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MAGNETO-HYDRODYNAMIC FORCED CONVECTIVE BOUNDARY LAYER FLOW PAST A STRETCHING / SHRINKING SHEET

By

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Abstract

MHD boundary layer forced convection flow along a shrinking surface with variable heat and mass flux in the presence of heat source is studied. The flow is producedowing to linear shrinking of the sheet and is influenced by uniform transverse magnetic field. The boundary layer partial differential equations of momentum, heat and mass transfer equations are converted into nonlinear ordinary differential equations by similarity transformation. Numerical solution of the resulting boundary value problem is obtained using Nachtsheim-Swigert Shooting iteration scheme along with the sixth order Runge-Kutta method. The effects of different parameter on velocity, temperature and concentration are shown graphically. Skin friction coefficient, Nusselt number and Sherwood number are also for different values of the parameter are also involved in the study.

Nomenclature

B	heat source parameter	T_∞	temperature of the free stream fluid
B_0	magnetic field strength	u	velocity in the x direction
C_f	skin friction coefficient	v	velocity in the y direction
C_p	specific heat at constant pressure	x	dimensional distance along the sheet
D	positive constant	y	dimensional distance normal to the sheet
F	dimensionless velocity	—	dimensionless temperature
k	thermal conductivity of the fluid	ξ	similarity variable
M^2	magnetic parameter	η	Dimensionless coordinate
n	heat flux parameter	2_w	Shear stress at the wall
Pr	Prandtl number	E	stretching/shrinking parameter
q_w	heat flux at the surface	ρ	density of the fluid

Q	internal heat generation	μ	dynamic viscosity of the fluid
S	suction parameter	¬	kinematic viscosity of the fluid
T	temperature of the fluid	σ	electrical conductivity of the fluid
T_w	temperature at the wall	·	Stream function

1. Introduction

In engineering processes boundary layer behavior over a continuous moving solid surface is an important type of flow. The heat and mass transfer due to a continuously stretching surface through an ambient fluid is the latest interest of current research. Chen and Char (1988) investigated the effects of variable surface temperature and variable surface heat and mass flux over the heat transfer characteristics of a continuous linear stretching surface [1]. Ali (1995) examined thermal boundary layer on a power law stretched surface with suction or injection was investigated by [2]. Elbashbeshy (1998) studied the heat transfer over a stretching surface with variable surface heat flux[3]. Liao (2005)acquired a new branch of solution of boundary layer flow over a permeable stretching plate[4].Bhargava et al. (2007) discussed the micro-polar transport phenomena over a stretching sheet [5]. MHD flow of a micro-polar fluid past a stretched permeable surface with heat generation or absorption was studied by Khedr et al. (2009)[6]. Anjali Devi et al. (2014) studied the numerical simulation of magneto-hydrodynamic forced convective boundary layer flow past a stretching/shrinking sheet prescribed with variable heat flux in the presence of heat source and constant suction [7]. We steady laminar, two dimensional boundary layer MHD flow of a viscous, incompressible electrically conducting fluid with heat and mass transfer over a stretching/shrinking sheet prescribed with variable heat. Nonlinear partial differential equations for momentum and energy are transformed into nonlinear ordinary differential equations by introducing suitable similarity transformations. The obtained equations are solved numerically utilizing Nachtsheim-Swigert shooting iteration scheme for the satisfaction of asymptotic boundary conditions together with the sixth order Runge-Kutta method.

Mathematical Analysis

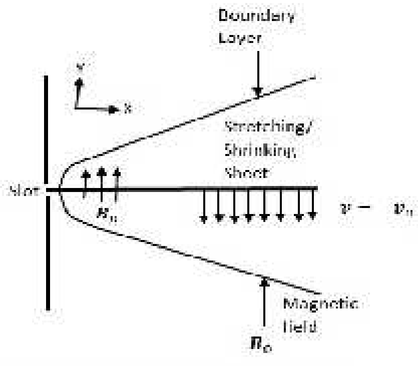

Fig. Schematic diagram of the problem

We consider a steady, two dimensional, laminar nonlinear hydro-magnetic boundary layer flow of a viscous, incompressible, electrically conducting fluid, caused by a stretching/shrinking sheet subjected to suction in the presence of uniform transverse magnetic field. The velocity components u and v are taken in x and y directions respectively. A magnetic field of strength B_0 is applied normal to the boundary of the flow. We consider the magnetic Reynolds number to be small so that the induced magnetic field is negligible. The effect of viscous and joules dissipation are assumed to be negligible in the energy equation. We consider the case when the variable heat flux is prescribed on the stretching/shrinking surface.

The governing equations of the problem under these assumptions,

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

(1) $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=¬\frac{\partial^2 u}{\partial y^2}-\frac{\pm {B_0}^2}{\circ}u$$

(2) $$\circ C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty)$$

(3) $$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}-R_C(c-c_\infty)$$ (4)

Boundary conditions pertaining to velocity are:

$$u=bx,\ v=-v_0,\ K\frac{\partial T}{\partial y}=q_w=Dx^n, C_w=0 \quad \text{at} \quad y=0 \tag{5}$$

$$u=0, T\to T_\infty, C\to C_\infty \quad \text{at} \quad y\to\infty \tag{6}$$

where, $b<0$ is the shrinking constant and $b>0$ is the stretching constant.

Flow Analysis

A stream function defined by: $u=\frac{\partial \cdot}{\partial y},\ v=-\frac{\partial \cdot}{\partial x}$ (7)

is such that the continuity equation is identically satisfied. A dimensionless stream function $F(¦)$ is given by: $· \left(x, y\right) = \sqrt{a³}\, xF\left(¦\right), ¦ = \sqrt{\frac{a}{³}},\ T - T_{\infty} = \frac{Dx^{n}}{k}\sqrt{\frac{³}{a}}¯\left(¦\right),\ ¤\left(¦\right) = \frac{c - c_{\infty}}{c_{w} - c_{\infty}}$ (8)

The velocity components become: $u = axF'\left(¦\right), v = -\sqrt{a³}\, F\left(¦\right)$ (9)

Equations (2-4) becomes

$$F''' + FF' - (F')^{2} - M^{2}F' = 0 \tag{10}$$

$$¯'' + \Pr F¯' - n \Pr F'¯ + \Pr B¯ = 0 \tag{11}$$

$$¤'' + Sc\, f¤' + Sc\, R¤ = 0 \tag{12}$$

with the boundary conditions

$$¦ = 0, F(0) = S, F`(0) = £, ¯`(0) = -1, ¤(0) = 1 \tag{13}$$

$$¦ \to \infty, F`(\infty) = 0, ¯(\infty) = 0, ¤(\infty) = 0 \tag{14}$$

where, the Prandtl number($\Pr$), heat source parameter(B), Schmidt parameter(Sc), Magnetic parameter(M^{2}), Reaction Parameter(R) and suction parameter(S) can be defined as follows:

$$\Pr = \frac{«C_{p}}{k}, B = \frac{Q}{a°C_{p}}, Sc = \frac{v}{D}, M^{2} = \frac{±B_{0}^{2}}{°a}, R = -\frac{R_{c}}{a}, S = \frac{³_{0}}{\sqrt{a³}}\left(³_{0} > 0\right) \ (15)\ £ = b/a$$

is the stretching/shrinking parameter and ε > 0 denotes the stretching sheet and ε < 0 denotes shrinking sheet.

Results and Discussion

Numerical values of the solution are obtained by fixing various values for the physical parameters involved in the problem namely suction parameter, magnetic parameter, Prandtl number, heat source parameter, stretching/shrinking parameter and heat flux parameter. The effect of parameters on the velocity, temperature, Skin friction coefficient, Nusselt number and Sherwood numberare presented. Velocity of fluid is at steady state for heat source parameter B ,heat flux parameter h in figs. 1-3,8-10,22-24,velocity of fluid increases with the increase of magnetic parameter M in figs.15-17 and the velocity of fluid decreases for the prandtl number $\Pr$ in figs.29-31.Temperature of fluid is at steady state for heat flux parameter h in figs.11-12,Temperature of fluid increases with the increase of heat source parameter B stretching/shrinking parameter W in figs.4-5,46-47and temperature of fluid decreases with the increase of magnetic parameter M ,prandtl number $\Pr$ suction parameter S in figs.18-19,25-26,39-40.Concentration of fluid is at steady state for heat source parameter B ,heat flux parameter h magnetic parameter M ,prandtl number $\Pr$,suction parameter S ,stretching shrinking parameter W in figs.7-8,13-14,20-21,27-28,34-35,48.

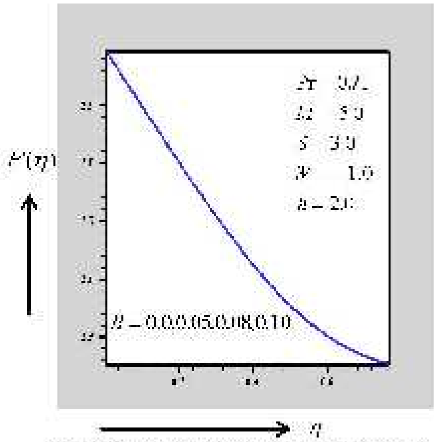

Fig.1.Effect of heat source parameter over dimensionless traverse velocity $F(\eta)$.

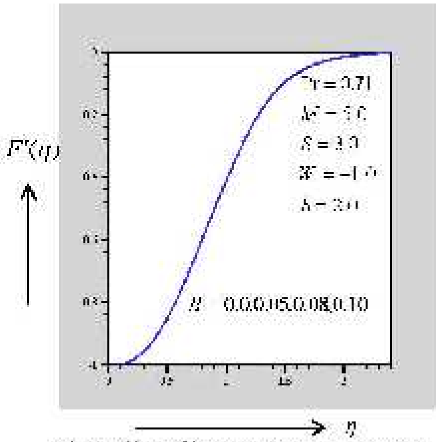

Fig.2.Effect of heat source parameter over dimensionless longitudinal velocity $F'(\eta)$.

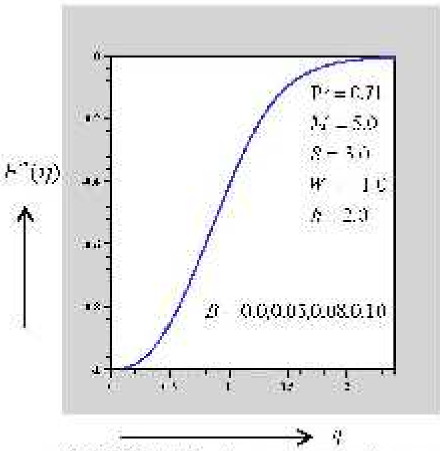

Fig.3.Effect of heat source parameter over velocity $F''(\eta)$.

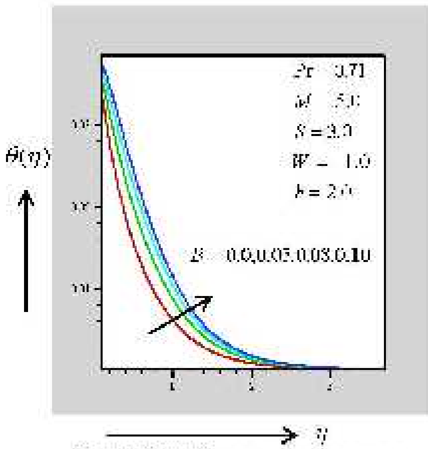

Fig.4.Effect of heat source parameter over dimensionless temperature $\theta(\eta)$.

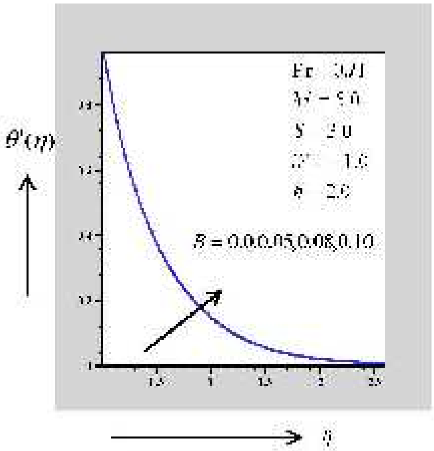

Fig.5.Effect of heat source parameter over dimensionless temperature $\theta'(\eta)$.

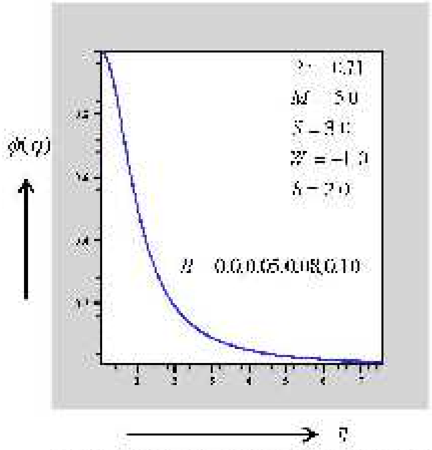

Fig.6.Effect of heat source parameter over concentration $\phi(\eta)$.

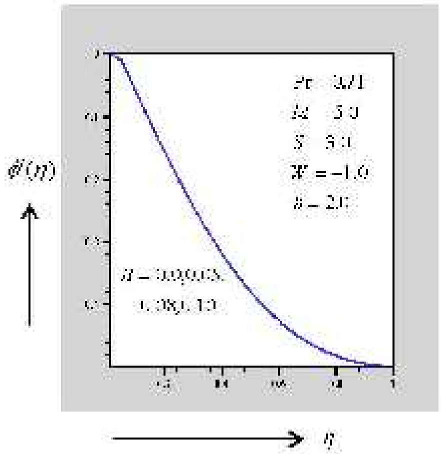

Fig.7.Effect of heat source parameter over concentration $\phi'(\eta)$.

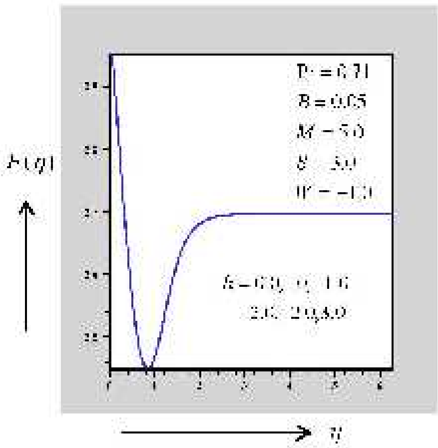

Fig.8. Effect of heat flux parameter over dimensionless traverse velocity $F(\eta)$.

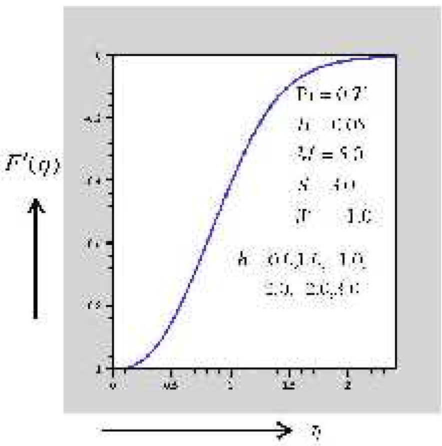

Fig.9. Effect of heat flux parameter over dimensionless longitudinal velocity $F'(\eta)$.

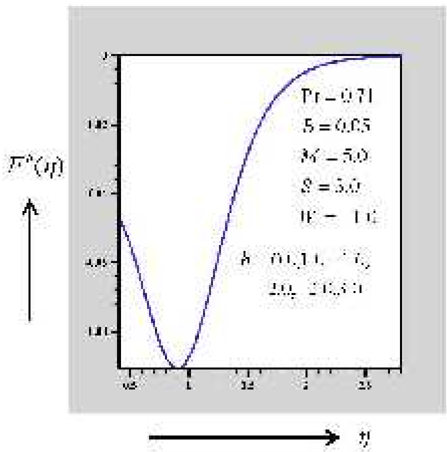

Fig.10. Effect of heat flux parameter over velocity $F''(\eta)$.

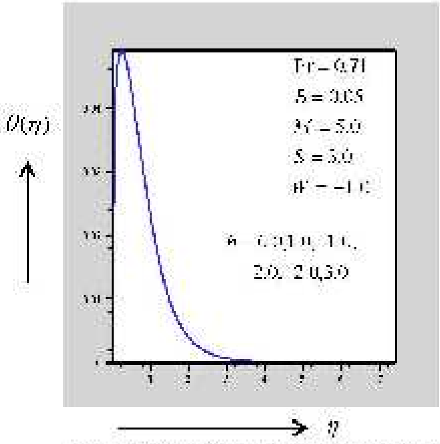

Fig.11.Effect of heat flux parameter over dimensionless temperature $\theta(\eta)$.

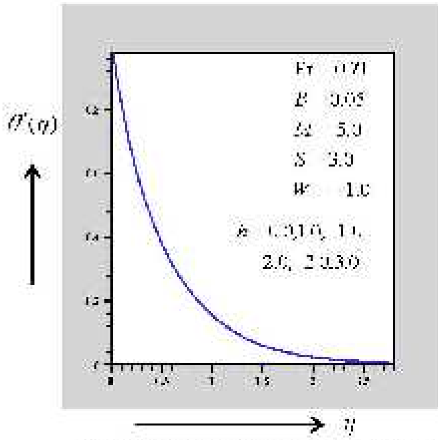

Fig.12.Effect of heat flux parameter over dimensionless temperature $\theta'(\eta)$.

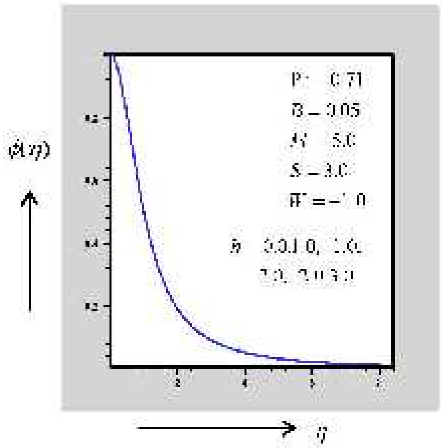

Fig.13.Effect of heat flux parameter over concentration $\phi(\eta)$.

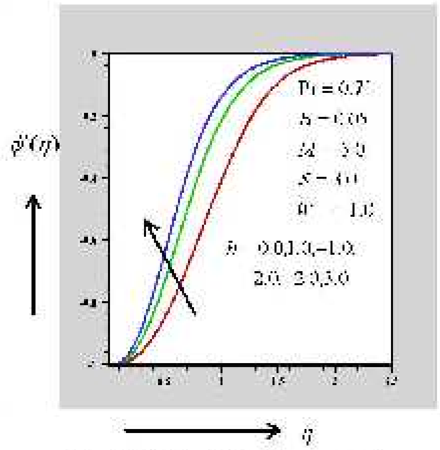

Fig.14.Effect of heat flux parameter over concentration $\phi'(\eta)$.

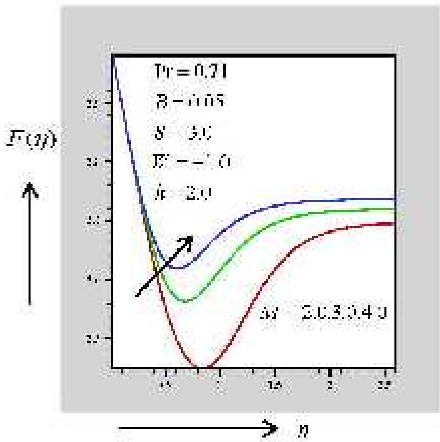

Fig.15. Effect of magnetic parameter over dimensionless traverse velocity $F(\eta)$.

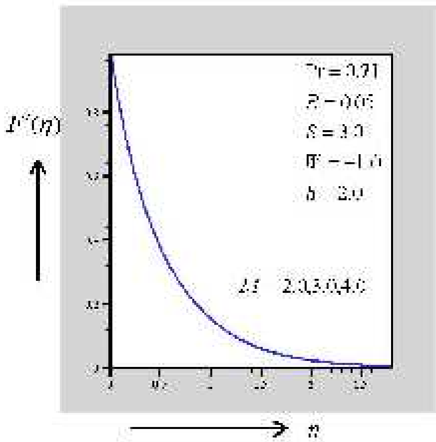

Fig.16. Effect of magnetic parameter over dimensionless longitudinal velocity $F'(\eta)$.

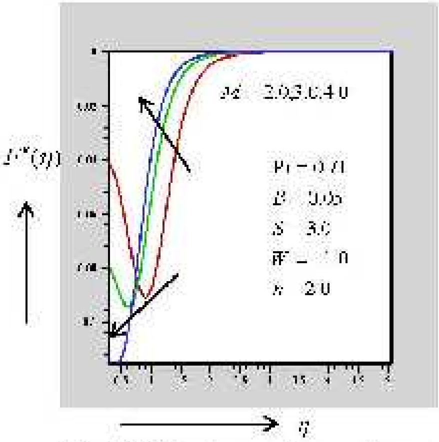

Fig.17. Effect of magnetic parameter over dimensionless temperature $F''(\eta)$.

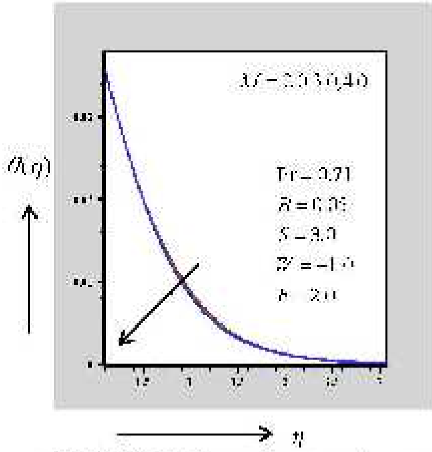

Fig.18. Effect of magnetic parameter over dimensionless temperature $\theta'(\eta)$.

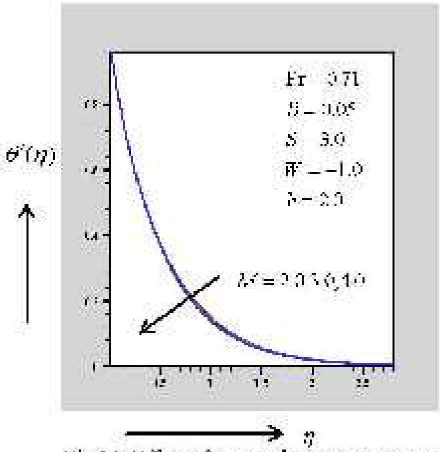

Fig.19. Effect of magnetic parameter over dimensionless temperature $\theta'(\eta)$.

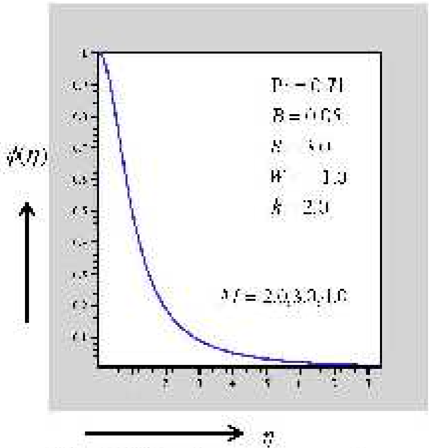

Fig.20. Effect of magnetic parameter over concentration $\phi(\eta)$.

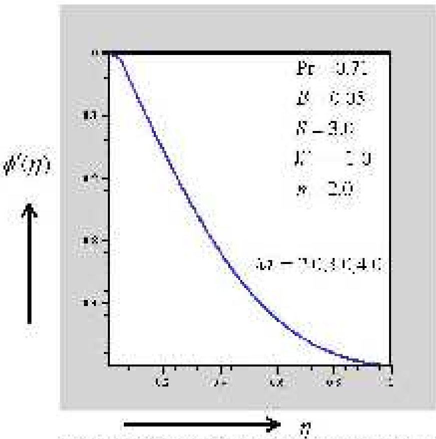

Fig.21. Effect of magnetic parameter over concentration $\phi'(\eta)$.

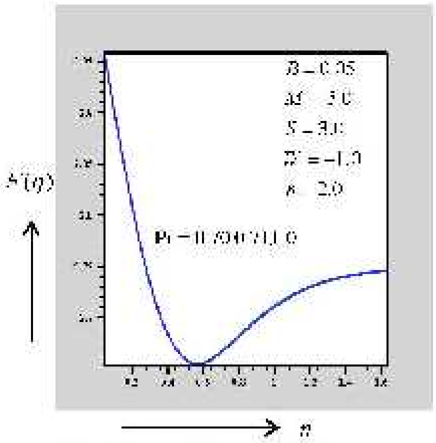

Fig.22. Effect of prandtl number over dimensionless traverse velocity $F(\eta)$.

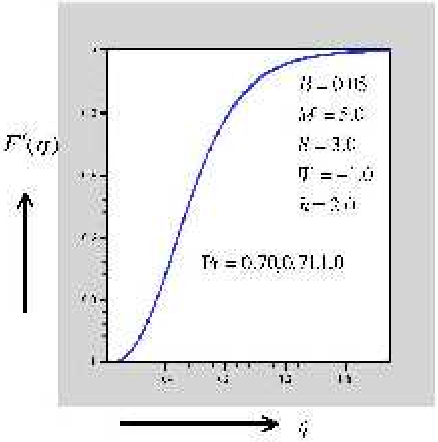

Fig.23. Effect of prandtl number over dimensionless longitudinal velocity $F'(\eta)$.

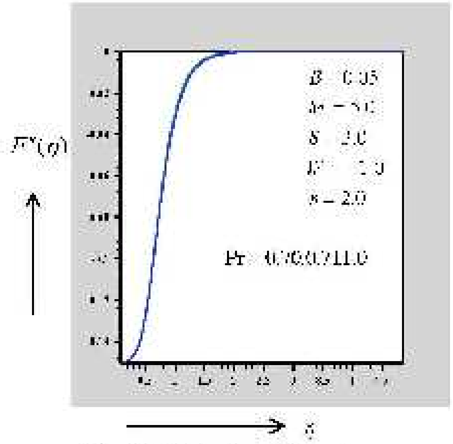

Fig.24. Effect of prandtl number over dimensionless velocity $F''(\eta)$.

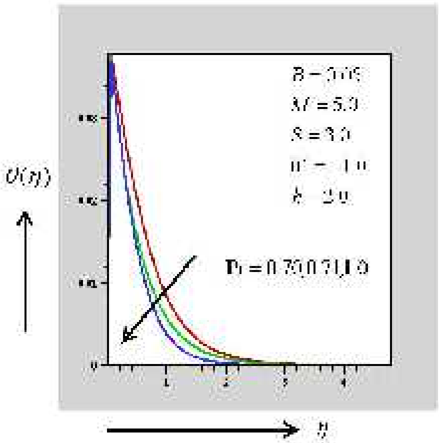

Fig.25. Effect of prandtl number over dimensionless temperature $\theta(\eta)$.

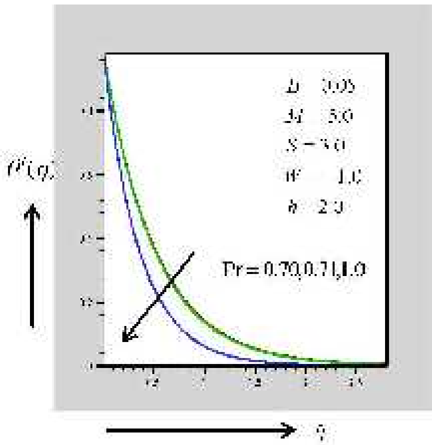

Fig.26. Effect of prandtl number over dimensionless temperature $\theta'(\eta)$.

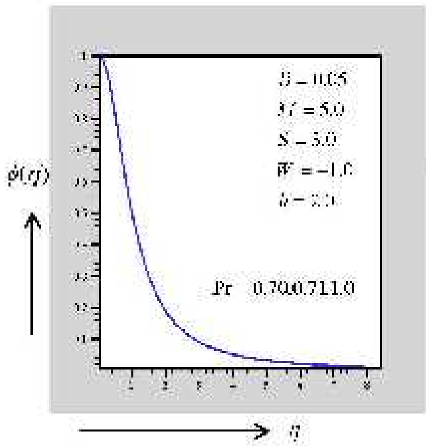

Fig.27. Effect of prandtl number over concentration $\phi(\eta)$.

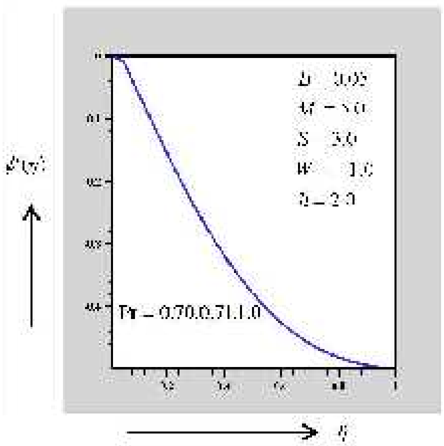

Fig.28. Effect of prandtl number over concentration $\phi'(\eta)$.

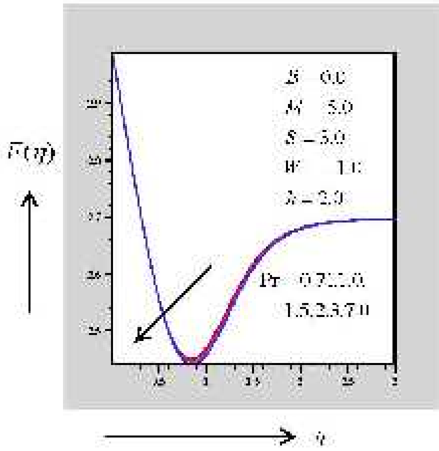

Fig.29. Effect of prandtl number over dimensionless traverse velocity $F(\eta)$.

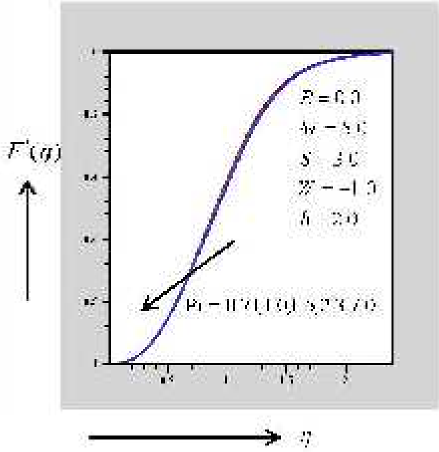

Fig.30. Effect of prandtl number over dimensionless longitudinal velocity $F'(\eta)$.

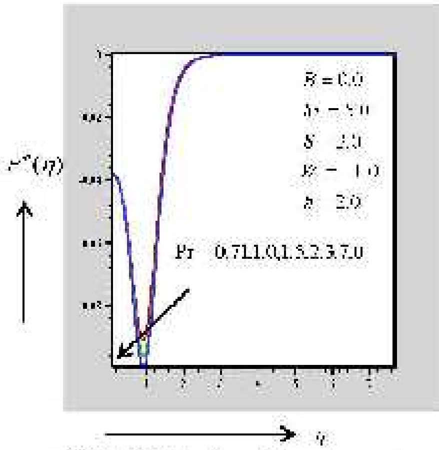

Fig.31. Effect of prandtl number over dimensionless velocity $F''(\eta)$.

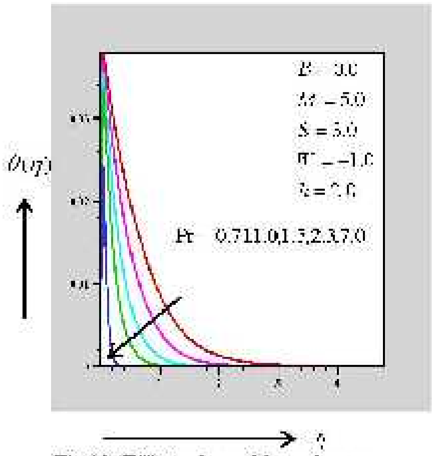

Fig.32. Effect of prandtl number over dimensionless temperature $\theta(\eta)$.

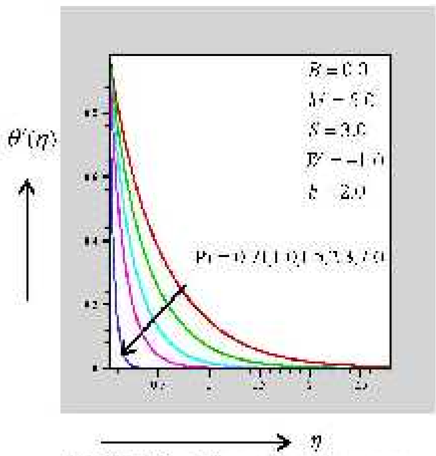

Fig.33. Effect of prandtl number over dimensionless temperature $\theta'(\eta)$.

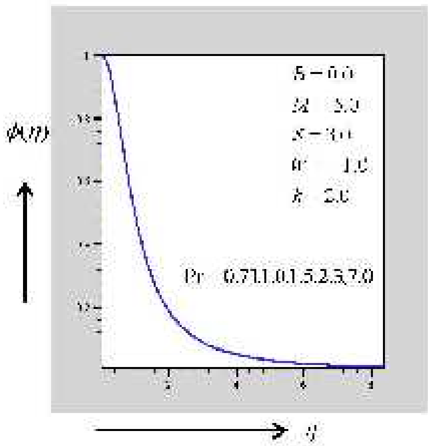

Fig.34. Effect of prandtl number over concentration $\phi(\eta)$.

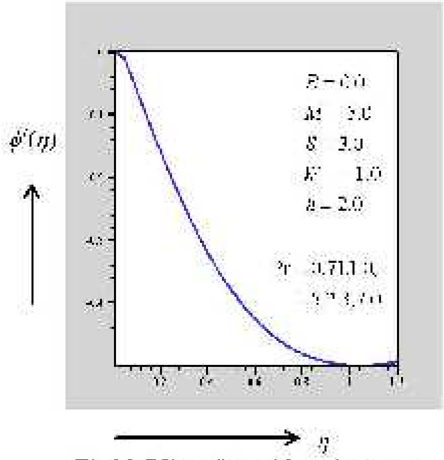

Fig.35. Effect of prandtl number over concentration $\phi'(\eta)$.

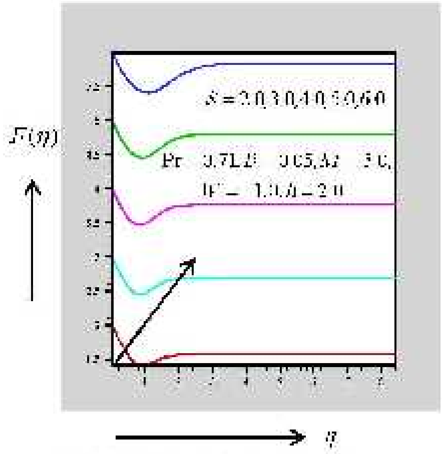

Fig.36. Effect of suction parameter over dimensionless traverse velocity $F(\eta)$.

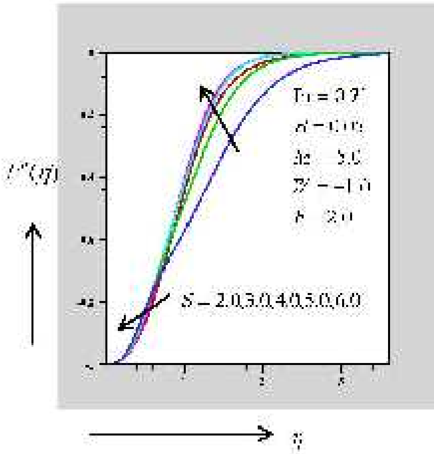

Fig.37. Effect of suction parameter over dimensinless longitudinal velocity $F'(\eta)$.

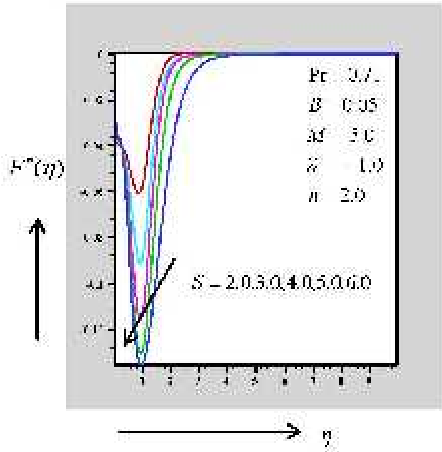

Fig.38. Effect of suction parameter over dimensionless velocity $F''(\eta)$.

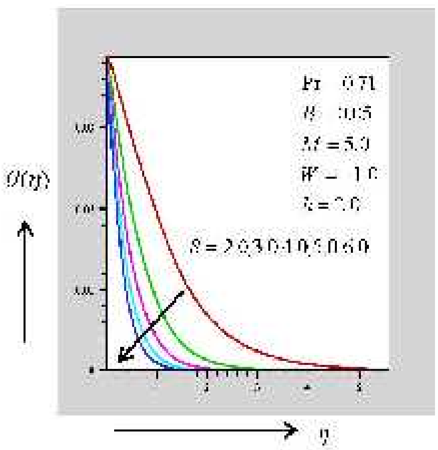

Fig.39. Effect of suction parameter over dimensionless temperature $\theta(\eta)$.

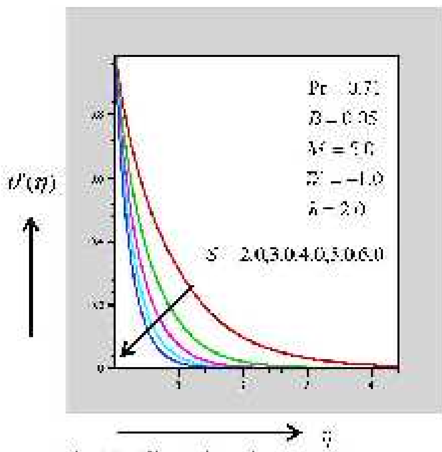

Fig.40. Effect of suction parameter over dimensionless temperature $\theta'(\eta)$.

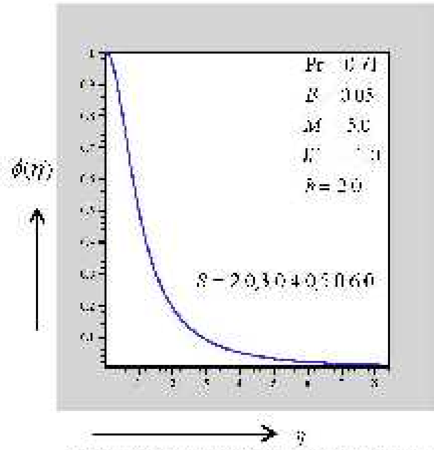

Fig.41. Effect of suction parameter over concentration $\phi(\eta)$

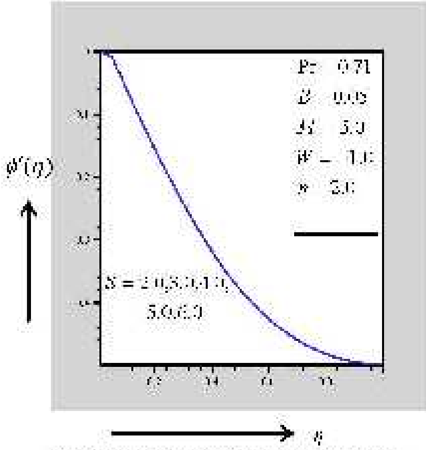

Fig.42. Effect of suction parameter over concentration $\phi'(\eta)$.

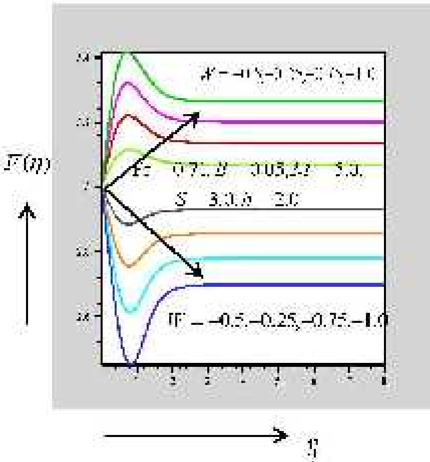

Fig.43. Effect of stretching/shrinking parameter over dimensionless traverse velocity $\phi'(\eta)$

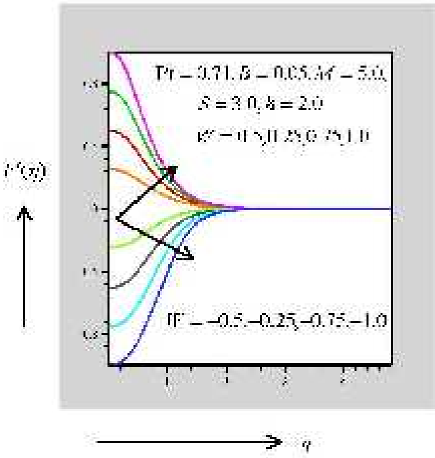

Fig.44. Effect of stretching/shrinking parameter over dimensionless longitudinal velocity $\phi(\eta)$

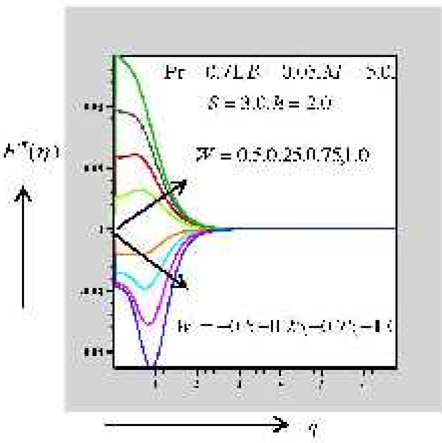

Fig.45: Effect of stretching/shrinking parameter over dimensionless velocity $F'(\eta)$

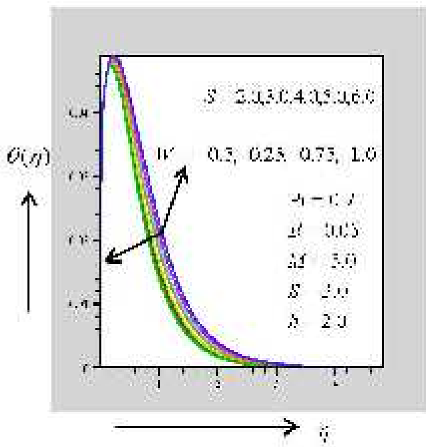

Fig.46. Effect of stretching/shrinking parameter over dimensionless temperature $\theta(\eta)$

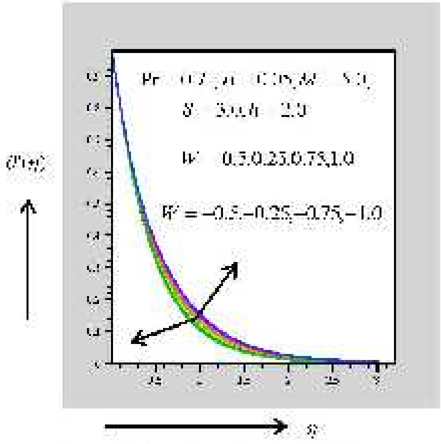

Fig.17. Effect of stretching/shrinking parameter over dimensionless temperature $\theta(\eta)$.

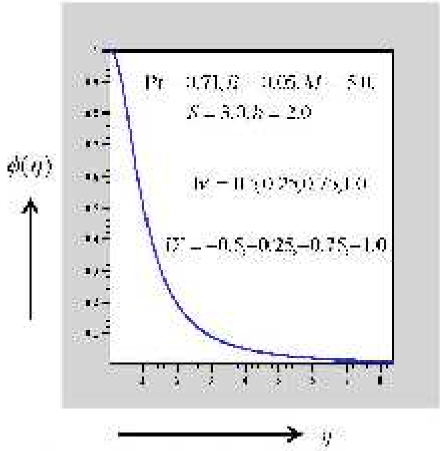

Fig.18. Effect of stretching/shrinking parameter over concentration $\phi(\eta)$.

Conclusion

From the above results and discussion, the following conclusions are arrived:

- The effect of suction parameter is to accelerate the transverse velocity and longitudinal velocity whereas the temperature of the flow field is decreased by the increasing value of suction parameter.
- Magnetic parameter increases both transverse and longitudinal velocity significantly. When the shrinking sheet is prescribed with variable surface heat flux temperature, the temperature of the fluid decreases with an increase in values of magnetic parameter.
- Suction parameter increases the skin friction with increase in magnetic parameter.
- The influence of Prandtl number is to decrease the temperature and hence the thermal boundary layer thickness reduces.
- The effect of heat source parameter is to increase the temperature of the fluid and thermal boundary layer thickness reduces with a decrease in heat source parameter.
- The heat flux parameter increases the temperature of the flow field significantly. It is noted that the thermal boundary layer thickness increases due to heat flux parameter n

- The Suction parameter, magnetic parameter and Prandtl number decreases the wall temperature whereas it is increased by the heat source parameter and heat flux parameter n .

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