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PULSATILE FLOW OF BLOOD IN AN ELASTIC TUBE WITH SLIP AT THE WALL

BY

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Abstract

Pulsatile flow of blood in an elastic circular tube with slip at the permeable walls is investigated in the present analysis solutions for axial and radial velocity has constructed. The volumetric rate of blood flow also measured in the axial direction. The expression for flow characteristic, velocity profile are obtained. Numerical results are shown in tabular form. The effect of slip velocity ,size of the artery ,viscosity on the flow are shown graphically and discussed briefly.

1. INTRODUCTION :

Blood flow is the study of motion of blood through vessels. Any disorder in the blood flow within artery causes problem in cardio – vascular system . Also vascular fluid dynamics plays important role in many arterial diseases. The fluid mechanics study of blood flow in the artery so get great interest in Medical and Bio-engineering study. Cardio Vascular system like other physiological system , has a complicated three dimensional structure and composition .Its behavior is time dependent and can be regulated. Blood flow is controlled by the constriction or dilation of vessel walls, whose action is regulated by the sympathetic nervous system and by local conditions within blood vessels and surrounding tissues. Thus Blood flow in the artery dominated by unsteady flow.

Pulsatile flow in a rigid body makes the fluid to oscillate in bulk. That is with each pulse applied to the flow there is a uniform increase and decrease in the flow for different position along the vessel . In blood flow the diameter of the artery controlled the cardiac output. Artery gradually becomes narrow and branches occur. The walls of the arteries are elastic and stretched and recoil with each pulse of blood . The present work is devoted to study the pulsatile flow of blood with a slip in the permeable wall. The model used here allow one to observe the effect of slip on the motion of blood within artery, though it is assumed that blood is a Newtonian fluid.

2. FORMULATION OF GOVERNING EQUATION

we consider the Pulsatile motion of a Newtonian incompressible fluid in an axi -symmetric circular tube. Also we assume that the arterial vessel is rectilinear, elastic, thick shell of isotropic incompressible material not becomes narrowing and blood flow restricted in an arterial region where no branching occurs and without longitudinal movements. Also we consider the blood as incompressible Newtonian fluid and the flow is axially symmetric. This assumption allow us ([1],[2]) to use Navier-Stokes equations and continuity equation in cylindrical coordinate (r,ɵ.x) :

$$\circ\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)+\frac{\partial p}{\partial x}=\text{«}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \quad (2.1)$$

$$\circ\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)+\frac{\partial p}{\partial x}=\text{«}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right) \quad (2.2)$$

And $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0 \quad (2.3)$$

Where (u,v) are the velocity components. p is the pressure , « is magnetic permeability and ° the constant density. Since the pressure gradient and velocities inside the tube are function of both x and t wave motion exits within the tube . Again we assume that the length of the propagating wave length L is much longer than the radius r , and wave speed is much higher than the average flow velocity then the equations (2.1), (2. 2) becomes

$$\circ\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=\text{«}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \quad (2.4)$$

$$\circ\frac{\partial v}{\partial t}+\frac{\partial p}{\partial x}=\text{«}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right) \qquad (2.5)$$

Taking the solution of the form ([3])

$$p(x,r,t)=P(r)e^{i\mu(t-x/c)} \qquad (2.6)$$

$$u(x,r,t)=U(r)e^{i\mu(t-x/c)} \qquad (2.7)$$

$$v(x,r,t)=V(r)e^{i\mu(t-x/c)} \qquad (2.8)$$

The differential equations (2.4) , (2.5) becomes

$$\frac{d^2U}{dr^2}+\frac{1}{r}\frac{dU}{dr}-\frac{i\mu\circ}{\text{«}}U=\frac{P}{\text{«}} \qquad (2.9)$$

$$\frac{d^2V}{dr^2}+\frac{1}{r}\frac{dV}{dr}+\frac{1}{r^2}\left(k^2r^2-1\right)V=\frac{i\mu P}{\text{«}} \qquad (2.10)$$

Where $k^2=-\frac{i\mu\circ}{\text{«}}$ (2.11)

The solution of the Differential equation (2.9) [which is in the form of Bessel's equation]

$$U=AJ_0\left(i\sqrt{\frac{iw\circ}{\text{«}}}r\right)+BY_0\left(i\sqrt{\frac{iw\circ}{\text{«}}}r\right)-\frac{P}{i\mu\circ} \qquad (2.12)$$

Since U must be finite when r = 0

So B = 0 (2.13)

Again slip condition at the boundary r = a is

$$\text{«}\frac{\partial u}{\partial r}=-\ u \qquad \text{on } r = a \qquad (2.14)$$

The boundary condition (2.14) is the well known Beavers and Joseph (1967) slip condition [5],Where « is the dynamic viscosity coefficient , is the coefficient of sliding friction.

Using (2.13) , from (2.12) we get

$$A=\frac{-P}{\mu\circ}\left[\frac{\mu\circ a}{2}\left(1+\frac{a}{2\text{«}}\right)+i(1-\frac{\mu^2\circ^2a^2}{4\ \ ^2})\right] \qquad (2.15)$$

Therefore $U=\frac{-P}{\mu\circ}\left[\frac{\mu\circ a}{2}\left(1+\frac{a}{2\ll}\right)+i\left(1-\frac{\mu a^{2}\circ^{2}}{4\;{}^{2}}\right)\right]J_0\left(i\sqrt{\frac{iw\circ}{\ll}}r\right)-\frac{P}{i\mu\circ}$ (2.16)

From (7)

$$u=\left(\frac{-P}{\mu\circ}\left[i+\frac{\mu\circ}{2}\left(1+\frac{a}{2\ll}\right)-i\frac{\mu\circ^{2}a^{2}}{4\;{}^{2}}\right]J_0\left(i\sqrt{\frac{iw\circ}{\ll}}r\right)-\frac{P}{i\mu\circ}\right)e^{i\mu\left(t-\frac{x}{c}\right)}$$

Solution of (2.10) is $V=C_1J_1(kr)+C_2Y_1(kr)-\frac{i\mu P}{\ll k^2}$ (2.17)

Since V is finite at r = 0

Therefore $C_2=0$ (2.18)

When r = a , $V=V^{/}$

Then $C_1=\frac{1+V^{/}}{J_1\left(i\sqrt{\frac{iw\circ}{\ll}}a\right)}$ (2.19)

Therefore $V=\frac{1+V^{/}}{J_1\left(i\sqrt{\frac{iw\circ}{\ll}}a\right)}J_1\left(i\sqrt{\frac{iw\circ}{\ll}}r\right)-1$ (2.20)

The volumetric flow rate Q is given by

$$Q=\int_0^a u2®rdr$$

$$=2®e^{i\mu(t-x/c)}\int_0^a\left[ArJ_0\left(i\sqrt{\frac{i\mu\circ}{\ll}}r\right)-\frac{\Pr}{i\mu\circ}\right]dr \quad (2.21)$$

Where A is given by (2.15)

Now $\int_0^a\left[ArJ_0\left(i\sqrt{\frac{i\mu\circ}{\ll}}r\right)\right]dr$

$$= \int_0^a \left[ArJ_0\left(i^{3/2} \frac{r\sim}{a} \right) \right] dr \qquad \text{where} \qquad \sim\, = a\sqrt{\frac{\mu^{\circ}}{\text{«}}}$$

$$= A\int_0^a \left[rJ_0\left(\frac{r\ \ '}{a} \right) \right] dr \qquad \text{where} \qquad i^{3/2}\sim\, = \ \ '$$

$$= A\frac{a^2}{(\ \ ')^2}\int_0^{-} {}^{-}J_0(^{-})d^{-} \qquad \text{where} \qquad \frac{r\ \ '}{a} = {}^{-}$$

$$= A\frac{a^2}{(\ \ ')^2}\ \ 'J_1(\ \ ')$$

Therefore

$$Q = 2\text{®}\left(A\frac{a^2}{,}J_1(\ \ ') - \frac{p}{i\mu^{\circ}}\frac{a^2}{2} \right) e^{i\mu\left(t-\frac{x}{c}\right)}$$

$$= \left[\frac{-P}{\mu^{\circ}} \left\{ \frac{\mu^{\circ}a}{2}\left(1+\frac{a}{2\text{«}} \right) + i\left(1-\frac{\mu^2 a^2 \circ^2}{4\ ^2} \right) \right\} \frac{a^2}{,}J_1(\ \ ') - \frac{P}{i\mu^{\circ}}\frac{a^2}{2} \right] 2\text{®}e^{i\mu\left(t-\frac{x}{c}\right)}$$

$$= \frac{\text{®}Pa^2}{\mu^{\circ}}\left[\text{¡}_1 + i\text{¡}_2 \right] e^{i\mu\left(t-\frac{x}{c}\right)}, \qquad \text{where } \text{¡}_1 = \frac{\mu^2\circ^2}{\text{«}}\left[\frac{a}{2}\left(1+\frac{a}{2\text{«}} \right) - \frac{1}{8\text{«}}\left(1-\frac{\mu^2\circ^2 a^2}{4\ ^2} \right) \right.$$

$$\text{and} \qquad \text{¡}_2 = \frac{\mu^{\circ}}{\text{«}}\left(1-\frac{\mu^2\circ^2 a^2}{4\ ^2} \right) + \frac{\mu^3\circ^3 a}{16\text{«}^2}\left(1+\frac{a}{2\text{«}} \right) - 1$$

$$= \frac{-\text{®}Pa^2}{\mu^{\circ}}\left[G_1 + iG_2 \right]$$

Where

$$G_1 = \text{¡}_1\cos\mu\left(t-\frac{x}{c}\right) - \text{¡}_2\sin\mu\left(t-\frac{x}{c}\right) \tag{2.22}$$

$$G_2 = \text{¡}_1\sin\mu\left(t-\frac{x}{c}\right) + \text{¡}_2\cos\mu\left(t-\frac{x}{c}\right) \tag{2.23}$$

Therefore

$$Q = \frac{P\text{®}a^2}{\mu^{\circ}}\sqrt{G_1^2 + G_2^2} \quad = \frac{P\text{®}a^2}{\mu^{\circ}}\sqrt{\text{¡}_1^2 + \text{¡}_2^2} \tag{2.24}$$

3. RESULT AND DISCSSTION

The pressure gradient and blood viscosity being taken as 100 g $cm^{-1}s^{-2}$ and 0.04g. $cm^{-1}s^{-1}$ respectively
Considering the heart beats about 72 times a minute

$$\frac{2®}{\mu} = \frac{60}{72}$$

or, $w = 8 \text{ rad / sec}$

For blood we take density $(°)$ = 1.05 gm / sec. , the coefficient of viscosity $(«)$ = 0,04 gm / cm sec , pressure gradient (P) = 100 gm/ cm sec^2.

Table I

	u	v	Q
1	4341.645	432.4216	1630.978
1.5	4699.957	414.4141	1763.604
2	4651.999	402.3283	1745.453
2.5	4447.353	394.081	1668.976
3	4147.605	388.1746	1557.189
3.5	3773.857	383.754	1417.978
4	3334.924	380.3284	1254.707
4.5	2835.017	377.5988	1069.115
5	2276.354	375.3793	862.4141

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Table II

	a=.01	a=.02	a=.03	a =.04	a =.05	a=.06	a =.07	a =.08	a =.09	a =1.0
1	20.55	81.57	181.44	317.5296	486.29	683.21	902.87	1138.95	1384.29	1630.98
1.5	20.55	81.68	182.21	320.32	493.601	699.08	933.19	1191.816	1470.35	1763.71
2	20.53	81.55	181.75	319.22	491.44	695.31	927.13	1182.69	1457.2	1745.453
2.5	20.51	81.32	180.82	316.63	485.62	683.96	907.06	1149.65	1405.8	1668.98
3	20.48	81.04	179.6	313.13	477.594	668.02	878.48	1102.11	1331.189	1557.189
3.5	20.45	80.71	178.16	308.92	467.84	648.5	843.27	1043.29	1238.54	1417.978
4	20.42	80.34	176.52	304.09	456.5616	625.84	802.24	974.53	1130.02	1254.707
4.5	20.39	79.94	174.69	298.68	443.86	600.23	755.76	896.5	1006.71	1069.115
5	20.35	79.5	172.69	292.7	429.8	571.78	704.02	809.55	869.24	862.4141

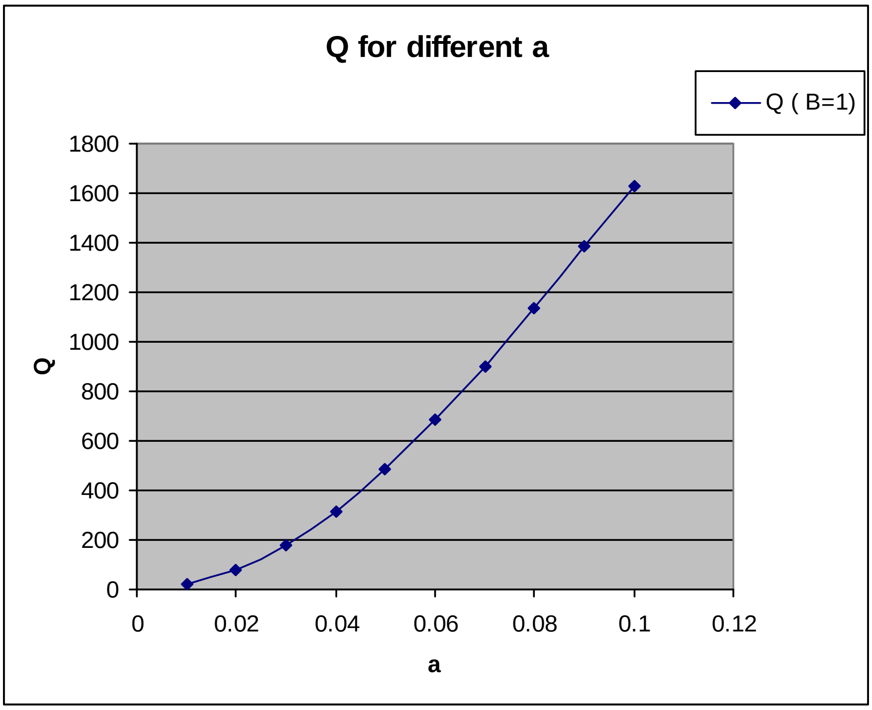
Q for different a
Q (B=1)
Q
a
1800
1600
1400
1200
1000
800
600
400
200
0
0
0.02
0.04
0.06
0.08
0.1
0.12

Fig. 1

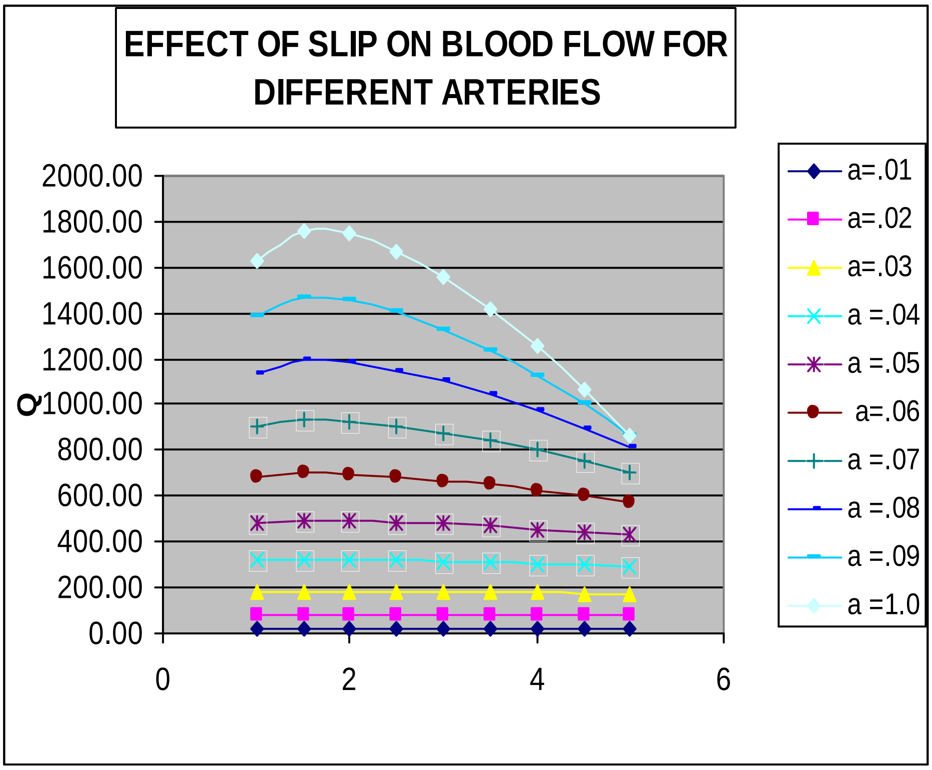
EFFECT OF SLIP ON BLOOD FLOW FOR DIFFERENT ARTERIES
Q
2000.00
1800.00
1600.00
1400.00
1200.00
1000.00
800.00
600.00
400.00
200.00
0.00
0
2
4
6
a=.01
a=.02
a=.03
a =.04
a =.05
a=.06
a =.07
a =.08
a =.09
a =1.0

Fig. 2

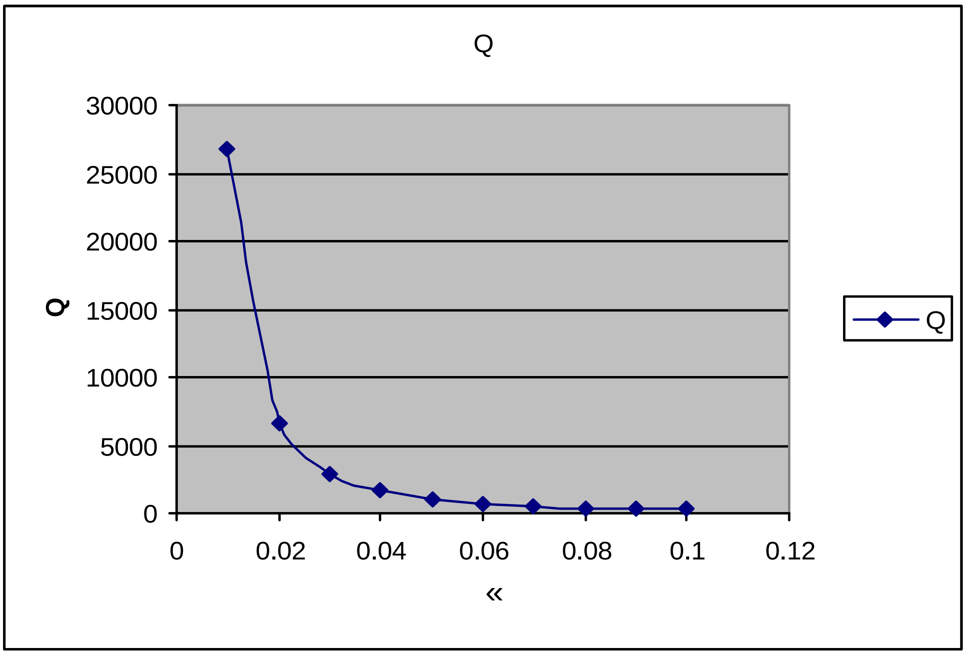
Q
30000
25000
20000
15000
10000
5000
0
Q
0
0.02
0.04
0.06
0.08
0.1
0.12
«
Q

Fig. 3

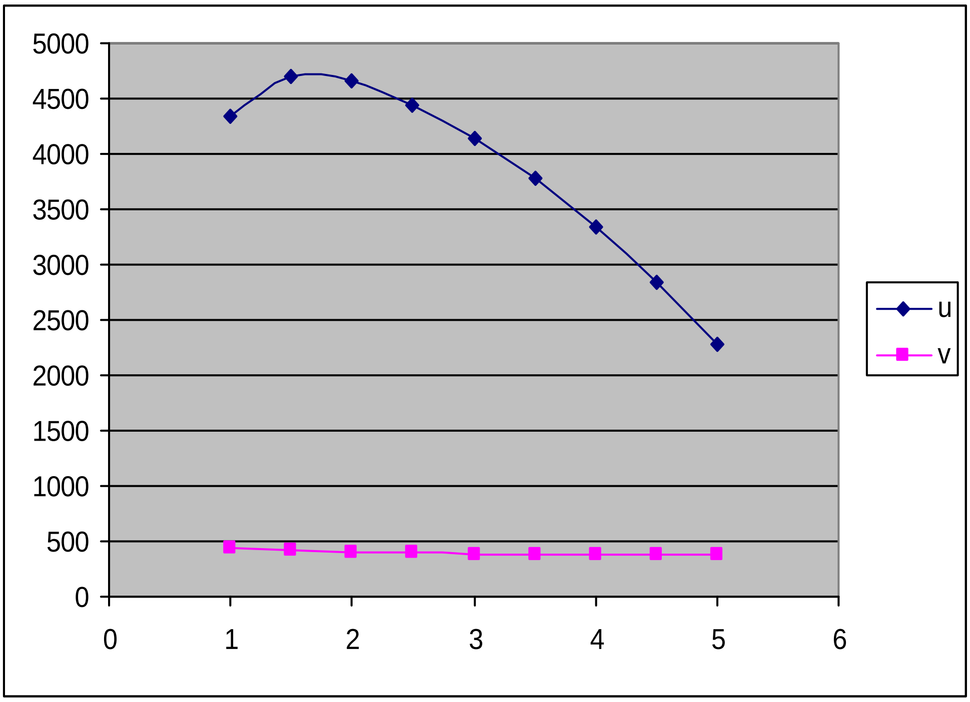
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
1
2
3
4
5
6
u
v

Fig . 4

To investigate the problems of combined effect of wall slip and magnetic field on blood flow in an artery we consider the pulsatile motion of a Newtonian incompressible fluid in an axisymmetric circular tube with slip at the permeable boundaries . The results revealed that

1) The volumetric flow rate (Q) of blood increases steadily as the size of the artery increases
   when slip(coefficient of sliding friction) , coefficient of dynamic viscosity « , remain constant
2) Fig.2 shows that Q remain constant as sliding friction increases when radius of the artery a lies within 01 < a < .04 . When .04 < a < .0.6 ,Q remain almost constant and when a > 0.7 , Q increases when lies between 1 to 1.5 and then decreases . If a > 0.6 then the curve is always convex upwards
3) Fig. 3 shows that as « (dynamic viscosity coefficient) increases (, a remaining the same) the flux decreases rapidly and approaches zero as m approaches 1.
4) Fig. 4 shows that the horizontal component of velocity along axial direction increases steadily when 1< < 1.5 and then decreases while the sliding friction has no effect on radial direction of the flow.

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