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A DIRECT ANALYTICAL METHOD FOR FINDING AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEMS

By

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Abstract

In this paper a direct analytical method is proposed for finding an optimal solution for a wide range of transportation problems. A numerical illustration is established and the optimality of the result yielded by this method is also checked. The most attractive feature of this method is that it requires very simple arithmetical and logical calculations. The method will be very worthwhile for those decision makers who are dealing with logistics and supply chain related issues. One can also easily adopt the proposed method among the existing methods for simplicity of the presented method.

Keywords and phrases: *TP=Transportation problem, SS=Stepping Stone, MODI= Modified Distribution, NCM= North-West Corner Method, LCM= Least Cost Method, VAM= Vogel's Approximation Method, Numerical Example.*

1. Introduction

The transportation problems are concerned with finding an optimal distribution plan for a single commodity. A given supply of the commodity is available at different number of sources for which there is a specified demand for the commodity at each of the various numbers of destinations, and the unit transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for transporting the products from different sources to several destinations that minimizes the total transportation cost. TP can also be formulated as linear programming problem that can be solved using either dual simplex or Big-M method. Sometimes this can also be solved by using interior approach method. However, it is difficult to get the optimal solution of TP using all these methods. There are many methods for solving TP. Vogel's approximation method (VAM) gives approximate solution while MODI and Stepping Stone (SS) methods are considered as standard techniques for obtaining optimal solution of TP. These two methods are widely used for solving TP. Goyal [1] has improved Vogel's approximation method (VAM) for the unbalanced transportation problem. Ramakrishna [2] has discussed some improvement of Goyal's Modified VAM for unbalanced transportation problem. Moreover, Sultan [3], Sultan and Goyal [4] have studied initial basic feasible solutions and resolution of degeneracy in transportation problem. Few researchers have tried to give their alternate method for overcoming major obstacles over MODI and SS methods. Adlakha and Kowalski [5,6] have suggested an alternative solution algorithm for solving certain TP based on the theory of absolute point. Ji and Chu [7] have discussed a new approach to so called Dual Matrix Approach to solve the transportation problem which gives also an optimal solution. Recently, Pandian and Natarajan [8] and Sudhakar *et al.* [9] have proposed two different methods in 2010 and 2012 respectively for finding an optimal solution directly.

In this paper, a simple heuristic approach is proposed for finding an optimal solution of a transportation problem directly with less number of iterations and very easy computations. The stepwise procedure of the proposed method is carried out as follows and a numerical example is given for testing the optimality.

2. Direct Analytical Method

Step 1: Select the first column (destination) and verify the row (source) which has minimum unit cost. Write that destination under column 1 and corresponding source under column 2. Continue this process for each destination. However, if any destination has more than one same minimum value in different sources then write all these sources under column 2.

Step 2: Select those destinations under column-1 which have unique source. For example, under column-1, destinations are D1, D2, D3 have minimum unit cost which represents the sources K1, K1, K3 written under column 2 respectively. Here K3 is unique and hence allocate cell (K3, D3) a minimum of demand and supply. For example, if corresponding to that cell supply is 8, and demand is 6, then allocate a value 6 for that cell. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand is exhausted.

Step 3: If source under column-2 is not unique then select those destinations where sources are identical. Next find the difference between minimum and next minimum unit cost for all those destinations where sources are identical.

Step 4: Check the destination which has maximum difference. Select that destination and allocate a minimum of supply and demand to the corresponding cell with minimum unit cost. Delete that row/column where supply/demand is exhausted.

If the maximum difference for two or more than two destinations appear to be same then find the difference between minimum and next to next minimum unit cost for those destinations and select the destination having maximum difference. Allocate a minimum of supply and demand to that cell. Next delete that row/column where supply/demand is exhausted.

Step 5: Repeat step 1 to step 4 until all the demand and supply are exhausted.

Step 6: Total cost is calculated as the sum of the product of unit cost and corresponding allocated number of units of supply / demand. That is,

$$\text{Total cost } = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, X_{ij},$$

where C_{ij} = Transportation cost per unit of commodity and X_{ij}= Number of allocated units.

3. Numerical Example

Consider the following transportation problem with four sources and four destinations.

Destination → / Source ↓	E	F	G	H	Supply
A	2	4	5	8	52
B	5	7	6	7	59
C	16	20	10	12	28
D	19	18	17	28	94
Demand	40	55	68	70	233(Total)

Step 1: The minimum cost value for the corresponding destinations E, F, G, and H are 2, 4, 5 and 7 which represent the sources A, A, A and B respectively which is shown in Table 3.1

Table 3.1

Column 1	Column 2
E	A
F	A
G	A
H	B

Step 2: Here the source B is unique for destination H and allocate the cell (B, H), min(59,70)=59. This is shown in table 3.2.

Table 3.2

Destination / Source	E	F	G	H	Supply
A	2	4	5	8	52
B	5	7	6	$\boxed{59}$ 7	59
C	16	20	10	12	28
D	19	18	17	28	94
Demand	40	55	68	70	

Step 3: Delete row B as this supply is exhausted and adjust demand as (70-59) =11. Next the minimum cost value for the corresponding destinations E, F, G and H are 2, 4, 5 and 8 which represent the sources A, A, A and A respectively which is shown in table 3.3.

Table 3.3

Column 1	Column 2
E	A
F	A
G	A
H	A

Step 4: Here the sources are not unique because destinations E, F, G and H have identical source A. So, we find the difference between minimum and next minimum unit cost for the destinations E, F, G and H. The differences are 14, 14, 5 and 4 respectively for the destinations E, F, G and H. Since the maximum difference 14 occurs for two destinations. Now find the difference between minimum and next minimum unit cost for the destinations E and F. The differences are 17 and 16 for the destinations E and F respectively. Here the maximum difference is 17 which represents destination E. Now allocate the cell (A, E), min(40,52)=40 which is shown in table 3.4.

Table 3.4

Destination / Source	E	F	G	H	Supply
A	40 2	4	5	8	52
C	16	20	10	12	28
D	19	18	17	28	94
Demand	40	55	68	11	

Step 5: Delete the column E as this demand is exhausted and adjust supply as (52-40) =12. Next the minimum unit cost for the corresponding destinations F, G and

H are 4, 5 and 8 which represent the sources A, A and A respectively which is shown in table 3.5.

Table 3.5

Column 1	Column 2
F	A
G	A
H	A

Step 6: Repeat step 3 and step 4, we get the following table 3.6

Table 3.6

Destination / Source	F	G	H	Supply
A	$\boxed{12}$ 4	5	8	12
C	20	10	12	28
D	18	17	28	94
Demand	55	68	11	

Step 7: Delete row A as this supply is exhausted and adjust demand as (55-12) = 43. Next the minimum cost value for the corresponding destinations F, G and H are 18, 10, and 12 which represent the sources D, C and C respectively which is shown in table 3.7.

Table 3.7

Column 1	Column 2
F	D
G	C
H	C

Step 8: Repeat step 2 we get the following table 3.8

Table 3.8

Destination / Source	F	G	H	Supply
C	20	10	12	28
D	$\boxed{43}$ 18	17	28	94
Demand	43	68	11	

Step 9: Delete the column F as this demand is exhausted and adjust supply as (94-43) =51. Next the minimum unit cost for the corresponding destinations G and H are 10 and 12 which represent the sources C and C respectively which is shown in table 3.9

Table 3.9

Column 1	Column 2
G	C
H	C

Step 10: Repeat step 3 and step 4 we get the following table 3.10.

Table 3.10

Destination / Source	G	H	Supply
C	10	$\boxed{11}$ 12	28
D	17	28	51
Demand	68	11	

Step11: Delete column H as this demand is exhausted and adjust supply as (28-11) =17. However, only one destination remains. So allocate the remaining supply 17 and 51 to the corresponding cells (C, G) and (D, G) which is shown in table 3.11.

Table 3.11

Destination / Source	G	Supply
C	$\boxed{17}$ 10	17
D	$\boxed{51}$ 17	51
Demand	68	

Step 12: Since all the demand and supply are exhausted. So, we calculate total cost as the sum of the product of the unit cost and its corresponding allocated number of units of supply or demand which is shown in table 3.12

Table 3.12 Optimum solution applying proposed method

Destination / Source	E	F	G	H	Supply
A	$\boxed{40}$ 2	$\boxed{12}$ 4	5	8	52
B	5	7	6	$\boxed{59}$ 7	59
C	16	20	$\boxed{17}$ 10	$\boxed{11}$ 12	28
D	19	$\boxed{43}$ 18	$\boxed{51}$ 17	28	94
Demand	40	55	68	70	233(Total)

The total cost associated with these allocations is

$$= (40\times 2) + (12\times 4) + (59\times 7) + (17\times 10) + (11\times 12) + (43\times 18) + (51\times 17) = 2484.$$

4. *Optimality Check*

To find initial basic feasible solution for the above example by using NCM, LCM and VAM then the allocations are obtained as follows:

Table 4.1 Initial basic feasible solution using NCM

Destination / Source	E	F	G	H	Supply
A	$\boxed{40}$ 2	$\boxed{12}$ 4	5	8	52
B	5	$\boxed{43}$ 7	$\boxed{16}$ 6	7	59
C	16	20	$\boxed{28}$ 10	12	28
D	19	18	$\boxed{24}$ 17	$\boxed{70}$ 28	94
Demand	40	55	68	70	233(Total)

The initial transportation cost associated with these allocations is

$$= (40\times 2) + (12\times 4) + (43\times 7) + (16\times 6) + (28\times 10) + (24\times 17) + (70\times 28) = 3173.$$

Table 4.2 Initial basic feasible solution using LCM

Destination / Source	E	F	G	H	Supply
A	$\boxed{40}$ 2	$\boxed{12}$ 4	5	8	52
B	5	7	$\boxed{59}$ 6	7	59
C	16	20	$\boxed{9}$ 10	$\boxed{19}$ 12	28
D	19	$\boxed{43}$ 18	17	$\boxed{51}$ 28	94
Demand	40	55	68	70	233(Total)

The initial transportation cost associated with these allocations is

$$= (40\times 2)+(12\times 4)+(59\times 6)+(9\times 10)+(19\times 12)+(43\times 18)+(51\times 28) = 3002.$$

Table 4.3 Initial basic feasible solution using VAM

Destination / Source	E	F	G	H	Supply
A	$\boxed{40}$ 2	$\boxed{12}$ 4	5	8	52
B	5	$\boxed{43}$ 7	6	$\boxed{16}$ 7	59
C	16	20	10	$\boxed{28}$ 12	28
D	19	18	$\boxed{68}$ 17	$\boxed{26}$ 28	94
Demand	40	55	68	70	233(Total)

The initial transportation cost associated with these allocations is

$$= (40\times 2)+(12\times 4)+(43\times 7)+(16\times 7)+(28\times 12)+(68\times 17)+(26\times 28) = 2761.$$

To get an optimal solution by adopting Modified Distribution Method, the optimal solution is obtained as 2484. It can be seen that the value of the objective function obtained by proposed method is same as the optimal value obtained by MODI method. Thus, the total transportation cost obtained by direct analytical method is also optimal.

5. Conclusions

It can be concluded that A Direct Analytical Method provides an optimal solution directly, in fewer iterations for the transportation problems. So our proposed method consumes less time and is very easy to understand and applicable to transportation problems. So it will be very helpful for decision makers who are dealing with logistic and supply chain problems.

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