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AN ALTERNATIVE APPROACH OF GREEDY SUMMATION APPLIED TO NUMBERS AND ARRAYS: THEORITICAL IMPLEMENTATION

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Abstract

Mathematical treatment for numbers and arrays in the field of functional analysis need special interest. In the present paper, we will focus on a new alternative approach of greedy summation approach of unordered numbers and arrays. A theoretical background is firstly presentedregarding the numbers and arrays and their importance in the field of functional analysis, then the alternative approach for the greedy summation based on absolute values is presented. Some theoretical proofs regarding the relation between theoretical greedy summation and the Dirichlet series is presented in brief details. At the end of the present paper, some important conclusions are listed due to their importance and their effect for the upcoming research works.

Keywords : Numbers, Arrays, unordered sum, Numerical arrays, Greedy sum of numbers, Greedy sum of arrays, convergence of series, Dirichlet series of array.

I. Introduction

Numbers and arrays are found in wide range of science, engineering, and industrial applications[I,VII]. Due to this importance, careful should be taken into consideration when dealing with numbers and arrays in these applications. In the present paper, we will focus on new alternative approach of greedy summation approach of unordered numbers and arrays. A theoretical background is firstly presented regarding the numbers and arrays and their importance in the field of functional analysis, then the alternative approach for the greedy summation based on absolute values is presented. Some theoretical proofs regarding the relation between theoretical greedy summation and the Dirichlet series is presented in brief details. At the end some important conclusions are listed due to their importance and their effect for the upcoming research works.

Arrays: Any set of real or complex numbers has an index j and have an arbitrary nature, and written mathematically as $\{a_j\}_{j\in J}$ is well known as numerical array [IV].

II. Direct product of two indexed numbers

Suppose that we have two numerical arrays, $\{a_k\}_{k\in K}$ and $\{b_l\}_{l\in L}$, we can state that their direct product can be defined as the array product $\{a_k b_j\}_{(k,l)\in K\times L}$, particularly, this direct product forms a double series[V].

Greedy sum

The Greedy sum for any numeric array, $\{a_j\}_{j\in J}$ and the existence of any positive number, δ, we can define some of its partial sum as $\sum_{|a_j|\geq\delta} a_j$, this sum exists under satisfaction of the condition that if and only if (iff) this sum contains a finite number of summands. Now we are ready to define greedy summation $\sum_{|q_k|\geq\delta} q_k$, defined as[VI]:

$$\textbf{greedy sum} = \lim_{\delta\to 0}\sum_{|q_k|\geq\delta} q_k \quad (1)$$

Simply this greedy sum takes the following simple notation;

$$\textbf{greedy sum} = \sum_{k\in K} q_k \quad (2)$$

It is important to remember that, the greedy sum of the absolute convergent series is the same as its usual sum. Also, remember that the sum of any conditionally convergent series may or may not differs from its greedy, and finally, if the array $\{a_j\}_{j\in J}$ have a greedy sum, then the array is called a greedy summable [V].

III. Theorem

In this section, we will make use of the concept of greedy zeta-function of an array to prove the direct product of the greedy summable array. Assume that we have two numerical arrays $\{d_m\}_{m\in M}$, $\{q_n\}_{n\in N}$ and assume that their direct product is defined as:

$$\sum_{(m,n)\in M\times N}\{d_m\}_{m\in M}\{q_n\}_{n\in N} = \sum_{m\in M}\{d_m\}_{m\in M}\sum_{n\in N}\{q_n\}_{n\in N} \quad (3)$$

Proof

To prove this theorem, let us make use the concept of greedy zeta-function of an array, as follows:

Assume that the complex variable function for a numerical array $\{d_m\}_{m\in M}$ defined as $D(z)$ and the later written as:

$$D(z)=\sum_{j\in J} d_m |d_m|^z \tag{4}$$

Equation (4) is called the greedy zeta-function of the array $\{d_m\}_{m\in M}$, therefore,

$$d_m|d_m|^z . q_n|q_n|^z = d_m q_n |d_m q_n|^z \tag{5}$$

Therefore, the direct product of two arrays, which simple takes the name greedy zeta-function will equal to the greedy zeta-function of their product individually.

IV. A numerical array and its Dirichlet series

In this subsection, we will focus the sights to what is called, the Dirichlet series of a given numerical array. First let us remember the generalized form for Dirichlet series which can take the following series form defined as [I]:

$$D.S. = \sum_{m=1}^{m\to\infty} v_m \exp(-\eta_m z) \tag{6}$$

In equation (6):

η_m : An increasing sequence of real numbers of a monotone matter,

v_m : The series coefficients,

Finally, the series coefficients v_m and the variable z in this equation are in complex variable.

Now, for any numerical array $\{a_j\}_{j\in J}$, then its Dirichlet series will take the following form:

$$D.S. = \sum v_k \exp(-\eta_k z) \tag{7}$$

In equation (7):

$-\eta_k$: is the kth value element of the set $\{\ln|a_j| \,|\, j\in J\}$ and in a particular general case $-\eta_1$ is the maximal element of the set $\{\ln|a_j| \,|\, j\in J\}$ and the coefficient v_j is defined as follows:

$$v_k = \sum_{\ln|a_j|=-\eta_k} a_j \tag{8}$$

V. Dirichlet series Convergencey

Back to the given numerical array defined as $\{a_j\}_{j\in J}$, the convergence of its Dirichlet series for a given value of the complex variable z will be equivalent to the existence of its greedy sum defined before as $\sum_{j\in J} a_j |a_j|^z$ for the specific value of the complex variable $z = 0$.

Therefore, one can attain the following conclusion that the greedy zeta-function of a given numerical array will coincide with its Dirichlet series [I].

VI. Formal product theorem

Assume that we have two Dirichlet series, given as:

$$\sum v_k \exp(-\eta_k z)$$
&
$$\sum w_k \exp(-\zeta_k z) \tag{9}$$

Then, their formal product can be defined as:

$$\sum q_k \exp(-\xi_k z) \tag{10}$$

Such that

$$\xi_k = v_i + w_j$$
&
$$q_k = \sum_{\xi_k = v_i + w_j} v_i w_k \tag{11}$$

One can conclude that Dirichlet series resulted from the direct product of given two arrays will equal to the formal product of Dirichlet series of the factors.

Difference between arrays and series

Assume that we have the following infinite number terms:

$$a_1 + a_2 + a_3 + ... + a_n + ...\infty \tag{12}$$

This summation is called an infinite number of terms, or infinite series, simply it can be written as:

$$a_1 + a_2 + a_3 + ... + a_n + ...\infty = \sum_{i=1}^{n\to\infty} a_i \tag{13}$$

There are two types of sums they are consecutive sum, defined as:

$$C.S. = \lim_{n\to\infty} \sum_{i=1}^{n\to\infty} a_i \tag{14}$$

The 2nd one is called is the greedy sum, defined before. The main difference between the two series is that the later is handled as a number over an array over set M of +ve integer.

VII. Monotone greedy series

Back to the primary definition of the greedy series, if the limit of its partial sums coincides with its ε cut sums, one then can say the greedy series convergent monotone and generallytakes the name greedily summable.Continuing this subtopic, let us turn to the absolutely summability, the following result will be valid, that is if we have series, absolutely convergent, then its sum still the sameby rearrange its terms in an decreasing order, thereforethis sumrelated to series of criteria absolutely convergencewill coincide with the consecutive sum of the same series.

VIII. Additive greedy sum

In this subsection, we will define the meaning of the additive greedy sum $\sum B$ for any set A numbers, in such a way that it is indexed itself using the identity map, i.e.,

$$(b_0 = b): \sum B = \sum_{b\in B} b_0 \tag{15}$$

Subsequently, if we have two sets C , D and they are disjoint, then:

$$\sum C \cup D = \sum C + \sum D \tag{16}$$

Therefore, one can conclude that greedy summation is not countable additive.

IX. Countable additive sum

In the previous section, we reached to the conclusion that the greedy sum is not countable, unless certain condition will be achieved. This will be illustrated through the following example [I].

Example

Assume that we have:

$$A_n = \left\{ \frac{1}{2\times n} \pm i \frac{1}{2\times n} \right\}, \quad n = \textbf{Integers from 1 to } \infty \tag{i}$$

Represent pairs of complex conj. numbers. And assume we have number sequence of the form:

$$A_0 = \left\{-\frac{1}{2n+1}\right\},\ n = 1,2,3,......,\infty \tag{ii}$$

Then, the greedy sum of (i) and (ii) takes the form:

$$A = \bigcup_{n=0}^{\infty} A_n \tag{iii}$$

This greedy sum will differs from array:

$$A^{/} = \left\{\frac{1}{n}\right\} \cup A_0 \tag{v}$$

In equation (v), the pairs $A_n, n > 0$ are replaced by their sums, and this ensures the previous conclusion.

X. Nonlinearity

The greedy sum does not have the linear behavior property; the proof will be derived from what is called A-sum. The A-sum is a special type of greedy sum, and to achieve this sum, an extra condition should be satisfied, that is if we have two indexed functions a_j and b_j, then the A-sum have the linear behavior as follows:

$$\sum_{j \in J} a_j + \sum_{j \in J} b_j = \sum_{j \in J} \left(a_j + b_j\right) \tag{17}$$

XI. Cartesian (Direct) product of arrays

In this section, we will introduce the basic concepts of the direct product, or Cartesian product of two arrays and this product will carry on the greedy sum. Assume that we have two arrays $\{a_l\}_{l \in L}$ and $\{b_m\}_{m \in M}$, then their direct product takes the form:

$$\{a_l\}_{l \in L} . \{b_m\}_{m \in M} = \{a_l b_m\}_{(l,m) \in L \times M} \tag{18}$$

From the above explanation, we have now two types of sums, the 1st is called absolute sum and the 2nd is called greedy sum. Therefore, one can say that the product of the product of two types of arrays, the first one is absolute summable and the second one is greedy summable, the result will be greedily summable.

XII. Conclusions

In the present paper, the concept of the greedy sum for set of numbers and arrays is present on some brief details. The paper started by introducing the basic definition for the arrays and the greedy sums.

From this theoretical treatment of the greedy sum for numbers and arrays, one can conclude the following remarkable results:

1- Know well the main difference between greedy sums for set of numbers or an array and the condition to say greedy summable.

2-The basic properties of greedy sums had been discussed well.

3-The main difference between series and array from greedy sums point of view.

4-Convergent conditions studied well.

5-Determination the real meaning of greedy zeta-function.

6-The greedy sum is not countable, unless certain condition will be achieved.

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