J.Mech.Cont.& Math. Sci., Vol.-10, No.-2, January (2016) Pages 1535-1550

A CREDIT POLICY APPROACH OF AN INVENTORY MODEL FOR DETERIORATING ITEM WITH PRICE AND TIME DEPENDENT DEMAND

By

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Abstract

In this paper, we have developed an inventory model for deteriorating items with price and time dependent demand considering inflation effect on the system. Shortages if any are allowed and partially backlogged with a variable rate dependent on the duration of waiting time up to the arrival of next lot. The corresponding problem has been formulated as a nonlinear constrained optimization problem, all the cost parameters are crisp valued and solved. A numerical example has been considered to illustrate the model and the significant features of the results are discussed. Finally, based on these examples, a sensitivity analyses have been studied by taking one parameter at a time keeping the other parameters as same.

Keywords Inventory, deterioration, partially backlogged shortages, permissible delay in payment

1. Introduction

Due to highly competition in marketing policy permissible delay in payment is one of the important factors to increase their business. As a result, wholesalers/suppliers offer different types of facilities to their retailers to promote their business. In that case, wholesalers/suppliers offer a certain credit period to their retailer. In this period, no interest is charged by the supplier to their retailer. However, after this period, a low rate of interest is charged by the supplier to the next credit period and after this time period, a high rate of interest is charged by the supplier under certain terms and conditions. This is known as inventory problem with permissible delay in payments. It is also known as trade credit financing inventory problem. This type of idea was first introduced by Haley and Higgins [1]. Thereafter, Goyal [2] formulated an EOQ model under the conditions of permissible delay in payments. Then, Aggarwal and Jaggi [3] extended the Goyal's model for deteriorating items. Shortages are not considered in their model. Jamal et al. [4] developed the general EOQ model, considering fully backlogged shortages. After Jamal et al. [4], a number of works have been done by several researchers to their research. The detailed of some of the works have been shown in **Table 1**.

Table 1: Summary of related literature for inventory model with permissible delay in payments

Author(s) and year	Deterioration	Demand Rate	Shortages	Level of permissible delay in payments	Inventory policies
Hwang and Shinn[5]	Yes	Constant	No	Single	--
Chang, Ouyang and Teng[6]	Yes	Constant	No	Single	--
Abad and Jaggi[7]	No	Linearly time dependent	No	Single	--
Ouyang, Wu and Yang[8]	Yes	Constant	No	Single	--
Huang [9]	No	Linearly time dependent	No	Two level	---
Huang[10]	No	Constant	No	Single	--
Huang[11]	No	Constant	No	Two level	--
Sana and Chaudhuri[19]	Yes	Selling price dependent	No	Single	--
Huang and Hsu Yang[20]	No	Constant	No	Two level partial trade credit	--
Ho and Ouyang [21]	No	Constant	No	Two level	--
Jaggi and Khanna [24]	Yes	Inventory level dependent	Complete backlogging	Single	IFS
Jaggi and Kausar [25]	No	Selling price dependent	Complete backlogging	Single partial trade credit	--
Jaggi and Mittal [29]	Yes	Annual	Complete backlogging	Single	IFS
Shah, Patel and Lou[37]	Yes	Inventory level dependent	No	Single	--

Das, Maity and Maiti[12]	No	Inventory level dependent	No	Single	--
Niu and Xie[13]	Yes	Constant	Complete backlogging	Single	IFS
Rong, Mahapatra and Maiti[14]	Yes	Selling price dependent	Partial and complete backlogging	Single	IFS
Dey, Mondal and Maiti[15]	No	Dynamic	No	Single	--
Hsieh, Dey and Ouyang[16]	Yes	Constant	Complete backlogging	Single	IFS
Maiti[17]	No	Inventory level dependent	No	Single	--
Jaggi and Verma [18]	No	Selling price dependent	Complete backlogging	No	--
Lee and Hsu [22]	Yes	Linearly time dependent			
Jaggi, Aggarwal and Verma[23]	Yes	Selling price dependent	Partial backlogging	No	IFS
Bhunia and Shaikh[26]	Yes	Price and time dependent	Partial Backlogging	No	IFS
Bhunia, Pal and Chattopsdhyay[27]	Yes	Inventory level dependent	Partial backlogging	No	IFS
Jaggi, Khanna and Verma[28]	Yes	Linearly time dependent	Partial backlogging	No	IFS
Yang [30]	Yes	Constant	Partial backlogging	No	IFS&SFI
Bhunia, Shaikh, Maiti and Maiti [31]	Yes	Linearly time dependent	Partial Backlogging	No	IFS
Bhunia, Shaikh and Gupta[32]	Yes	Linearly time dependent	Partial backlogging	No	IFS&SFI
Yang and Chang [33]	Yes	Constant	Partial backlogging	Single	SFI
Chung and Huang [34]	Yes	Constant	No	Single	--
Liang and Zhou [35]	Yes	Constant	No	No	--
Bhunia, Jaggi, Sharma and Sharma [36]	Yes	Constant	Partial backlogging	Alternative approach Single	IFS
Present paper	Yes	Selling price dependent demand	Partial Backlogging	Single	IFS

Assumptions

The following assumptions and notations are used to develop the proposed model:

(i) The entire lot is delivered in one batch.

(ii) Inflation effect of the system.

(iii) The demand rate $D(p,t)$ is dependent on time. It is denoted by $D(p,t) = a - bp + ct,\ a,b,c > 0$.

(iv) The deteriorated units were neither repaired nor refunded.

(v) The inventory system involves only one item and one stocking point and the inventory planning horizon is infinite.

(vi) Replenishments are instantaneous and lead time is constant.

(vii) The replenishment cost (ordering cost) is constant and transportation cost does not include for replenishing the item.

Notations:

$q(t)$	Inventory level at time t
S	Highest stock level at the beginning of stock-in period
R	Highest shortage level
‾	Deterioration rate ($0 < {}^{-} << 1$)
C_o	Replenishment cost per order
¢	Backlogging parameter
C_p	Purchasing cost per unit
p	Selling price per unit of item
$D(t)$	Time dependent demand
C_h	Holding cost per unit per unit time
C_b	Shortage cost per unit per unit time
C_{ls}	Opportunity cost due to lost sale
M	Credit period offered by the supplier
I_e	Interest earned by the retailer
I_p	Interest charged by the suppliers to the retailers
T	Time at which the highest shortage level reaches to the lowest point
r	Inflation rate
$Z(.)$	The total average cost

Inventory model with shortages

In this model, it is assumed that after fulfilling the backorder quantity, the on-hand inventory level is S at $t=0$ and it declines continuously up to the time $t = t_1$ when it reaches the zero level. The decline in inventory during the closed time interval $0 \le t \le t_1$ occurs due to the customer's demand and deterioration of the item. After the time $t = t_1$ shortage occurs and it accumulates at the rate $\left[1+\text{¢}\left(T-t\right)\right]^{-1}$, ($\text{¢} > 0$) up to the time $t = T$ when the next lot arrives. At time $t = T$, the maximum shortage level is *R*. This entire cycle then repeats itself after the cycle length *T*.

Let $q\left(t\right)$ be the instantaneous inventory level at any time $t \ge 0$. Then the inventory level $q\left(t\right)$ at any time *t* satisfies the differential equations as follows:

$$\frac{dq(t)}{dt}+{}^{-}q(t)=-D\left(p,t\right), \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dq(t)}{dt}=\frac{-D(p,t)}{[1+\text{¢}(T-t)]}, \; t_1 < t \le T \tag{2}$$

with the boundary conditions

$$q\left(t\right)=S \text{ at } t=0 \text{ , } q(t)=0 \text{ at } t=t_1 \text{ .} \tag{3}$$

$$\text{and } q\left(t\right)=-R \text{ at } t=T \text{.} \tag{4}$$

Also, $q(t)$ is continuous at $t = t_1$.

Using the conditions (3) and (4), the solutions of the differential equations (1)-(2) are given by

$$q(t)=\frac{a}{-}+b\left\{\frac{t}{-}-\frac{1}{-^{2}}\right\}-\left\{\frac{a}{-}-\frac{b}{-^{2}}\right\}e^{-(t_1-t)}, \quad 0 \le t \le t_1$$

$$=\frac{\left\{a\text{¢}+b(1+\text{¢}T)\right\}\log\left|1+\text{¢}(T-t)\right|}{\text{¢}^{2}}+\frac{bt}{\text{¢}}-\left(R+\frac{bT}{\text{¢}}\right), \; t_1 < t \le T$$

Now the total inventory holding cost for the entire cycle is given by

$$C_{hol}=C_h\int_0^{t_1} e^{-rt}q(t)dt$$

Again, the total shortage cost C_{Sho} over the entire cycle is given by

$$C_{sho}=C_b\int_{t_1}^{T}\left\{-e^{-rt}q(t)\right\}dt$$

Cost of lost sale OCLS over the entire cycle is given by

$$OCLS=C_{ls}\int_{t_2}^{T} e^{-rT}\left\{1-\frac{1}{1+\text{¢}(T-t)}\right\}D(t)dt$$

Interest earned and interest charged depends upon the length of the cycle and allowable credits time *M*. The flowing two cases arise:

Case 6.1: $0 < M < t_1$

Case 6.2: $t_1 < M < T$

Now, we shall discuss all the cases in details.

Case 6.1: $M < t_1$

In this scenario, the total interest earned during the period $\left[0, M\right]$ is given by

$$IE1 = pI_e \int_0^{t_1} \int_0^{t} D du dt + pRI_e (T - t_1)$$

Again, the interest paid during the period $\left[M, t_1\right]$ is given by

$$IP1 = C_p I_p \left\{ \int_M^{t_1} q(t) dt \right\}$$

Hence, in this case, the average cost $Z_1\left(t_1, T\right)$ is given by

$$Z_1\left(t_1, T\right) = \frac{X}{T}$$

where *X*= <*setup cost*> + <*inventory holding cost*> + <*deterioration cost*> +<*Interest paid*>-<*Interest Earn*>

$$= C_o e^{-rT} + C_{hol} + IP1 - IE1$$

Case 6.2: $t_1 < M < T$

In this scenario, the total interest earned during the period $\left[0, M\right]$ is given by

$$IE1 = pI_e \int_0^{t_1} \int_0^{t} D du dt + pRI_e (T - t_1)$$

In this case, there is no interest charge

Hence, in this case, the average cost $Z_2\left(t_1, T\right)$ is given by

$$Z_2\left(t_1, T\right) = \frac{X}{T}$$

where *X*= <*setup cost*> + <*inventory holding cost*> + <*deterioration cost*> -<*Interest Earn*>

$$= C_o e^{-rT} + C_{hol} - IE1$$

4: Numerical Example

To illustrate the model with partially backlogged shortages, a numerical example with the following data has been considered.

$C_h = \$1.5$ per unit per unit time, C_b=*$15* per unit per unit time, $C_p = 30$ per unit, $C_o = 300$ per order, ‾ $= 0.05$, a = *45*, b =0. 5, c=15,*M=60/365,p=45,*
¢ $= 1.5, I_e = 0.09, I_p = 0.12, C_{ls} = 15$.

According to the solution procedure, the optimal solution has been obtained with the help of LINGO software for different values of m. The optimum values of t_1, T, S and R along with maximum average profit are displayed in **Table 1**.

Table 1: Optimal solution for different cases

Cases	S	R	T	t_1	Z
Case6.1	45.804	33.66	1.8436	0.1643	2396.241
Case6.2	49.28	25.44	1.7521	0.1523	2274.737

5: Sensitivity Analysis

For the given example mentioned earlier, sensitivity analysis has been performed to study the effect of changes (under or over estimation) of different parameters like demand, deterioration, inventory cost parameters. This analysis has been carried out by changing (increasing and decreasing) the parameters from – 20% to + 20%, taken one or more parameters at a time making the other parameters at their more parameters at a time and making the other parameters at their original values. The results of this analysis are shown in **Figures**.

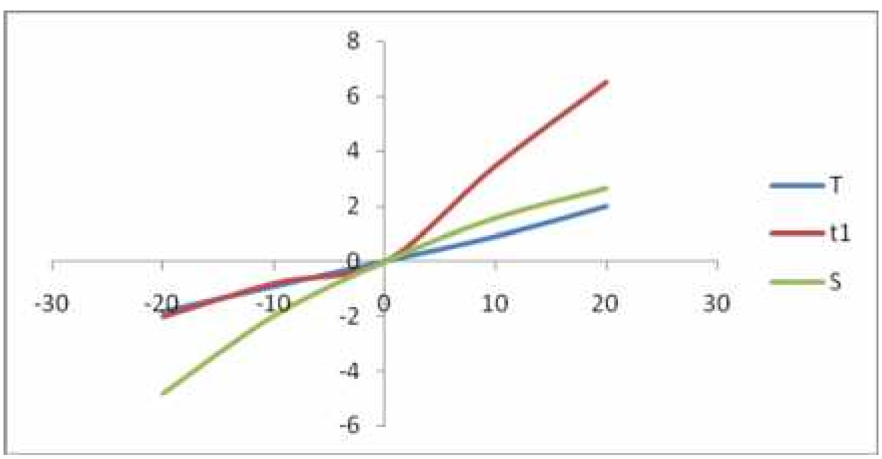

Fig-1: %change of parameter *Co* w.r.t *T*, t_1, *S*

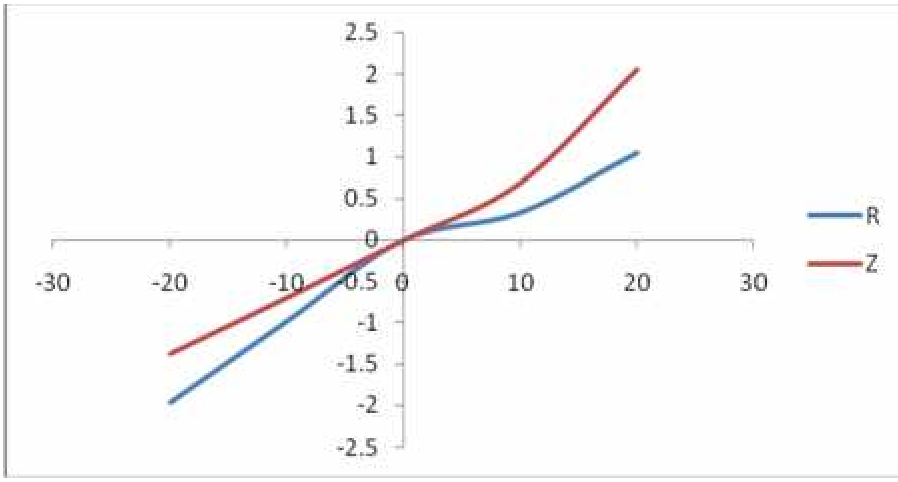

Fig-2: %change of parameter *Co* w.r.t *R,Z*

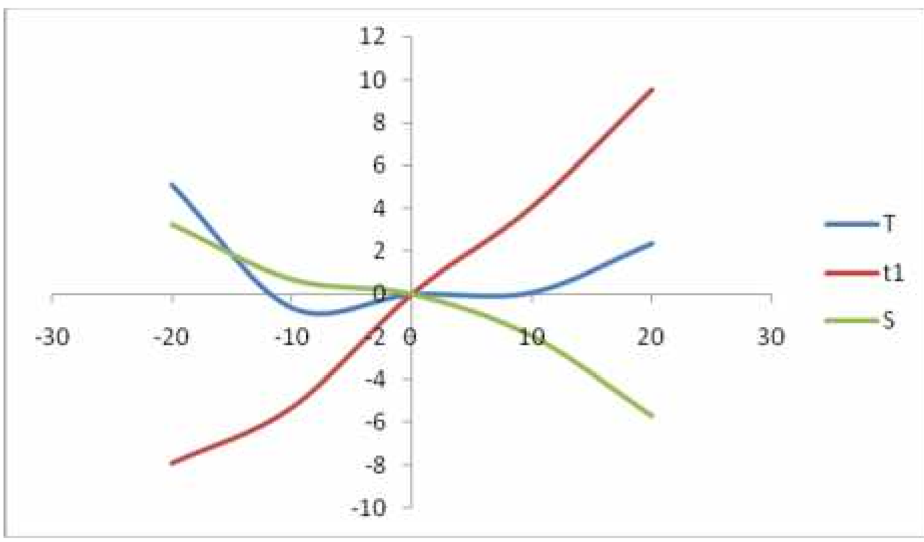

Fig-3: %change of parameter *a* w.r.t T,t_1,S

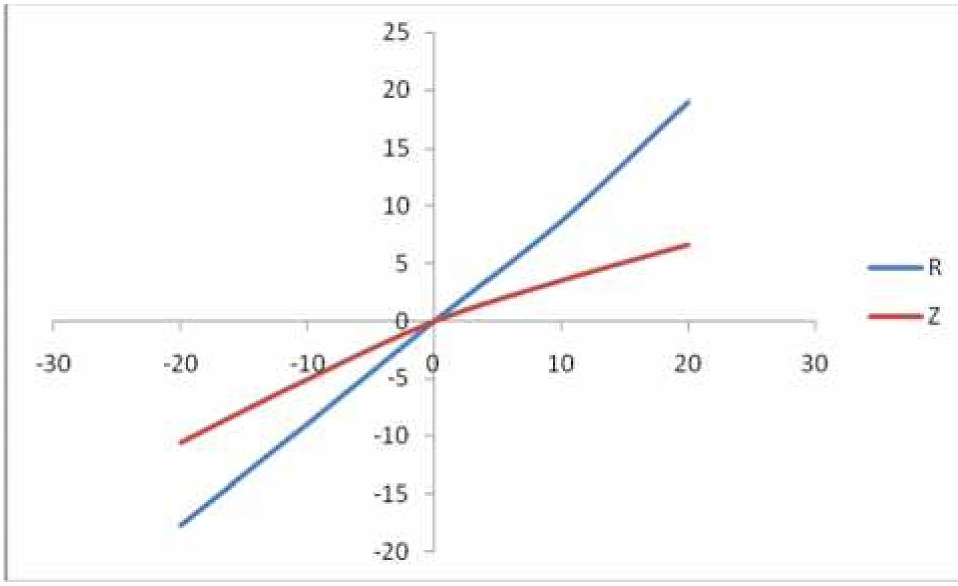

Fig-4: %change of parameter *a* w.r.t *R,Z*

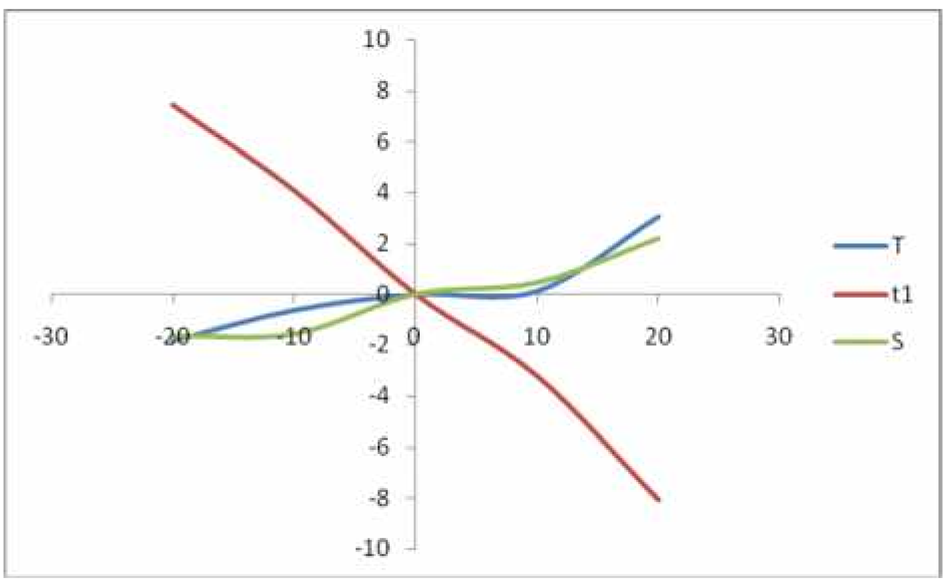

Fig-5: %change of parameter *b* w.r.t *T*,t_1,*S*

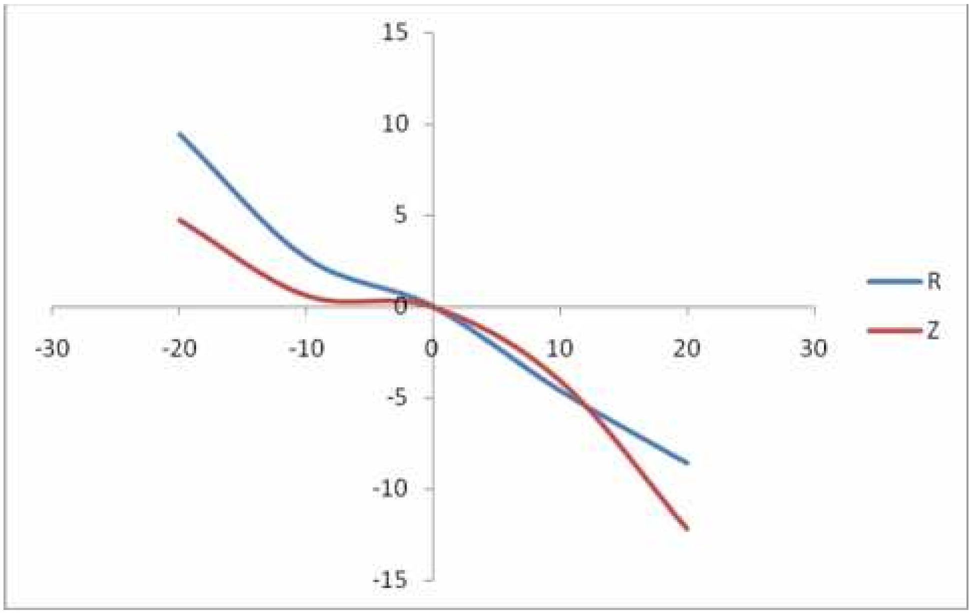

Fig-6: %change of parameter *b* w.r.t *R,Z*

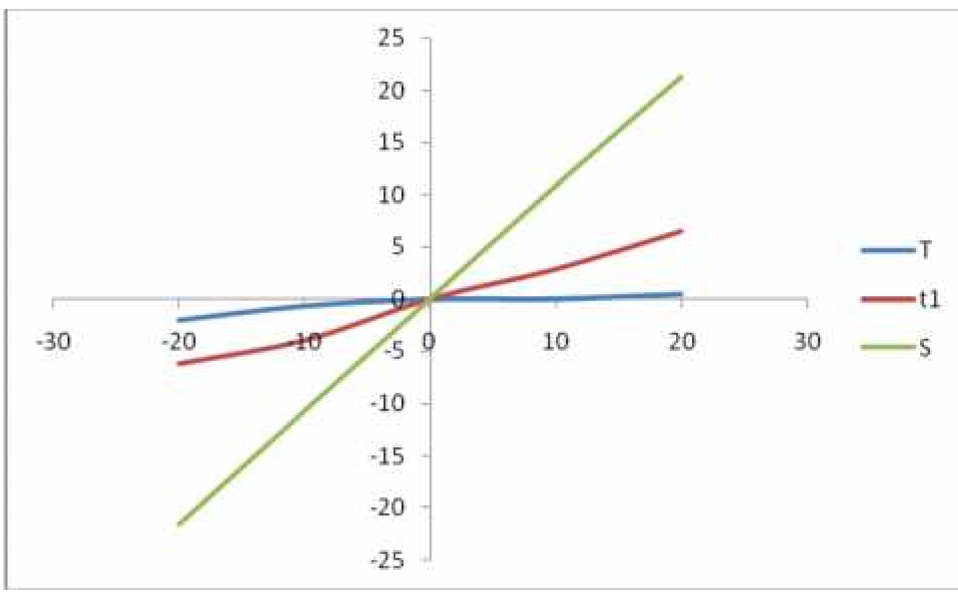

Fig-7: %change of parameter *c* w.r.t T, t_1, S

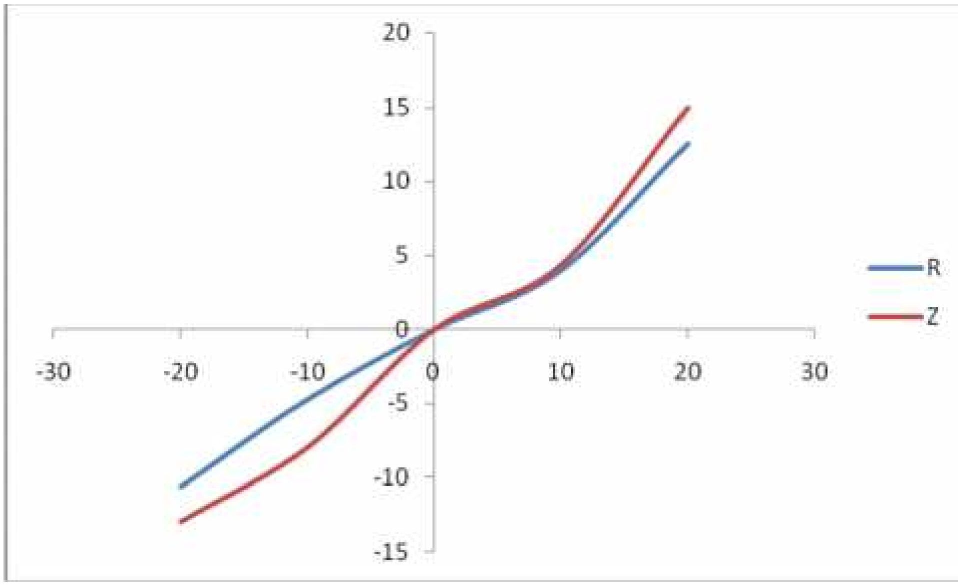

Fig-8: %change of parameter *c* w.r.t *R,Z*

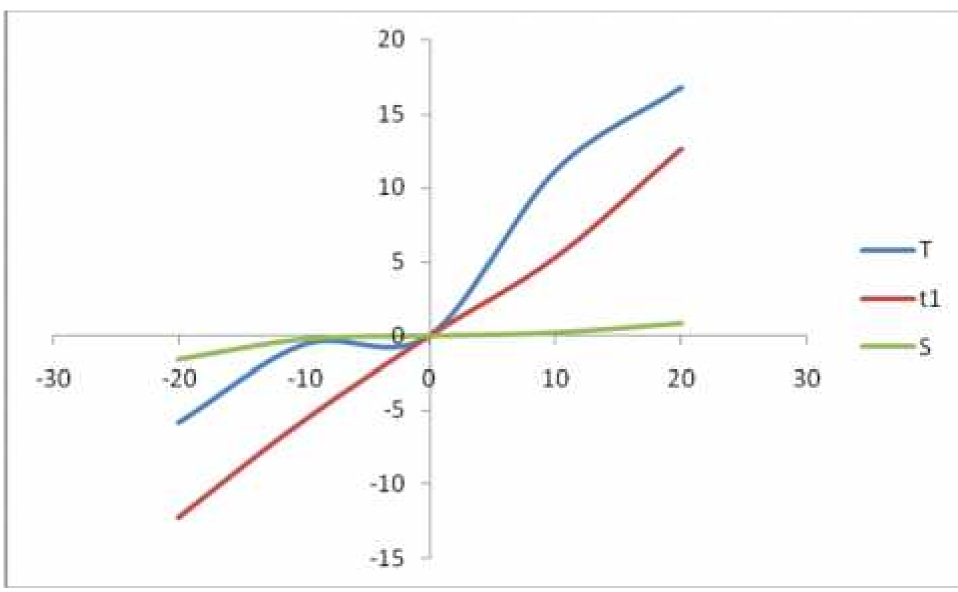

Fig-9: %change of parameter *p* w.r.t T, t_1, S

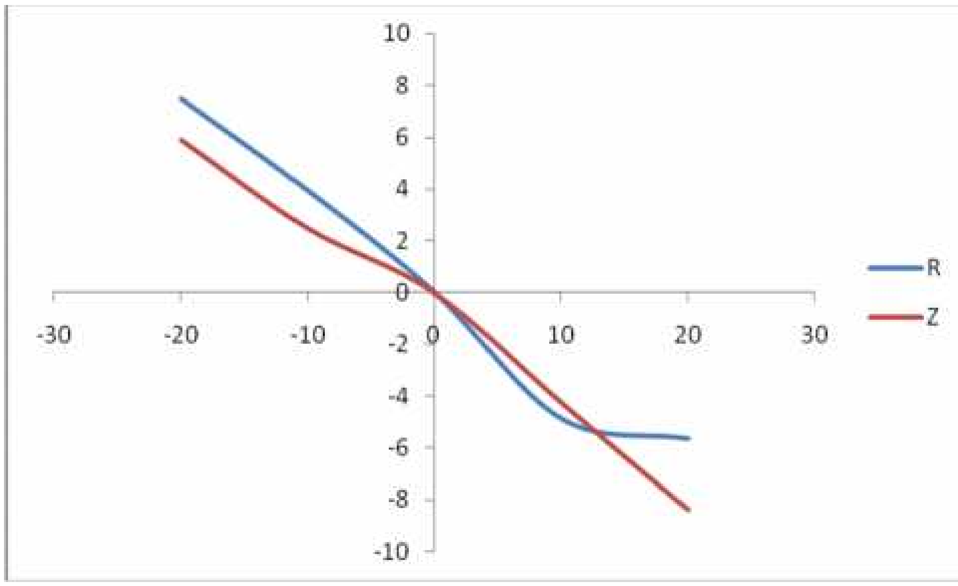

Fig-10: %change of parameter *p* w.r.t *R,Z*

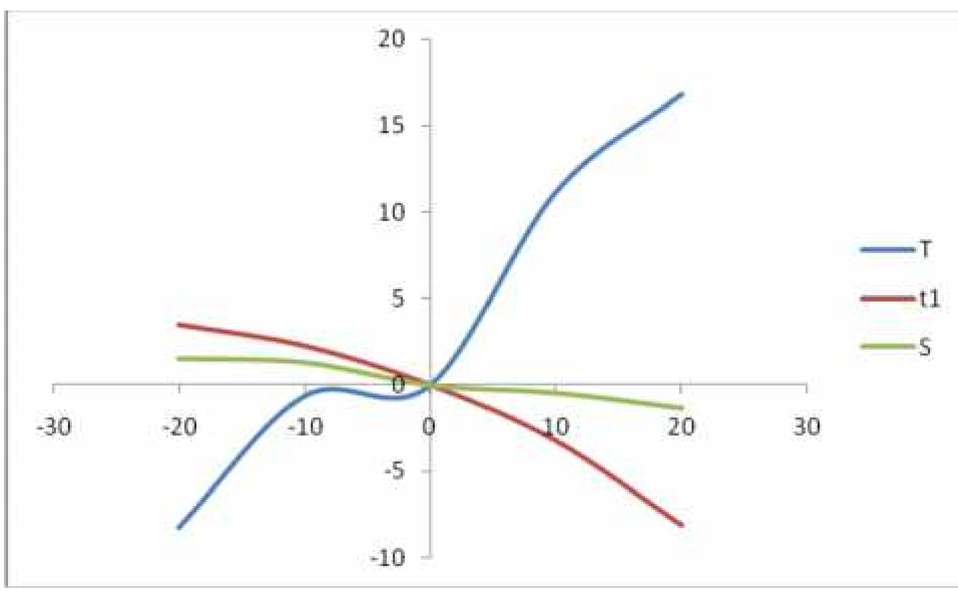

Fig-11: %change of parameter c_p w.r.t T,t_1,S

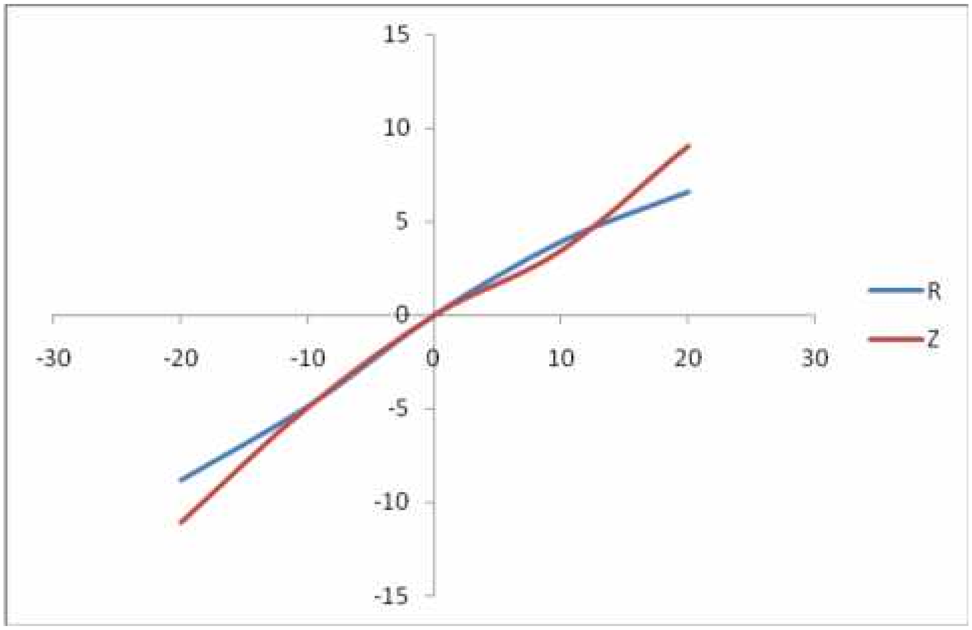

Fig-12: %change of parameter c_p w.r.t *R,Z*

Concluding Remarks

This paper deals with a deterministic inventory model for deteriorating items with variable demand dependent on price and time inflation effect of the system.

The present model is also applicable to the problems where the selling prices of the items as well as the advertisement of items affect the demand. It is applicable for fashionable goods, two level and single level credit policy approach also.

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