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PRESENT VALUE CALCULATION WITH DISIMILARITY IN EXPECTED RATE AND DISCOUNTING RATE

BY

Debashish Dutta[1], Sudipta Ghosh[2], Arpan Dutta[3], Shilpi Samajpati[3], Balaram Kar[4], Debasish Kundu[5]

[1]IT Research & AnalystTollygunge, Kolkata – 92, WB, India.

[2]Bengal Institute of Technology and Management, Bolpur, WB, India.

[3]Future institute of Engineering &Management Sonarpur, Kolkata - 150, WB, India

[4,5]Camellia School of Engineering &Technology ,Barasat,WB, India.

[6] Gobindapur Saphali Memorial Polytechnic, Gobindrapur, Guskara, Burdwan, West Bengal, India

Email-debashishd75@gmail.com, ghoshsudipta12@yahoo.co.in, arpn_dut@live.com, iamshilpi7@gmail.com, b.kar.mail @ gmail.com , debasish.kundu1962@gmail.com

Abstract:

The sole aim of this paper is to calculate present value of periodical cash flows (fixed time interval) from an investment considering the effect of inflation, where cash flow is fixed (an ordinary annuity), or in a constant growth, or in a random nature. In common practice the discounting rate is equal to the expected rate of return of the investor for calculating present value, but in reality this is not equal. The reason is that the expected rate of return is affected by the change of price level, that's inflation, and that's why this directly affects the purchasing power of money of the investor. To prevent the purchasing power from the inflation, some adjustment is required for calculating actual discounting rate. This paper provides the method of calculating inflation adjusted discounting rate and calculating adjusted present value of future cash flows. It also provides the correct investment decisions from various investment opportunities.

Keywords: Present value of cash flows, Annuity, Discounting rate, Expected rate of return, Value of money, Inflation, Purchasing power of money.

1. INTRODUCTION

In finance, valuation is the very important process to take correct investment decisions, that's informing to the investor the actual present value of a financial asset (business, company, bond, share etc.). This also provides scope of comparisons with other investment opportunities.

In modern study, there were lots of evaluation models available to valuing the asset. But one thing is common, all models involve in finding the present value of the asset's expected future cash flows (profit, dividend, etc.) using the investor's expected rate of return. The expected rate is that rate where an investor expects the return from the investment (cost of project) after adjusting all the uncertainty (risk). Or in other words the expected rate of return actually means the change of purchasing power of the investor from an investment. That's why the discounting rate for calculating present value is not equal to expected rate. This is the biggest challenge for any investment analyst is to determine the appropriate discounting rate to fulfil investor's expectations.

The discussion about some existing ash flow discount model is given in section II, Existing Model and to find out the appropriate discounting rate is provided in section III, The Theoretical Model for clear understanding of present value and methods of valuation.

2. THE EXISTING MODELS

The discounting cash flow valuation is the foundation of which all other valuation approaches are built. This approach stands in the present value rule, where invested amount tally with the present value of all future cash flows from the investment. What is the meaning of present value? If someone is committed to pay Rs. 100 after one year, then what is the present value of Rs. 100? In other word what amount of discount is allowed for repay the amount today? If the banking interest rate is 6% (discounting rate or expected rate of return), then from the well known simple interest formula P x $100/(1+r)^n$ answer is Rs. 94.34$\left(\frac{100}{1.06^1}\right)$. Because if today Rs.94.34 is invested in a bank, then after one year the bank will pay Rs. 100 $(94.34 * 1.06^1)$. That means after one year Rs. 100 is equivalent to today's Rs. 94.34. If cash flows in a random nature, then this model can be defined mathematically as follows:

$$PVCF = \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t} \tag{1}$$

Where PVCF = Present value of cash flows
N = Life of the project.
CF = Expected future Cash flow in period t = 1,. . ., N
i = The discounting rate or the expected rate of return.

If all periodical cash flows from an investment are fixed (not random), then it will be easy to calculate the present value of all cash flows by applying well known ordinary annuity formula. For instance, an annuity which makes periodic cash flows of Rs. CF for N periods is depicted in Figure II.1 where today is depicted as time 0.

Figure II.1

	CF	CF	CF	....	CF	CF
0	1	2	3	....	N - 1	N

If the discounting rate or expected rate of return is *i*, then the present value of cash flow is calculated as follows:

$$PVCF = \left[\frac{\left(1-\left[\frac{1}{(1+i)^{N}}\right]\right)}{i}\right] * CF \qquad (2)$$

For example, if an investment opportunity committed to pay CF= Rs.26 for next N=5 years and discounting rate is i=0.06 or 6% then present value of all cash flow (or annuity) is calculated by applying the above formula, is Rs.109.5215. That's means, if today Rs.109.5215 is deposited in a bank with 6% interest, then from next year for 5 year Rs. 26 is withdrawn every year, and after 5 years the bank balance is zero. This is illustrated in table I.

N	Balance with interest in bank	Withdraw from bank	Balance after withdraw
0	Rs. 109.5215		
1	Rs. 116.0928	Rs. 26	Rs. 90.0928
2	Rs. 95.4984	Rs. 26	Rs. 69.4984
3	Rs. 73.6683	Rs. 26	Rs. 47.6683
4	Rs. 50.5284	Rs. 26	Rs. 24.5284
5	Rs. 26.0000	Rs. 26	Rs. 0.0000

Table-I

Theoretically if the invested amount for this particular project (cost of project) is below Rs.109.5215 then this project is accepted, otherwise rejected.

Now consider cash flows is in a growth, therefore, CF is not fixed nor random, i.e. it is changing in a constant multiplier every time interval. For instance, first cash flow is Rs. CF with the growth of *r* for N periods is depicted in Figure II.2 where today is depicted as time 0.

Figure II.2

	CF	$CF * (1+r)^1$	$CF * (1+r)^2$	. . . .	$CF * (1+r)^{N-2}$	$CF * (1+r)^{N-1}$
0	1	2	3	. . . .	N - 1	N

Now calculate the present value of all future cash flows is the little modification of formula 1, as follows.

$$PVCF = \begin{cases} \left[\left[\dfrac{\left(1-\left(\frac{1+r}{1+i}\right)^N\right)}{(i-r)}\right]\right] * CF & \text{for } i \neq r \\ \left(\dfrac{N}{(1+i)}\right) * CF & \text{for } i = r \end{cases} \tag{3}$$

For example, if an investment opportunity is committed to pay Rs.26 (CF) for first year and from next year to pay Rs.26 with 10% or 0.1 growth rate (r) up to 5 years (N) and discounting rate (expected rate) is 0.06 or 6% (i), then present value of all cash flow is calculated by applying the above formula, is Rs.132.2534. That's means, if today Rs.132.2534 is deposited in a bank with 6% interest, then from next year for 5 years Rs. 26 is withdrawn with 10% growth per year, and after 5 years the bank balance is zero. This is illustrated in Table-II

N	Balance with interest in bank	Withdraw from bank	Balance after withdraw
0	Rs. 132.2534		
1	Rs. 140.1886	Rs. 26	Rs. 114.1886
2	Rs. 121.04	Rs. 28.6	Rs. 92.4399
3	Rs. 97.986	Rs. 31.46	Rs. 66.5263
4	Rs. 70.5179	Rs. 34.606	Rs. 35.9119
5	Rs. 38.0666	Rs. 38.0666	Rs. 0.0000

Table-II

3. THE THEORETICAL MODEL

Till now only discounting rate that is equal to expected rate of return is considered, that means the change of price level is not taken into account. This expected rate is called the *nominal rate.* It only measures monetary gain or loss. But the problem lies when an investor wants to know the change of purchasing power from the investment, instead of in terms of money.

Say today Rs 20/- is the price of 1kg rice and Rs 100/- is invested in a bank by sacrificing 5 kg (Rs.20 * 5 = Rs. 100) of rice and bank has committed to repay Rs. 106

after 1 year, this amount will buy 5.3kg rice if the price of rice is unchanged. In terms of money, bank has returned with 6% interest (nominal rate), and in terms of goods, to buy 6% more rice. That means this investment increase 6% more purchasing power of money, this is called *real rate*.

In the reality, price levels are not constant. Fluctuation of purchasing power of money is called *inflation*. From the previous example of rice, if the price of rice goes to Rs. 21 per kg after one year, and other parameter remains same, then what is the scenario? In terms of money, the bank has been returned the same with 6% interests (nominal rate) after one year, but in terms of goods, the amounts will buy 5.05kg rice (Rs.106 / Rs.21), that means the purchasing power changed (real rate) only 0.95%.

Now if the investor wants to increase the purchasing power or real rate of return in future from the investment by 6% instead of expected rate of return or nominal rate of return, then what is the discounting rate applied for fulfilling the expectation, where expected yearly inflation is 4%. This is the challenge for any investment analyst, that is to determine the discounting rate where the calculation of present value tells actual change of purchasing power from the investment.

The discounting rate always is a nominal rate, because this tells only monetary gain or loss. For calculating change of purchasing power or real rate, then the real rate must be converted to nominal rate or discounting rate by the adjustment of inflation. If the nominal rate is *i*, inflation rate is π and real rate is *r*, then the modified formula for calculation of rate is given below in equation (iv)

$$r = \frac{(1+i)}{(1+\pi)} - 1 \tag{4}$$

Again recall example of rice, where nominal rate is 6% and inflation is 5%$\left(\frac{21}{20} - 1\right)$, then calculate the real rate of return from the equation(iv) is 0.95%$\left(\left(\frac{1+0.06}{1+0.05}\right) - 1\right)$.

Now if the nominal rate (= *i*) with adjustment of inflation is calculated for applying a discounting rate and to calculate the present value, then the equation (iv) is modified as follows

$$i = \left[\left((1+r) * (1+\pi)\right) - 1\right] \tag{5}$$

If inflation (π) is 4% and real rate (r) is 6% then nominal rate (i) or inflation adjusted discount rate is calculated by applying the above formula, is 10.24%.

For the last time again recall the example of rice, if the investor want to buy 6% (r) more rice after one year when price of rice increase 5% (π) from now, then bank should be pay minimum 11.3% (i) to fulfil the expectations. This is explaining step by step, when invested amount is Rs.100 and now price of rice per kg is Rs20.

If price of rice is unchanged for the next year, that means inflation is zero (π = 0), then nominal rate is equal to real rate from above formula, that's 6%. So that Rs106 (Rs100 * 1.06) is capable to purchase 6% more rice. Now consider the effect of inflation. If price of rice is increased 5% (π) or Rs.21 per kg of rice, that means Rs.106 also increase 5% to buy 6% more rice or 5.3kg (5kg * 1.06) rice. So now minimum Rs.111.3 (Rs.106 * 1.05) is require to buy 5.3kg rice. If so then the investor get 6%

more the purchasing power or real rate of return (r) from the investment, and the discounting rate or nominal rate is 11.3% (Rs100 * 1.113).

4. Conclusion

This paper discusses various types of cash flow, and brief explained each types of cash flow, and methods of calculating present value of each cash flow for valuation of any financial asset. This also focussed on five types of rate, nominal rate, real rate, discounting rate, expected rate and inflation rate, and explain the relation between them.The main aim of this paper is to proof that discounting rate and expected rate are not equal all time. If inflation is not equal to zero then discounting rate differ from the expected rate. The equations also provide effect of inflation in discounting rate while calculating present value of future cash flows. The methods explained are also applicable to valuation of various types of financial assets like bond, share, company etc.

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