ISSN (Online) : 2454 -7190 ISSN (Print) 0973-8975
J.Mech.Cont.& Math. Sci., Vol.-13, No.-4, September-October (2018) Pages 50-66

Kink and Periodic Solutions to the Jimbo-Miwa Equation and the Calogero-Bogoyavlenskii-Schiff Equation

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Abstract

In this article, we form the exact wave solutions of the Jimbo-Miwa equation and the Calogero-Bogoyavlenskii-Schiff equation by applying the new generalized (G'/G)*-expansion method. We explained the new generalized* (G'/G)*-expansion method to look for more general traveling wave solutions of the above mentioned equations. The traveling wave solutions attained by this method are in terms of hyperbolic, trigonometric and rational functions. The graphical representation of the obtained solutions is kink soliton, singular kink soliton, singular soliton and singular periodic solution. This method is very significant for extracting exact solutions of NLEEs which habitually occur in mathematical physics, engineering sciences and applied mathematics.*

Keywords: Exact traveling wave solutions; Jimbo-Miwa equation; Calogero-Bogoyavlenskii-Schiff equation; new generalized (G'/G)-expansion method.

I. Introduction

In the real world phenomena, most of the evolution equations are nonlinear. Nonlinear evolution equations (NLEEs) i.e., a partial differential equation which contains time derivative has become significant tool for investigation the natural phenomena of science and engineering. NLEEs which explain nonlinear phenomena, appear in an extensive diversity of applications in solitary wave theory, water waves, propagation of shallow water waves, tsunami waves, theoretical physics, nuclear physics, plasma physics, chemical physics, hydrodynamics, fluid dynamics, theory of turbulence, meteorology, optical fibers, quantum mechanics, chaos theory, coastal engineering, ocean engineering, biomathematics and such many other applications. Soliton is one of the natural phenomena which appear everywhere in daily life. The dynamics of solitons have changed the daily lifestyle of all people across the world. For example, all internet activities, phone conversations are due to soliton transmission, over transcontinental and transoceanic distances, through optical fiber cables. Investigating exact solutions of NLEEs is an important background in the area of nonlinear science and theoretical physics. Thorough exploratory solutions of nonlinear evolution equations by applying the suitable methods have shown an enormous dynamism. Searching the exact traveling wave solutions by implementing satisfactory techniques and constructive methods has also shown massive essentiality. In recent years, for obtaining exact traveling solutions of NLEEs have been developed different influential methods, such as the generalized Kudryashov method [IX, XXXVI, XXXVII], the (G'/G)-expansion method [V], the Backlund transformation method [XX], the Hirota's bilinear transformation method [XXI, XXIII, XV], the Darboux transformation method [XXII], the Cole-Hopf transformation method [VII, XI], the tanh method [XXXIV], the tanh-coth method [XXV, XXXV, XIX], the exp-function method [XVII, XVIII, XIV], the *F*-expansion method [XVI], the Jacobi elliptic function method [XXXVIII, VI], the extended Jacobi elliptic function method [XXXII], the Riccati equation method [XXVI], the simple equation method [XXX], the modified simple equation method [III, IV, XXXI, XIII, XII], the exp(-ϕ(ξ))-method [X], the multiple exp-function method [XXIV] etc. Lately, another important method namely, the (G'/G)-expansion method was introduced by Wang et al. [XXXIII] to look for analytic solutions of NLEEs. Consequent Wang et al. many researchers have investigated various NLEEs to explore exact traveling wave solutions [XL, VIII, XXIX, I, XXVII]. After that, various researchers applied this method to obtain exact solutions of quite a lot of NLEEs [XXXIX]. Later, diverse extensions of the (G'/G)-expansion method have been proposed and developed by different class of researcher. Several extension and improvement shows the effectiveness and authenticity of this method. Zhang et al. [XLI] extended the (G'/G)-expansion method which is called the improved (G'/G)-expansion method. Akbar et al. [II] established the generalized and improved (G'/G)-expansion method with additional parameter. In recent times, Naher and Abdullah [XXVIII] proposed the new generalized (G'/G)-expansion method that is more understandable and straightforward for a group of NLEEs to produce the further general and abundant solutions with more parameters.

The aim of this article is that, we give attention to construct exact traveling wave solutions together with solitons, kink, and periodic, rational solutions and to show the effect of these new terms on the structures of the obtained solutions of the Jimbo-Miwa and the Calogero-Bogoyavlenskii-Schiff equation by implementing the generalized (G'/G)-expansion method.

II. Explanation of method

In this portion, we outline the generalized (G'/G)-expansion method to search the traveling wave solutions of nonlinear evolution equations.

Let us consider the general nonlinear evolution equation is of the form

$$H(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \dots) = 0, \quad (1)$$

where $u = u(x,t)$ is an anonymous function, H is a polynomial in $u(x,t)$, x is the spatial variable and t temporal variable and its partial derivatives in which highest order derivatives and nonlinear terms are included and the order of this method is as follows:

Step 1: We regard as the arrangement of real variables x and t be the compund variable η as follows:

$$u(x,t) = u(\eta), \eta = x + y + z \pm ct \tag{2}$$

where c is the speed of the traveling wave. By way of equation (2), equation (1) can be transformed into an ordinary differential equation (ODE) in the following form:

$$S(u, u', u'', u''', \dots) = 0, \tag{3}$$

where S is a polynomial of u and its derivatives and the superscripts signify the derivatives with respect to η.

Step 2: According to possibility, equation (3) can be integrated term by term one or more times. The integral constant may be zero for simplicity.

Step 3: We regard as the traveling wave solution of equation (3) can be develop in the form of a polynomial of the following shape:

$$u(\eta) = \sum_{l=0}^{P} a_l (d + K)^l + \sum_{l=1}^{P} b_l (d + K)^{-l}, \tag{4}$$

where either a_l or b_l may be zero but both a_l and b_l could not be zero at a time. a_l $(l = 0, 1, 2, 3, \dots P$, b_l $(l = 1, 2, 3, \dots P)$ and d are arbitrary constant which are determined.

$$\text{Here } K = (G'/G), \tag{5}$$

where $G = G(\eta)$ satisfies the following auxiliary nonlinear ordinary differential equation

$$AGG'' - BGG' - EG^2 - C(G')^2 = 0, \tag{6}$$

where prime indicates the derivative with respect to η and A, B C, E are parameters.

Step 4: To determine the positive integer P, it desires to take the homogeneous balance between the highest order nonlinear term and the highest order derivative appearing in equation (3).

Step 5: Inserting equations (4) and (6) including equation (5) into equation (3) and utilizing the value of P obtained in step 4, we obtain polynomial in $(d + K)^P$, $(P = 0, 1, 2, \dots)$ and $(d + K)^{-P}$, $(P = 1, 2, \dots)$. Then we collect each coefficient of the resulted polynomial to zero. We obtain a class of algebraic equations for $a_l(l = 0, 1, 2, \dots, P)$,$b_l(l = 1, 2, \dots, P)$, d and c.

Step 6: The general solution of equation (5) is known to us, substitute the value of $a_l(l = 0, 1, 2, \dots, P)$, $b_l(l = 1, 2, \dots, \mathrm{P})$, d and c into equation (4), we then obtain more general type and new exact traveling wave solution of the nonlinear evolution equation (1).

Step 7: Using the general solution of equation (6), we obtain the following solution of equation (5).

Family 1: Hyperbolic function solution when $B \neq 0$, $\phi = A - C$ and $Q = B^2 + 4E(A - C) > 0$,

$$K(\eta) = \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\frac{m_1 \sinh(\frac{\sqrt{Q}}{2A}\eta) + m_2 \cosh(\frac{\sqrt{Q}}{2A}\eta)}{m_1 \cosh(\frac{\sqrt{Q}}{2A}\eta) + m_2 \sinh\left(\frac{\sqrt{Q}}{2A}\eta\right)}. \tag{7}$$

Family 2: Trigonometric solution when $B \neq 0, \phi = A - C$ and $Q = B^2 + 4E\ (A - C) < 0$,

$$K(\eta) = \frac{B}{2\phi} + \frac{\sqrt{-Q}}{2\phi}\frac{-m_1 \sin(\frac{\sqrt{-Q}}{2A}\eta) + m_2 \cos(\frac{\sqrt{-Q}}{2A}\eta)}{m_1 \cos(\frac{\sqrt{-Q}}{2A}\eta) + m_2 \sin\left(\frac{\sqrt{-Q}}{2A}\eta\right)} \tag{8}$$

Family 3: Rational form solution when $B \neq 0$, $\phi = A - C$ and $Q = B^2 + 4E(A - C) = 0$,

$$K(\eta) = \frac{B}{2\phi} + \frac{m_2}{m_1 + m_2\,\eta}. \tag{9}$$

Family 4: Hyperbolic function solution when $B = 0$, $\phi = A - C$ and $P = \phi E > 0$

$$K(\eta) = \frac{\sqrt{P}}{\phi}\frac{m_1 \sinh(\frac{\sqrt{P}}{A}\eta) + m_2 \cosh(\frac{\sqrt{P}}{A}\eta)}{m_1 \cosh(\frac{\sqrt{P}}{A}\eta) + m_2 \sinh\left(\frac{\sqrt{P}}{A}\eta\right)}. \tag{10}$$

Family 5: Trigonometric solution when $B = 0$, $\phi = A - C$ and $P = \phi E < 0$,

$$K(\eta) = \frac{\sqrt{-P}}{\phi}\frac{-m_1 \sin(\frac{\sqrt{-P}}{A}\eta) + m_2 \cos(\frac{\sqrt{-P}}{A}\eta)}{m_1 \cos(\frac{\sqrt{-P}}{A}\eta) + m_2 \sin\left(\frac{\sqrt{-P}}{A}\eta\right)}. \tag{11}$$

These are the general solutions of equation (5) that can be applied to establish the solutions of the Jimbo-Miwa equation and the Calogero-Bogoyavlenskii-Schiff equation.

III. Formation of the Solutions

In this segment, we have introduced some more general and new exact traveling wave solutions to the Jimbo-Miwa equation and the Calogero-Bogoyavlenskii-Schiff equation through the new generalized (G'/G)-expansion method.

IIIa. Jimbo-Miwa equation

The Jimbo-Miwa equation is the second equation in the KP hierarchy of integrable systems, which is applied to clarify certain interesting (3+1)-dimensional waves in physics. The Jimbo-Miwa equation has been investigated on many extract for instance its solutions, inegrability properties and consistencies.

We consider (3+1)-dimensional Jimbo-Miwa equation as follows:

$$u_{xxxy} + 3u_x u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0. \quad (12)$$

By applying the conversion $u(x,t) = u(\eta)$, where $\eta = x + y - ct$, we reduce the equation (12) into the following ODE as,

$$u'''' + 6u'u'' - (2c+3)u'' = 0. \quad (13)$$

Integrating equation (13) with respect to η once, we have

$$u''' + 3(u')^2 - (2c+3)u' + l_1 = 0, \quad (14)$$

where l_1 is an integrating constant which is determine later. To attain the value of P, using the homogeneous balance between the highest order nonlinear term $(u')^2$ and linear term of highest derivative u''' look through in equation (14), we obtain $P = 1$. Accordingly, we obtain the solution of equation (14) in the succeeding form

$$u(\eta) = a_0 + a_1(d+K) + b_1(d+K)^{-1}, \quad (15)$$

where a_0, a_1, b_1 and d are constant which are to be determined. We will determine these constants later replacing equation (15) along with equations (5) and (6) into equation (14), we notice that the left hand side of equation (14) is converted into the polynomials$(G'/G)^P$, $(P = 0,1,2,3,\ldots)$ and$(G'/G)^{-P}$,$(P = 1,2,3,\ldots)$. Then, we collect each coefficient of these resulted polynomials equal to zero and attain the system of algebraic equations and solve the algebraic equations by the algebraic computation software, like Maple, for the constants a_0, a_1, b_1, c, l_1, d and we found three sets of solutions as follows:

Set 1:

$$d = -\frac{B}{2\phi}, a_0 = a_0, \qquad a_1 = \frac{2\phi}{A}, b_1 = \frac{B^2+4\phi E}{2\phi A},$$
$$c = -\frac{3A^2-16E\phi-4B^2}{2A^2}, l_1 = 0. \quad (16)$$

Set 2:

$$d = d, a_0 = a_0, a_1 = 0, b_1 = -\frac{2(d^2\phi - E + Bd)}{A},$$
$$c = -\frac{3A^2-4E\phi-B^2}{2A^2}, l_1 = 0. \quad (17)$$

Set 3:

$$d = d,\ a_0 = a_0,\ a_1 = \frac{2\phi}{A},\ b_1 = 0,$$
$$c = -\frac{3A^2 - 4E\phi - B^2}{2A^2},\ l_1 = 0. \tag{18}$$

Where $\phi = A - C$ and A, B, C, E are parameters.

For set 1: When $B \neq 0$, $\phi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, substituting the values of the constants arranged in equation (16) into equation (15), as well as equation (7) and simplifying, we obtain the following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ respectively

$$u_{1_1} = a_0 + \frac{2\phi}{A}\left\{\frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{1_2} = a_0 + \frac{2\phi}{A}\left\{\frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

In the similarly manner, by means of the values of the constants contained in equation (16) into (15), together with (8) to (11) and simplify, we attain respectively following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{1_3} = a_0 + \frac{2\phi}{A}\left\{\frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{1_4} = a_0 - \frac{2\phi}{A}\left\{\frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\} - \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{1_5} = a_0 + \frac{2\phi}{A}\left(\frac{m_2}{m_1 + m_2\,\eta}\right) + \frac{B^2+4\phi E}{2\phi A}\left(\frac{m_2}{m_1 + m_2\,\eta}\right)^{-1},$$

$$u_{1_6} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{1_7} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{1_8} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1},$$

$$u_{1_9} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1}.$$

For set 2: Setting the values of the constants hold in equation (17) into (15), along with equation (7) and simplifying, we obtain following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{2_1} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{2_2} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

Similarly, by means of the values of the constants contained in equation (17) into (15), together with equation (8) to (11) and simplifying, we obtain respectively following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{2_3} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{2_4} = a_0 - \frac{B^2 + 4\phi E}{2\phi A}\left\{d + \frac{B}{2\phi} - \frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{2_5} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left(d + \frac{B}{2\phi} + \frac{m_2}{m_1 + m_2\eta}\right)^{-1},$$

$$u_{2_6} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{d + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{2_7} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{d + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{2_8} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{d + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1},$$

$$u_{2_9} = a_0 - \frac{2(d^2\phi - E + Bd)}{A}\left\{\mathrm{d} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1}.$$

For set 3: Inserting the values of the constants scheduled in equation (18) into (15), as well as equation (7) to (11) and simplifying, we get following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{3_1} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\},$$

$$u_{3_2} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\},$$

$$u_{3_3} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\},$$

$$u_{3_4} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} - \frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\},$$

$$\mathrm{u}_{3_5} = a_0 + \frac{2\phi}{A}\left(d + \frac{B}{2\phi} + \frac{m_2}{m_1 + m_2\eta}\right),$$

$$u_{3_6} = a_0 + \frac{2\phi}{A}\left\{d + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\},$$

$$u_{3_7} = a_0 + \frac{2\phi}{A}\left\{d + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\},$$

$$u_{3_8} = a_0 + \frac{2\phi}{A}\left\{d + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\},$$

$$u_{3_9} = a_0 + \frac{2\phi}{A}\left\{\mathrm{d} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}.$$

III b. The (2+1)-Dimensional Calogero-Bogoyavlenskii-Schiff equation

The Calogero-Bogoyavlenskii-Schiff (CBS) equation was primarily constructed by Bogoyavlenskii and Schiff in several ways. The CBS equation has some physical application in mathematical physics. A diverse group of researchers across the globe show that quite a lot of effectual methods for obtaining exact solutions of the CBS equation.

The potential form of (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff equation is given by

$$u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z = 0. \tag{19}$$

Now, we use the wave transformation $u(x,t) = u(\eta)$, where $\eta = x + z - ct$ into the (19), which yields,

$$u'''' + 6u'u'' - cu'' = 0, \tag{20}$$

which is integrality. Therefore, integrating equation (20) with respect η once yields,

$$u''' + 3(u')^2 - cu' + l_2 = 0 \tag{21}$$

where l_2 is an integrating constant which is to be determined. To find the integer value of P, taking the homogeneous balance between u''' and $(u')^2$ of equation (21), we get $P = 1$. Thus, the solution of equation (21) is of the form,

$$u(\eta) = a_0 + a_1(d + K) + b_1(d + K)^{-1}, \tag{22}$$

where a_0, a_1, b_1 and d are arbitrary constant which are to be determined. Inserting equation (22) along with equations (5) and (6) into equation (21), the left-hand side of equation (21) is switched into polynomials in $(G'/G)^P$, $(P = 0, 1, 2, 3, \dots)$ and$(G'/G)^{-P}$,$(P = 1, 2, 3, \dots)$. We collect each coefficient of these evolved polynomials to zero yields a set of contemporary algebraic equations for $a_0, a_1, b_1\ l_2$ and d. Solving these algebraic equations with the help of algebraic computation software, like Maple, for the constants a_0, a_1, b_1, c, l_2, d and found three sets of solutions as follows:

Set 1:

$$d = -\frac{B}{2\phi},\ a_0 = a_0,\ a_1 = \frac{2\phi}{A},\ b_1 = \frac{B^2 + 4\phi E}{2\phi A},\ c = \frac{16E\phi + 4B^2}{A^2},\ l_2 = 0. \tag{23}$$

Set 2:

$$d = d,\ a_0 = a_0,\ a_1 = 0,\ b_1 = -\frac{Bd - E + d^2\phi}{A},\ c = \frac{4E\phi + B^2}{A^2},\ l_2 = 0. \quad (24)$$

Set 3:

$$d = d,\ a_0 = a_0,\ a_1 = \frac{2\phi}{A},\ b_1 = 0,\ c = \frac{4E\phi + B^2}{A^2},\ l_2 = 0. \quad (25)$$

Where $\phi = A - C$ and A, B, C, E are parameters.

For set 1: When $B \neq 0$, $\phi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, substituting the values of the constants arranged in equation (23) into equation (22), as well as equation (7) and simplifying, we obtain the following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ respectively

$$u_{4_1} = a_0 + \frac{2\phi}{A}\left\{\frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{4_2} = a_0 + \frac{2\phi}{A}\left\{\frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

In the similarly manner, by means of the values of the constants contained in equation (23) into (22), together with (8) to (11) and simplify, we attain respectively following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{4_3} = a_0 + \frac{2\phi}{A}\left\{\frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{4_4} = a_0 - \frac{2\phi}{A}\left\{\frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\} - \frac{B^2+4\phi E}{2\phi A}\left\{\frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{4_5} = a_0 + \frac{2\phi}{A}\left(\frac{m_2}{m_1 + m_2\,\eta}\right) + \frac{B^2+4\phi E}{2\phi A}\left(\frac{m_2}{m_1 + m_2\,\eta}\right)^{-1},$$

$$u_{4_6} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{4_7} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{4_8} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1},$$

$$u_{4_9} = a_0 + \frac{2\phi}{A}\left\{-\frac{B}{2\phi} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\} + \frac{B^2+4\phi E}{2\phi A}\left\{-\frac{B}{2\phi} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1}.$$

For set 2: Setting the values of the constants hold in equation (24) into (22), along with equation (7) and simplifying, we obtain following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{5_1} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{5_2} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\}^{-1},$$

Similarly, by means of the values of the constants contained in equation (24) into (22), together with equation (8) to (11) and simplifying, we obtain respectively following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{5_3} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{5_4} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{B}{2\phi} - \frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\}^{-1},$$

$$u_{5_5} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left(d + \frac{B}{2\phi} + \frac{m_2}{m_1 + m_2\,\eta}\right)^{-1},$$

$$u_{5_6} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{5_7} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\}^{-1},$$

$$u_{5_8} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{d + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1},$$

$$u_{5_9} = a_0 - \frac{(d^2\phi - E + Bd)}{A}\left\{\mathrm{d} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}^{-1}.$$

For set 3: Inserting the values of the constants scheduled in equation (25) into (22), as well as equation (7) to (11) and simplifying, we get following traveling wave solutions for $m_1 = 0$ but $m_2 \neq 0$ and $m_2 = 0$ but $m_1 \neq 0$ as follows:

$$u_{6_1} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\coth\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\},$$

$$u_{6_2} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{Q}}{2\phi}\tanh\left(\frac{\sqrt{Q}}{2A}\eta\right)\right\},$$

$$u_{6_3} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} + \frac{\sqrt{-Q}}{2\phi}\cot\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\},$$

$$u_{6_4} = a_0 + \frac{2\phi}{A}\left\{d + \frac{B}{2\phi} - \frac{\sqrt{-Q}}{2\phi}\tan\left(\frac{\sqrt{-Q}}{2A}\eta\right)\right\},$$

$$u_{6_5} = a_0 + \frac{2\phi}{A}\left(d + \frac{B}{2\phi} + \frac{m_2}{m_1 + m_2\,\eta}\right),$$

$$u_{6_6} = a_0 + \frac{2\phi}{A}\left\{d + \frac{\sqrt{P}}{\phi}\coth\left(\frac{\sqrt{P}}{A}\eta\right)\right\},$$

$$u_{6_7} = a_0 + \frac{2\phi}{A}\left\{d + \frac{\sqrt{P}}{\phi}\tanh\left(\frac{\sqrt{P}}{A}\eta\right)\right\},$$

$$u_{6_8} = a_0 + \frac{2\phi}{A}\left\{d + \frac{\sqrt{-P}}{\phi}\cot\left(\frac{\sqrt{-P}}{A}\eta\right)\right\},$$

$$u_{6_9} = a_0 + \frac{2\phi}{A}\left\{\mathrm{d} - \frac{\sqrt{-P}}{\phi}\tan\left(\frac{\sqrt{-P}}{A}\eta\right)\right\}.$$

IV. Graphical representation and physical explanations

In this section, we have to do with the graphical representation and the physical explanation of the attained solutions of the NLEEs by means of the Jimbo-Miwa and the Calogero-Bogoyavlenskii-Schiff equation. Graph is an essential tool to introduce the problems and explain properly the solutions of the phenomena. The graphical representation of the solutions which we generated is provided below.

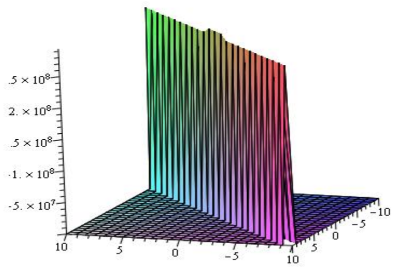

Figure 1: Singular soliton of u_{1_1} within the interval $-10 \le x \le 10$ and $-10 \le t \le 10$ for the parameters
$A = 15.21110255, B = 1, C = 1, E = 1, a_0 = 2$.

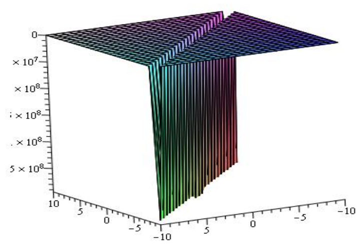

Figure 2: Singular kink soliton of u_{1_2} within the interval $-10 \le x \le 10$ and $-10 \le t \le 10$ for the parameters $A = 15.21110255, B = 1, C = 1, E = 3, a_0 = 2$.

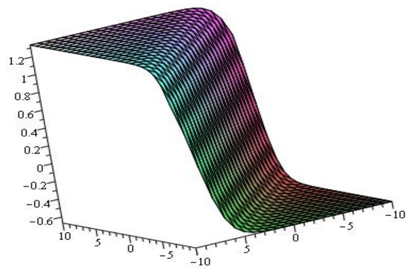

Figure 3: Kink soliton of u_{2_2} within the interval $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$ for the parameters $A = 3$, $B = 1$, $C = 1$, $E = 1$, $a_0 = 2$.

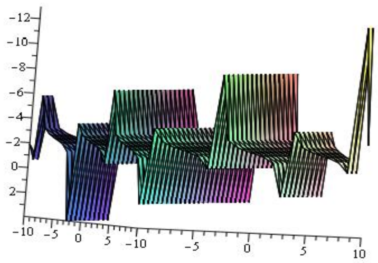

Figure 4: Periodic soliton of u_{5_3} within the interval $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$ for the parameters $a_0 = 2$, $A = 1$, $B = 1$, $C = 6$, $E = 1$.

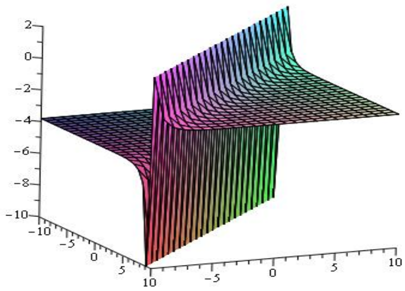

Figure 5: Singular kink soliton of u_{2_6} within the interval $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$ for the parameters $a_0 = 2$, $A = 3$, $B = 1$, $C = 1$, $E = 1$.

Solutions $u_{1_1} - u_{1_8}$, u_{2_5}, u_{2_6}, u_{3_1}, $u_{3_3} - u_{3_6}$, $u_{4_1} - u_{4_5}$, u_{4_6}, u_{4_8}, u_{5_1}, u_{5_5}, u_{5_6}, u_{6_3}, u_{6_5} and u_{6_8} are the singular kink soliton. Fig.1, fig.2 and fig.5 shows the shape of the exact singular kink-type solution u_{1_1}, $u_{1_2}$and , u_{2_6}. The shape of figure of above other solutions is similar to the figure of solutions u_{1_1}, $u_{1_2}$and , $u_{2_6}$of the Jimbo-Miwa equation and the figures of these solutions are omitted for convenience.

Solutions u_{1_9}, u_{2_3}, u_{2_4}, u_{3_4}, u_{4_9}, u_{5_3}, u_{5_4}, u_{5_8}, u_{5_9}, u_{6_4} and u_{6_9} represent the exact periodic traveling wave soliton. The graphical shape of fig. 4 shows the periodic solution of u_{5_3} the Calogero-Bogoyavlenskii-Schiff equation. The graphical representation of above other solutions is same to the shape of solution u_{5_3}. For convenience the figure is omitted.

The graphical representation of the solutions u_{2_2}, u_{2_7}, u_{2_9}, u_{3_2}, u_{3_7}, u_{3_9}, u_{4_7}, u_{5_2}, u_{5_7}, u_{6_2} and u_{6_7} are kink soliton. Kink waves are traveling waves which take place from one asymptotic position to another. The kink solutions are move toward to a constant at infinity. Fig. 3 shows the shape of the exact kink-type solution u_{2_2}. Other figures are omitted for convenience.

V. Conclusion

In this article, the new generalized (G'/G)-expansion method with nonlinear auxiliary equation has effectively been performed to explore various new and further general exact solutions the Jimbo-Miwa and the Calogero-Bogoyavlenskii-Schiff equation. The graphical description of the solutions shows that the formation of the obtained solutions is singular kink soliton, singular soliton, singular periodic solution and kink shaped soliton. The obtained solution of these equations has many efficient inflictions in applied mathematics, mathematical physics and engineering. The solutions include free parameters and might be constructive in miscellaneous physical applications. It is examined that the new generalized (G'/G)-expansion method is sufficient, effective, suitable and vital mathematical tool for solving other nonlinear evolution equations.

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