ $h_1(q_{11}), h_1(q_{12}), h_1(q_{13})$}= $h_1(q_{13})$; Return $q_{13} = r_{31}$ =0.8 and k=3.
Evaluate q_{ij} for all j except k =3. q_{11} = Min(r_{31}, r_{32}, r_{33}) = 0.2; q_{12} = Min(r_{31}, r_{32}, r_{33}) = 0.2.

At i =2

Evaluate_ q_{ik} (i)
With $q_{21} = r_{12}$ = 0.5; $h_1(q_{21}) = q_{21} - \{(q_{21} \wedge r_{11}) + (q_{21} \wedge r_{13})\}/2 = 0.1$ With $q_{22} = r_{22}$ =0.6; $h_1(q_{22}) = q_{22} - \{(q_{22} \wedge r_{21}) + (q_{22} \wedge r_{23})\}/2 = 0.1$ With $q_{23} = r_{32}$ = 0.7; $h_1(q_{23}) = q_{23} - \{(q_{23} \wedge r_{31}) + (q_{23} \wedge r_{33})\}/2 = 0.25$
Since, Max{ $h_1(q_{21})$, $h_1(q_{22})$, $h_1(q_{23})$ }= $h_1(q_{23})$; Return q_{23} = r_{23} = 0.7 and k=3.
Evaluate q_{ij} for all j except k = 3.
q_{21} = Min(r_{31}, r_{32}, r_{33}) = 0.2; q_{22} = Min(r_{31}, r_{32}, r_{33}) = 0.2.

At i =3

Evaluate_ q_{ik} (i)
With $q_{31} = r_{13}$=0.6; $h_1(q_{31}) = q_{31}$-{($q_{31} \wedge r_{11}$)+($q_{31} \wedge r_{12}$)}/2 = 0.2 With $q_{32} = r_{23}$=0.9; $h_1(q_{32}) = q_{32}$-{($q_{32} \wedge r_{21}$)+($q_{32} \wedge r_{22}$)}/2= 0.4 With $q_{33} = r_{33}$=0.2; $h_1(q_{33}) = q_{33}$-{($q_{33} \wedge r_{31}$)+($q_{33} \wedge r_{32}$)}/2 = 0.0
Since, Max{ $h_1(q_{31})$, $h_1(q_{32})$, $h_1(q_{33})$}= $h_1(q_{32})$; Return $q_{32} = r_{23}$ = 0.9 and k=2.
Evaluate q_{ij} for all j except k = 2. q_{31} = Min(r_{21}, r_{22}, r_{23}) = 0.4; q_{33} = Min(r_{21}, r_{22}, r_{23}) = 0.4.

***Step* 2**: The $\mathbf{R_i}$ for $8 \le i \le 9$ will be the identity matrix. Thus $\mathbf{R_{f\,m}}$ in the present context will be a (27 x 27) matrix, whose diagonal blocks will be occupied by $\mathbf{R_i}$. Further, since all the non-diagonal block matrices are null matrix, the $\mathbf{R_f}^{-1}$ can be constructed by substituting $\mathbf{R_i}$ in $\mathbf{R_{fm}}$ by $\mathbf{R_i}^{-1}$.

The $\mathbf{P'_{f\,m}}$ and $\mathbf{Q'_{f\,m}}$ in the present context are also (27 x 27) matrices.

$$\mathbf{P'_{f\,m}} = \begin{bmatrix} \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & I & \Phi & \Phi & I & I & \Phi \\ \Phi & \Phi & \Phi & \Phi & I & I & \Phi & \Phi & I \end{bmatrix},$$

$$\mathbf{Q'_{f\,m}} = \begin{bmatrix} I & I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I \end{bmatrix}$$

The $\mathbf{P'_{f\,m}}^{-1}$ and $\mathbf{Q'_{f\,m}}^{-1}$ are now estimated using the Algorithm I.

$$\mathbf{P'_{f\,m}}^{-1} = \begin{bmatrix} \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I \end{bmatrix},$$

$$\mathbf{Q'_{f\,m}}^{-1} = \begin{bmatrix} I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I & \Phi \\ \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & \Phi & I \end{bmatrix}$$

$\mathbf{N_{ini}}$ is a (27 x 1) vector, given by

$\mathbf{N_{ini}}$ = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.2 0.1 0.0 0.4 0.5 0.6]$^{T}_{27x1}$

Using the equation of Petri-net's dynamic, we write

$$\mathbf{N(t+1) = P'_{f\,m} \circ R_{fm} \circ (Q'_{f\,m} \circ N^{c}(t))^{c}}$$
$$\mathbf{P'_{f\,m}{}^{-1} \circ N(t+1) = R_{fm} \circ (Q'_{f\,m} \circ N^{c}(t))^{c}}$$
$$\mathbf{R_{f}{}^{-1} \circ (P'_{f\,m}{}^{-1} \circ N(t+1)) = (Q'_{f\,m} \circ N^{c}(t))^{c}}$$
$$\mathbf{(R_{f}{}^{-1} \circ (P'_{f\,m}{}^{-1} \circ N(t+1)))^{c} = Q'_{f\,m} \circ N^{c}(t)}$$
$$\mathbf{N(t) = [Q'_{f\,m}{}^{-1} \circ (R_{f}{}^{-1} \circ (P'_{f\,m}{}^{-1} \circ N(t+1)))^{c}]}$$

Now, the algorithm for backward reasoning is then invoked using above formula and the value of $\mathbf{Q'_{f\,m}{}^{-1}}$, $\mathbf{R_f{}^{-1}}$, $\mathbf{P'_{f\,m}{}^{-1}}$ and $\mathbf{N(t+1) = N_{ini}}$.

Sample run

1st step: **N (t)** = [0 0 0 0 0 0 0 0 0 .2 .2 .2 .6 .6 .5 .6 .6 .5 .2 .2 .2 0.2 0.1 0.0 0.4 0.5 0.6]^{T}

2nd step: **N (t)** = [0.2 0.2 0.2 0.2 0.2 0.2 0.5 0.5 0.6 0.2 0.2 0.2

0.6 0.6 0.5 0 .6 0.6 0.5 0.2 0.2 0.2 0.2 0.1 0.0 0.4 0.5 0.6]^T

3rd step: **N (t)** = [0.2 0.2 0.2 0.2 0.2 0.2 0.5 0.5 0.6 0.2 0.2 0.2 0.6 0.6 0.5 0.6 0.6 0.5 0.2 0.2 0.2 0.2 0.1 0.0 0.4 0.5 0.6]^T

The steady-state belief distribution obtained after 3 iterations is given by

$\mathbf{N}_{S.S}$ **= [0.2 0.2 0.2 0.2 0.2 0.2 0.5 0.5 0.6 0.2 0.2 0.2 0.6 0.6 0.5 0.6 0.6 0.5 0.2 0.2 0.2 0.2 0.1 0.0 0.4 0.5 0.6]^T**

The output of rectifier is near to 9 volts because in $\mathbf{N}_{ini}$ only $\mathbf{n}_9$ was set as higher value. $\mathbf{N}_{ini}$ is input of FPN for backward reasoning. After performing abductive reasoning in FPN we got: $\mathbf{n}_3$, $\mathbf{n}_5$ and $\mathbf{n}_6$ has been triggered. $\mathbf{n}_3$ indicates diode has been damaged. $\mathbf{n}_5$ indicates only one diode has been damaged. And $\mathbf{n}_6$ indicates transformer's secondary coil which is connected to damaged-diode may also be damaged. This inference is logically true and valid.

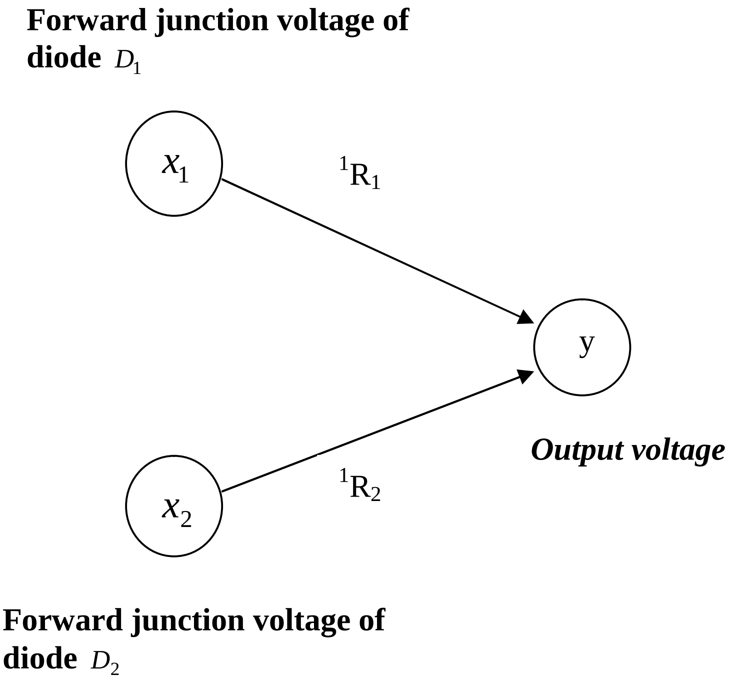


Fig. 4: $^1R_1$ and $^1R_2$ are relational matrices

Step **3**: **Abductive Reasoning:**

Now, we assume the primary membership distribution of "output voltage as MEDIUM" is: Source 1: (0.2 0.9 0.1), source 2: (0.1 1.0 0.2) and source 3: (0.1 1.0 0.1). Furthermore, the secondary membership distribution of "output voltage as MEDIUM" is: Q' = [1.0 0.7 0.7 0.6 0.9 0.8 0.9 0.5 0.8]. Thus, the primary membership distribution for "output voltage as MEDIUM" is obtained using the above primary membership distributions from all sources correspond to the best secondary membership value as, ${}^{best}«_{Output\_Volt\_MEDIUM'}(y) = {}^{best}«_{B'}(x_1)$ = [0.2 1.0 0.1]. Now, we have the primary distribution for diode D_1 as ${}^{best}«_{A'}(x_1)$ = ${}^{best}«_{B'}(x_1)$ o $({}^{1}R_1)^T$ = [0.2 1.0 0.1] o $\begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.9 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$ = [0.9 0.3 0.1]. Similarly, we obtain the primary distribution for diode D_2 as ${}^{best}«_{c'}(x_1)$ = ${}^{best}«_{B'}(x_1)$ o $({}^{1}R_2)^T$ = [0.2 1.0 0.1] o $\begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.1 & 0.2 & 0.9 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$ = [0.1 0.2 0.9].

Thus, when the output voltage is MEDIUM, then we have the primary distribution for diode D_1 as [0.9 0.3 0.1], i.e., diode D_1 is not defective and the primary distribution for diode D_2 as [0.1 0.2 0.9], i.e., diode D_2 is defective.

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