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ON THE FLOW OF TWO IMMISCIBLE VISCO-ELASTIC FLUIDS THROUGH A RECTANGULAR CHANNEL

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Abstract

This paper is concerned with the flow of two immiscible visco-elastic fluids through a rectangular channel. The flow takes place due to a time-variant pressure gradient which is transient in character. The visco-elastic fluids are of Oldroyd type. In finding solutions of the problem variable separation technique appropriate to the boundary conditions and pressure gradient is applied. In the bulk of the paper, some interesting results such as interface velocity, flux, skin-friction and mean velocity are presented.

1. Introduction

Bagchi [1] investigated the flow of two immiscible visco-elastic Maxwell fluids between two plates when the upper plate is moving with a transient velocity and the lower one remains fixed. The flow of two immiscible viscous fluids between two fixed plates was investigated by Kapur and Sukhla [2]. The problem of periodic flow of two immiscible conducting fluids between two parallel plates in presence of uniform magnetic field has been studied by Das [3]. Sengupta and Ray Mahapatra [4] have investigated the flow of two immiscible visco-elastic Maxwell fluids through a rectangular channel when a transient pressure gradient is applied to both the fluids.

In the present paper the authors have made an attempt to investigate the flow of two immiscible visco-elastic fluids through a rectangular channel. The visco-elastic fluids are of Oldroyd [5] type. The flow takes place due to a transient pressure gradient. The analytical expressions for velocity components, interface velocity, flux, skin friction and mean velocity have been derived.

2. Basic Equations and Formulation of the Problem

For slow motion, the equation of state relating to the stress tensor τ_{ik} and the rate of strain tensor e_{ik} for visco-elastic fluid of Oldroyd type are of the form

$$\tau_{ik} = -pg_{ik} + \tau'_{ik}\,, \tag{1}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau'_{ik} = 2\mu\left(1+\lambda'\frac{\partial}{\partial t}\right)e_{ik}\,, \tag{2}$$

$$e_{ik} = \frac{1}{2}\left(v_{i,k}+v_{k,i}\right); \tag{3}$$

where τ'_{ik} is the part of the stress-tensor associated with the change of shape of the material element, p an arbitrary isotropic pressure, g_{ik} the metric tensor, μ (>0) the coefficient of viscosity and v_i the velocity vector; λ and λ' ($\lambda > \lambda' > 0$) are the stress relaxation time and rate of strain retardation time respectively.

Let us consider a rectangular cartesian co-ordinate system in which the walls of channel are the planes $x=\pm a$ and $y=\pm b$. We choose z-axis on the interface of the fluids and in the direction of motion of both the fluids, the x-axis perpendicular to the interface drawn into the upper fluid and the y-axis in the plane of the interface. Let ρ_1, μ_1, λ_1 , λ'_1 respectively be the density, viscosity, relaxation time and retardation time of the lower fluid and ρ_2, μ_2, λ_2 , λ'_2 those of the upper fluid. Let each of the fluids occupies a height '*a*' . We suppose that a time-variant transient pressure gradient $-p_0e^{-\beta t}$ is applied to both the fluids. The components of velocity of the lower fluid are $\{0,0,w_1(x,y,t)\}$ and those of the upper fluid are $\{0,0,w_2(x,y,t)\}$.

The equations of motion can be written, with the help of (1), (2) and (3), as

$$\left(1+\lambda_i\frac{\partial}{\partial t}\right)\frac{\partial w_i}{\partial t} = -\frac{1}{\rho_i}\left(1+\lambda_i\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z}+\vartheta_i\left(1+\lambda'_i\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w_i}{\partial x^2}+\frac{\partial^2 w_i}{\partial y^2}\right),\ (\,i=1,2\,) \tag{4}$$

where $\vartheta_i = \frac{\mu_i}{\rho_i}$ is the kinematical coefficient of viscosity.

3. Solution of the Problem

We assume the solution of (4) as

$$w_i = u_i \cos my\; e^{-\beta t} \tag{5}$$

where u_i is a function of x alone.

The boundary conditions of the lower fluid are

$$\left.\begin{aligned} u_1 &= 0 \ \ when\ \ x=-a, -b\le y\le b \\ u_1 &= u_0 \ \ when\ \ x=0, -b\le y\le b \\ w_1 &= 0 \ \ when\ \ y=\pm b, -a\le x\le a \end{aligned}\right\} \tag{6}$$

The boundary conditions of the upper fluid are

$$\left.\begin{aligned} u_2 &= 0 \ \ when\ \ x=a, -b\le y\le b \\ u_2 &= u_0 \ \ when\ \ x=0, -b\le y\le b \\ w_2 &= 0 \ \ when\ \ y=\pm b, -a\le x\le a \end{aligned}\right\} \tag{7}$$

Now, the assumed solution (5) will satisfy the last boundary conditions of (6) and (7) provided that $\cos mb = 0$ and hence we get

$$m = (2n+1)\frac{\pi}{2b} \tag{8}$$

Therefore, we may take the possible solution of (4) in the form

$$w_i = \sum_{n=0}^{\infty} u_i(x)\cos my\, e^{-\beta t} \tag{9}$$

where m is given by (8).

We suppose that

$$\frac{1}{\mu_i}\frac{\partial p}{\partial z} = -p_0 e^{-\beta t}\,,\ \beta > 0 \tag{10}$$

Then using (9) and (10) we get from (4)

$$\sum_{n=0}^{\infty}\frac{d^2 u_i}{dx^2}\cos my - \sum_{n=0}^{\infty}\left\{m^2 - \frac{\beta(1-\lambda_i\beta)}{\vartheta_i(1-\lambda_i'\beta)}\right\}u_i\cos my + \frac{p_0(1-\lambda_i\beta)}{\mu_i(1-\lambda_i'\beta)} = 0 \tag{11}$$

We now express $\frac{p_0(1-\lambda_i\beta)}{\mu_i(1-\lambda_i'\beta)}$ as a Fourier series in y in the interval $-b \le y \le b$ and equate the coefficients of $\cos my$ to zero. Then the values of u_i can be obtained from

$$\frac{d^2 u_i}{dx^2} - \frac{k_i^2}{a^2}u_i + (B_n)_i = 0\,, \tag{12}$$

where $k_i^2 = \left\{m^2 - \frac{\beta(1-\lambda_i\beta)}{\vartheta_i(1-\lambda_i'\beta)}\right\}a^2$ and $(B_n)_i = \frac{(-1)^n 4p_0(1-\lambda_i\beta)}{(2n+1)\pi\mu_i(1-\lambda_i'\beta)}$.

Therefore, the solution of (12), with the boundary condition (6), for the lower fluid is obtained as

$$u_1(x) = \frac{(-1)^n 4p_0 a^2(1-\lambda_1\beta)}{(2n+1)\pi\mu_1 k_1^2(1-\lambda_1'\beta)}\left[1 - \frac{\sinh k_1\left(1+\frac{x}{a}\right) - \sinh\frac{k_1 x}{a}}{\sinh k_1}\right] + u_0\frac{\sinh k_1\left(1+\frac{x}{a}\right)}{\sinh k_1}\quad for\ -a \le x \le 0 \tag{13}$$

Again, the solution of (12), with the boundary condition (7), for the upper fluid is found out as

$$u_2(x) = \frac{(-1)^n 4p_0 a^2(1-\lambda_2\beta)}{(2n+1)\pi\mu_2 k_2^2(1-\lambda_2'\beta)}\left[1 - \frac{\sinh k_2\left(1-\frac{x}{a}\right) + \sinh\frac{k_2 x}{a}}{\sinh k_2}\right] + u_0\frac{\sinh k_2\left(1-\frac{x}{a}\right)}{\sinh k_2}\quad for\ 0 \le x \le a \tag{14}$$

Therefore, from (9), (13) and (14) we finally get the velocity components as

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4p_0 a^2(1-\lambda_1\beta)}{(2n+1)\pi\mu_1 k_1^2(1-\lambda_1'\beta)}\left\{1 - \frac{\sinh k_1\left(1+\frac{x}{a}\right) - \sinh\frac{k_1 x}{a}}{\sinh k_1}\right\} + u_0\frac{\sinh k_1\left(1+\frac{x}{a}\right)}{\sinh k_1}\right]\cos(2n+1)\frac{\pi y}{2b}.e^{-\beta t}$$

$$for\ -a \le x \le 0 \tag{15}$$

and

$$w_2 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4p_0 a^2(1-\lambda_2\beta)}{(2n+1)\pi\mu_2 k_2^2(1-\lambda_2'\beta)}\left\{1-\frac{\sinh k_2\left(1-\frac{x}{a}\right)+\sinh\frac{k_2 x}{a}}{\sinh k_2}\right\}+u_0\frac{\sinh k_2\left(1-\frac{x}{a}\right)}{\sinh k_2}\right]\cos(2n+1)\frac{\pi y}{2b}.e^{-\beta t}$$

$$for\ 0 \le x \le a \tag{16}$$

4. Determination of Interface Velocity, Flux and Skin Friction on the Walls

(a) Interface Velocity

Using the condition that the tangential stress is continuous at the interface for both the fluids, i.e.

$$\left[\sum_{n=0}^{\infty}\frac{\mu_1(1-\lambda_1'\beta)}{(1-\lambda_1\beta)}\frac{\partial u_1}{\partial x}.\cos my\right]_{x=0} = \left[\sum_{n=0}^{\infty}\frac{\mu_2(1-\lambda_2'\beta)}{(1-\lambda_2\beta)}\frac{\partial u_2}{\partial x}.\cos my\right]_{x=0}$$

we get from (13) and (14)

$$u_3 = \frac{\sum_{n=0}^{\infty}\frac{(-1)^n.2p_0a^2}{(2n+1)\pi}\left[\frac{\tanh\frac{k_1}{2}}{k_1/_2}+\frac{\tanh\frac{k_2}{2}}{k_2/_2}\right]\cos(2n+1)\frac{\pi y}{2b}}{\sum_{n=0}^{\infty}\left[\frac{\mu_1k_1(1-\lambda_1'\beta)}{(1-\lambda_1\beta)}\coth k_1+\frac{\mu_2k_2(1-\lambda_2'\beta)}{(1-\lambda_2\beta)}\coth k_2\right].\cos(2n+1)\frac{\pi y}{2b}} \tag{17}$$

(b) Flux

If Q be the total flux, then

$$Q = Q_1 + Q_2 = \int_{-b}^{b}\int_{-a}^{0} w_1\,dx\,dy + \int_{-b}^{b}\int_{0}^{a} w_2\,dx\,dy \tag{18}$$

From (15), (16) and (18) we obtain

$$Q = \sum_{n=0}^{\infty}\left[\frac{16p_0a^2b(1-\lambda_1\beta)}{(2n+1)^2\pi^2\mu_1k_1^2(1-\lambda_1'\beta)}\left\{1-\frac{\tanh\frac{k_1}{2}}{k_1/_2}\right\}+\frac{(-1)^n.2abu_0}{(2n+1)\pi}.\frac{\tanh\frac{k_1}{2}}{k_1/_2}\right].e^{-\beta t}$$
$$+\sum_{n=0}^{\infty}\left[\frac{16p_0a^2b(1-\lambda_2\beta)}{(2n+1)^2\pi^2\mu_2k_2^2(1-\lambda_2'\beta)}\left\{1-\frac{\tanh\frac{k_2}{2}}{k_2/_2}\right\}+\frac{(-1)^n.2abu_0}{(2n+1)\pi}.\frac{\tanh\frac{k_2}{2}}{k_2/_2}\right].e^{-\beta t} \tag{19}$$

(c) Skin Friction

The skin friction on the wall $x = -a$ is found out as

$$(\tau')_{x=-a} = \frac{\mu_1(1-\lambda_1'\beta)}{(1-\lambda_1\beta)}e^{-\beta t}\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}.4p_0a(1-\lambda_1\beta)}{(2n+1)\pi k_1\mu_1(1-\lambda_1'\beta)}\tanh\frac{k_1}{2}+\frac{u_0k_1}{a\sinh k_1}\right]\cos(2n+1)\frac{\pi y}{2b} \tag{20}$$

The skin friction on the wall $x = a$ is obtained as

$$(\tau'')_{x=a} = \frac{\mu_2(1-\lambda_2'\beta)}{(1-\lambda_2\beta)} e^{-\beta t} \sum_{n=0}^{\infty} \left[\frac{(-1)^{n+1}.4p_0 a(1-\lambda_2\beta)}{(2n+1)\pi k_2 \mu_2 (1-\lambda_2'\beta)} \tanh\frac{k_2}{2} + \frac{u_0 k_2}{a \sinh k_2} \right] \cos(2n+1)\frac{\pi y}{2b} \tag{21}$$

The total skin friction on the wall $y = b$ is

$$(\tau)_{y=b} = (\tau')_{y=b} + (\tau'')_{y=b}$$

$$= \sum_{n=0}^{\infty} \left[\begin{array}{l} \frac{2p_0 a^2}{b^2} \left\{ \frac{1}{k_1^2} \left(1 - \frac{\sinh k_1\left(1+\frac{x}{a}\right) - \sinh\frac{k_1 x}{a}}{\sinh k_1} \right) + \frac{1}{k_2^2} \left(1 - \frac{\sinh k_2\left(1-\frac{x}{a}\right) + \sinh\frac{k_2 x}{a}}{\sinh k_2} \right) \right\} \\ + u_0 \left\{ \frac{(-1)^n.(2n+1)\pi}{2b} \left(\frac{\mu_1(1-\lambda_1'\beta)}{(1-\lambda_1\beta)} . \frac{\sinh k_1\left(1+\frac{x}{a}\right)}{\sinh k_1} + \frac{\mu_2(1-\lambda_2'\beta)}{(1-\lambda_2\beta)} . \frac{\sinh k_2\left(1-\frac{x}{a}\right)}{\sinh k_2} \right) \right\} \end{array} \right] . e^{-\beta t} \tag{22}$$

We can obtain the skin friction on the wall $y = -b$ from (22) on replacing b by $-b$.

The mean velocity of the fluids through the rectangular channel is $\frac{Q}{2ab}$, where Q is given by (19).

5. Conclusion

It follows from the above analytical results that the velocity profile of a fluid particle and the other important results are transient in character and therefore die out after a long interval of time from the start of the motion.

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