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SOME FEATURES OF PAIRWISE $\alpha - T_0$ SPACES IN SUPRA FUZZY BITOPOLOGY

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Abstract

Four concepts of supra fuzzy pairwise T_0 bitopological spaces are introduced and studied in this paper. We also establish some relationships among them and study some other properties of these spaces.

Keywords: Fuzzy set, Supra topology, Supra fuzzy bitopological space, Good extension.

I. Introduction

American Mathematician Zadeh [XIX] first time in 1965 introduced the concepts of fuzzy sets. Chang [VII] and Lowen [XII] developed the theory of fuzzy topological space using fuzzy sets. Mashhour et al [XIII] introduced supra topological spaces and studied s-continuous functions and s^*- continuous functions Next time much research has been done to extend the theory of fuzzy topological spaces in various directions. Wong C. K. [XVII], Srivastava A. K. and Ali D. M [XVI] have developed the fuzzy topological spaces as well as fuzzy subspace topology. Hossain M. S. and Ali D. M. [VIII] worked on T_0 - fuzzy topological spaces.

The research for fuzzy bitopological spaces started in the early 1990s. The fuzzy bitopological spaces with separation axioms have become attractive as these spaces possess many desirable properties and can be found throughout various areas in fuzzy topologies. Fuzzy pairwise T_0 bitopological space has been introduced by Kandil and El-Shafee [X, XI]. Abu Sufiya et al [II] and Nouh [XV] have introduced fuzzy pairwise T_0 separation axioms. R. M. Amin et al [V] has also introduced two notions of fuzzy pairwise T_0 bitopological spaces in quasi- coincidence sense.

In this paper, we study, some features of $\alpha - T_0$-spaces in supra fuzzy bitopological spaces and establish relationship among them. As usual I= [0, 1] and $I_1 = [0,1]$

II. Preliminaries

In this section, we review some concepts, which will be needed in the sequel. Through the present paper X and Y are always presented non -empty sets.

Definition 2.1: For a set X, a function $u: X \to [0, 1]$ is called a fuzzy set in X. For every $x \in X, u(x)$ represents the grade of membership of x in the fuzzy set u. Some authors say u is a fuzzy subset of X. Thus a usual subset of X is a special type of a fuzzy set in which the ranges of the function is {0, 1}. The class of all fuzzy sets from X into the closed unit interval I will be denoted by I^X (Zadeh 1965)

Definition 2.2: Let X be a nonempty set and A be a subset of X. The function $I_A: X \to [0, 1]\{0, 1\}$ defined by $I_A(x) = \begin{cases} 1 \ \ if \ x \in A \\ 0 \ if \ \ x \notin A \end{cases}$ is called the characteristic function of A. The present authors also write 1_x for the characteristic function of {x}. The characteristic functions of subsets of a set X are referred to as the crisp sets in X (Zadeh 1965).

Definition 2.3: Let X be a non-empty set and t be the collection of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1, \ 0 \in t$
(ii) If $u_i \in t$ for each $i \in A$, then $\cup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, \ u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t-open(or simply open) fuzzy sets. If u is an open fuzzy set, then the fuzzy sets of the form 1-u are called t-closed(or simply closed) fuzzy sets(Chang 1968).

Definition 2.4: Let X be a nonempty set and t be the collection of fuzzy sets in I^X such that

(i) $1, \ 0 \in t$
(ii) If $u_i \in t$ for each $i \in A$, then $\cup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, \ u_2 \in t$ then $u_1 \cap u_2 \in t$.
(iv) All constants fuzzy sets in X belong to t.

Then t is called a fuzzy topology on X (Lowen 1976)

Definition 2.5: Let X be a non-empty set. A subfamily t^* of I^X is said to be a supra topology on X if and only if

(i) $1, \ 0 \in t^*$
(ii) If $u_i \in t^*$ for each $i \in A$, then $\cup_{i \in \Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological spaces. The elements of t^* are called supra open fuzzy sets in (X, t^*) and complement of a supra open fuzzy set is called supra closed fuzzy set (Mashhour et al. 1983).

Definition 2.6: Let (X, s) and (Y, t) be two topological spaces. Let s^* and t^* are associated supra fuzzy topologies with s and t respectively and $f: (X, s^*) \to (Y, t^*)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each i.e., if for any $v \in t^*, f^{-1}(v) \in s^*$. The function f is called supra fuzzy homeomorphic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous(Mashhour et al. 1983).

Definition 2.7: Let (X, s^*) and (X, t^*) be two supra fuzzy topological spaces. If u_1 and u_2 are supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1 \times u_2$ is a supra fuzzy subsets of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min\,[u_1(x), u_2(y)]$, for each pair $(x, y) \in X \times Y$(Azad 1981)

Definition 2.8: Suppose $\{X_i, i \in \Lambda\}$, be any collection of sets and X denoted the Cartesian product of these sets, i.e.,$X = \prod_{i\in\Lambda} X_i$. Here X consists of all points $p =< a_i, i \in \Lambda >$, where $a_i \in X_i$. For each $j_0 \in \Lambda$, the authors defined the projection π_{j_0} by $\pi_{j_0}(a_i: i \in \Lambda) = a_{j_0}$. These projections are used to define the product supra fuzzy topology (Wong 1974).

Definition 2.9: Let $\{X_\alpha\}_{\alpha\in\Lambda}$ be a family of nonempty sets. Let $X = \prod_{\alpha\in\Lambda} X_\alpha$ be the usual products of X_α's and let $\pi_\alpha: X \to X_\alpha$ be the projection. Further, assume that each X_α is a supra fuzzy topological space with supra fuzzy topology t_α^*. Now the supra fuzzy topology generated by $\{\pi_\alpha^{-1}(b): b_\alpha \in t_\alpha^*, \alpha \in \Lambda\}$ as a sub basis, is called the product supra fuzzy topology on X. Thus if w is a basic element in the product, then there exists $\alpha_1, \alpha_2, \dots\dots.\alpha_n \in \Lambda$ such that $w(x) = \min\{b_\alpha(x_\alpha): \alpha = 1, 2, 3, \dots., n\}$, where $x = (x_\alpha)_{\alpha\in\Lambda} \in X$(Wong 1974).

Definition 2.10: Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f: X \to R$ is a lower semi-continuous if and only if $\{x \in X: f(x) > \alpha\}$ is open for all $\alpha \in R$ (Abd EL-Monsef et al. 1987).

Definition 2.11: Let (X, T) be a topological space and T^* be associated supra topology with T. Then the lower semicontinuous topology on X associated with T^* is $\omega(T^*) = \{\mu: X \to [0, 1], \mu$ is supra lsc}. If $\omega(T^*): (X, T^*) \to [0, 1]$ be the set of all lower semicontinuous (LSC) functions. We can easily show that $\omega(T^*)$ is a supra fuzzy topology on X(Ming *et al*. 1980).

Definition 2.12: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) are two fuzzy bitopological spaces and $f: (X, s_1^*, t_1^*) \to (Y, s_2^*, t_2^*)$ be a function. Then the function f is a fuzzy pairwise continuous if both the function $f: (X, s_1^*) \to (Y, s_2^*)$ and $f: (X, t_1^*) \to (Y, t_2^*)$ are fuzzy continuous.(A. Mukherjee)

Definition 2.13: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) are two fuzzy bitopological spaces and $f: (X, s_1^*, t_1^*) \to (Y, s_2^*, t_2^*)$ be a function. Then the function f is a fuzzy pairwise open if both the function $f: (X, s_1^*) \to (Y, s_2^*)$ and $f: (X, t_1^*) \to (Y, t_2^*)$ are fuzzy open. i.e. for every open set $u \in s_1^*$, $f(u) \in s_2^*$ and for every $v \in t_1^*, f(v) \in t_2^*$.(A. Mukherjee)

Definition 2.14: Let $\{(X_i, s_i, t_i): i \in \Lambda\}$ is a family of fuzzy bitopological spaces. Then space $(\prod X_i, \prod s_i, \prod t_i)$ is called the product fuzzy bitopological space of the family $\{(X_i, s_i, t_i): i \in \Lambda\}$, where $\prod s_i$ *and* $\prod t_i$ denote the usual product fuzzy topologies of the families $\{\prod s_i : i \in \Lambda\}$ and $\{\prod t_i : i \in \Lambda\}$ of the fuzzy topologies respectively on X.(C.K. Wong)

Let S^* and T^* be two supra topologies associated with two topologies S and T respectively. Let P be the property of a supra bitopological space (X, S^*, T^*) and FP be its supra fuzzy topological analog. Then FP is called a 'good extension' of P 'if and only if the statement (X, S^*, T^*) has P if and only if $(X, \omega(S^*), \omega(T^*))$ has FP" holds good for every supra topological space (X, S^*, T^*).

3. $\alpha - T_0(I), \alpha - T_0(II), \alpha - T_0(\text{III})$ and $T_0(IV)$ SPACES IN SUPRA FUZZY BITOPOLOGICAL SPACE

In this section, we have given some new notions of $\alpha - T_0$ such as $\alpha - T_0(i), \alpha - T_0(ii), \alpha - T_0(\text{iii})$ and $T_0(iv)$ spaces in supra fuzzy bitopological spaces. We also discuss some properties of them and establish relationships among them by using these concepts.

Definition 3.1: Let (X, s^*, t^*) be a supra fuzzy bitopological spaces and $\alpha \in I_1$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$ or there exists $v \in t^*$ such that $v(x) \leq \alpha, v(y) = 1$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $u(x) = 0, u(y) > \alpha$ or there exists $v \in t^*$ such that $v(x) > \alpha, v(y) = 0$.

(c) (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $0 \leq u(y) \leq \alpha < u(x) \leq 1$ or there exists $v \in t^*$ such that $0 \leq v(x) \leq \alpha < v(y) \leq 1$.

(d) (X, s^*, t^*) is a pairwise $T_0(iv)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $u(x) \neq u(y)$ or $v \in t^*$ such that $v(x) \neq v(y)$.

Lemma 3.1: Let (X, s^*, t^*) be a supra fuzzy bitopological spaces and $\alpha \in I_1$. Then the following implications are true:

(a) (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$ implies (X, s^*, t^*) is a pairwise $T_0(iv)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_0(iv)$.

Proof: Suppose that (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$. Let x and y be two distinct elements in X. Since (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$, for $\alpha \in I_1$, by definition, there exists $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$ or there exists $v \in t^*$ such that $v(x) \leq \alpha, v(y) = 1$, which shows that there exists $u \in s^*$ such that $0 \leq u(y) \leq \alpha < u(x) \leq 1$ or there exists $v \in t^*$ such that $0 \leq v(x) \leq \alpha < v(y) \leq 1$. Hence by definition (c), (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$.

Suppose (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$. Then for $x, y \in X$, $x \neq y$ there exists $u \in s^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$ i.e. $u(x) \neq u(y)$or there exists $v \in t^*$

such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. i.e. $v(x) \neq v(y)$. Hence by definition (X, s^*, t^*) is a pair wise $T_0(iv)$

Let (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$. Then for $x, y \in X$, $x \neq y$ there exists $u \in s^*$ such that $u(x) = 0, u(y) > \alpha$ which implies that $0 \leq u(x) \leq \alpha < u(y) \leq 1$ or there exists $v \in t^*$ such that $v(x) > \alpha, v(y) = 0$ which implies that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. Hence by definition (X, s, t) is a pairwise $\alpha - T_0(iii)$ and hence (X, s^*, t^*) is a pairwise $T_0(iv)$. Therefore the proof is complete.

The non-implications among pairwise $\alpha - T_0(i), \alpha - T_0(ii), \alpha -$ T_0(iii) and $T_0(iv)$ are shown in the following examples:

Example 3.1: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.3, u(y) = 0.5$ and $v(x) = 0.5, v(y) = 0.7$. The supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. For $\alpha = 0.8$, we can easily show that (X, s^*, t^*) is a pairwise $T_0(iv)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_0(iii)$. It can also be shown that (X, s^*, t^*) is not a pair wise $\alpha - T_0(i)$ and a pairwise $\alpha - T_0(ii)$.

Example 3.2: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.5, u(y) = 0.8$ and $v(x) = 0.9, v(y) = 0.5$. The supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. For $\alpha = 0.7$, we have $0 \leq u(x) \leq 0.7 < u(y) \leq 1$ or $0 \leq v(y) \leq 0.7 < v(x) \leq 1$. This according to the definition (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_0(i)$. Also, it can be easily shown that (X, s^*, t^*) is not a pairwise $\alpha - T_0(ii)$.

Example 3.3: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.5$ and $v(x) = 0.4, v(y) = 1$. Consider the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. For $\alpha = 0.7$, we have $u(x) = 1$ and $u(y) \leq 0.7$ or $v(x) = 0.7$ and $v(y) = 1$. This according to the definition (X, s^*, t^*) is$a$ pairwise $\alpha - T_0(i)$ but (X, s^*, t^*) not a pairwise $\alpha - T_0(ii)$.

Example 3.4: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0, u(y) = 0.8$ and $v(x) = 0.6, v(y) = 0$. Consider the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. For $\alpha = 0.4$ it can be easily shown that (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$ but (X, s^*, t^*) not pairwise
$\alpha - T_0(i)$. This completes the proof.

Lemma 3.2: Let (X, s^*, t^*) is a supra fuzzy bitopological space and $\alpha, \beta \in I_1$ with $0 \leq \alpha \leq \beta < 1$, then

$(\boldsymbol{a})(X, s^*, t^*)$ is a pairwise $\alpha - T_0(i)$ implies (X, s^*, t^*) is a pairwise $\beta - T_0(i)$.
$(b)(X, s^*, t^*)$ is a pairwise $\beta - T_0(ii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$.
$(c)(X, s^*, t^*)$ is a pairwise $0 - T_0(ii)$ implies (X, s^*, t^*) is a pairwise $0 - T_0(iii)$.

Proof: Suppose (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$. We have to show that (X, s^*, t^*) is a pairwise $\beta - T_0(i)$. Let any two distinct points $x, y \in X$. Since

(X, s^*, t^*) is a pairwise $\alpha - T_0(i)$, for $\alpha \in I_1$, there exists $u \in s^*$ such that $u(x) = 1$ and $u(y) \le \alpha$. This implies that $u(x) = 1$and $u(y) \le \beta$, since $0 \le \alpha \le \beta < 1$ or there exists $v \in t^*$ such that $v(x) \le \alpha, v(y) = 1$. This implies that $v(x) \le \beta, v(y) = 1$, since $0 \le \alpha \le \beta < 1$. Hence by definition (X, s^*, t^*) is a pairwise $\beta - T_0(i)$.

Suppose (X, s^*, t^*) is a pair wise $\beta - T_0(ii)$. Then for $x, y \in X$, $x \ne y$ there exists $u \in s^*$ such that $u(x) = 0$ and $u(y) > \beta$, which implies that $u(x) = 0$ and $u(y) > \alpha$, since $0 \le \alpha \le \beta < 1$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$.

Example 3.5: Let X= $\{x,\ y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.6$ and $v(x) = 0.4, v(y) = 1$. . Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0,\ u,\ 1,\ constants\}$ and $\{0,\ v,\ 1,\ constants\}$ respectively. Then by definition for $\alpha = 0.3$ and $\beta = 0.8$; (X, s^*, t^*) is a pairwise $\beta - T_0(i)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_0(i)$.

Example 3.6: Let X= $\{x,\ y\}$ and $u, v \in I^X$ are defined by $u(x) = 0, u(y) = 0.65$ and $v(x) = 0.40, v(y) = 0$. . Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0,\ u,\ 1,\ constants\}$ and $\{0,\ v,\ 1,\ constants\}$ respectively. Then by definition for $\alpha = 0.35$ and $\beta = 0.75$; (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$ but (X, s^*, t^*) is not a pairwise $\beta - T_0(ii)$.

Theorem 3.1: Suppose (X, S^*, T^*) is a supra fuzzy bitopological space and $\alpha \in I_1$. Suppose the following statements:

(1) (X, S^*, T^*) be a pairwise T_0 space.
(2) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_0(i)$ space.
(3) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_0(ii)$ space.
(4) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_0(iii)$ space.
(5) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_0(iv)$ space.

The following implications are true:

(a) $(1) \Rightarrow (2) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)$.
(b) $(1) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)$.

Proof: Suppose (X, S^*, T^*) be a pairwise T_0 bitopological space. We have to prove that $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_0(i)$ space. Suppose x and y are two distinct elements in X. Since (X, S^*, T^*) be a pairwise T_0space, there exists $U \in S^*$ such that $x \in U, y \notin U$ or there exists $V \in T^*$ such that $y \in V, x \notin V$

By the definition of LSC, we have $I_U \in \omega(S^*) and\ I_U(x) = 1, I_U(y) = 0$ or $I_V \in \omega(T^*)$ and $I_V(x) = 0, I_V(y) = 1$.

Hence we have $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_0(i)$. Further it is easy to show that $(2) \Rightarrow (3)$, $(3) \Rightarrow (4)$ and $(4) \Rightarrow (5)$. We, therefore, prove that $(5) \Rightarrow (1)$

Suppose $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_0(iv)$ space. We have to prove that (X, S^*, T^*) be a pairwise T_0 space. Let $x, y \in X$, and $x \ne y$. Since $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_0(iv)$, there exists $u \in \omega(S^*)$ such that $u(x) < u(y)$ or $u(x) > u(y)$ or there exists $v \in \omega(T^*)$ such that $v(x) < v(y)$ or $v(x) > v(y)$. Suppose $u(x) < u(y)$ for $r_1 \in I_1$ such that $u(x) < r_1 < u(y)$. We observe that $x \notin u^{-1}(r_1, 1)$and $y \in u^{-1}(r_1, 1)$. By definition of LSC $u^{-1}(r_1, 1) \in S^*$. Suppose

$v(y) < v(x)$ and for $r_2 \in I_1$, such that $v(y) < r_2 < v(x)$. We observe that $y \notin v^{-1}(r_2, 1)$ and $x \in v^{-1}(r_2, 1)$ and by the definition of LSC $v^{-1}(r_2, 1) \in T^*$. Hence (X, S^*, T^*) be a pair wise T_0 space. Thus it seen that pair wise $\alpha - T_0(p)$ is a good extension of its bitopological counterpart(p=I, ii, iii, iv).

Theorem 3.2: Let (X, s^*, t^*) be a supra fuzzy bitopological space, $\alpha \in I_1$ and let $I_\alpha(s^*) = \{u^{-1}(\alpha, 1): u \in s^*\}$ and $I_\alpha(t^*) = \{v^{-1}(\alpha, 1): v \in t^*\}$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0.
(b) (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0.
(c) (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$ if and only if $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0.

Proof: (a) Let (X, s^*, t^*) be a supra fuzzy bitopological space and (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$. Suppose x and y be two distinct elements in X. Then for $\alpha \in I_1$, there exists $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$. Since $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $y \notin u^{-1}(\alpha, 1)$, and $x \in u^{-1}(\alpha, 1)$ or there exists $v \in t^*$ such that $v(x) \leq \alpha, v(y) = 1$. Since $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$, $x \notin v^{-1}(\alpha, 1)$, , and $y \in v^{-1}(\alpha, 1)$. So this implies that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0.

Similarly (b) can be proved.
(c)Suppose (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$. Let $x, y \in X, x \neq y$, then for $\in I_1$, there exists $u \in s^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$. Since $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $x \notin u^{-1}(\alpha, 1)$ and $y \in u^{-1}(\alpha, 1)$ or there exists $v \in t^*$ such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. Since $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$, $y \notin v^{-1}(\alpha, 1)$, and $x \in v^{-1}(\alpha, 1)$. So this implies that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0 space.
Conversely suppose that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0 space. Let $x, y \in X, x \neq y$. Then there exists $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$ such that $x \in u^{-1}(\alpha, 1)$ and $y \notin u^{-1}(\alpha, 1)$, where $u \in s^*$. Thus we have $u(x) > \alpha, u(y) \leq \alpha$. i.e. $0 \leq u(y) \leq \alpha < u(x) \leq 1$ or there exists $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$ such that $x \notin v^{-1}(\alpha, 1)$ and $y \in v^{-1}(\alpha, 1)$, where $v \in s^*$. Thus we have $v(x) \leq \alpha, v(y) > \alpha$. i.e. $0 \leq v(x) \leq \alpha < v(y) \leq 1$. This implies that (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$

Example 3.7: Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.7, u(y) = 0$ and $v(x) = 0.4, v(y) = 1$. . Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. Then by definition for $\alpha = 0.5$ (X, s^*, t^*) is not a pairwise $\alpha - T_0(ii)$. Now let $I_\alpha(s^*) = \{X, \varphi, \{x\}\}$ and let $I_\alpha(t^*) = \{X, \varphi, \{y\}\}$. Then we see that $I_\alpha(s^*)$ and $I_\alpha(t^*)$ are supra topology on X and $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_0 space. This completes the proof.

Theorem 3.3: Let (X, s^*, t^*) be a supra fuzzy bitopological space. $A \subseteq X$ and $s_A^* = \{u/A: u \in s^*\}$ and $t_A^* = \{v/A: v \in t^*\}$,then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_0(i)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_0(ii)$

(c) (X, s^*, t^*) is$a$ pairwise $\alpha - T_0(iii)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - (iii)$

(d) (X, s^*, t^*) is a pairwise $\alpha - T_0(iv)$ implies (A, s_A^*, t_A^*) is a pairwis $\alpha - T_0(iv)$

Proof: (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$. Let $x, y \in A$ with $x \neq y$. So that $x, y \in X$ as $A \subseteq X$. Then for $\alpha \in I_1$, there exists $u \in s^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$. For $A \subseteq X$, we have $u/A \in s_A^*$ and $0 \leq (u/A)(x) \leq \alpha < (u/A)\,(y) \leq 1$ as $x, y \in A$ or there exists $v \in t^*$ such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. For $A \subseteq X$, we have $v/A \in t_A^*$ and $0 \leq (v/A)(y) \leq \alpha < (v/A)\,(x) \leq 1$ as $x, y \in A$. Hence by definition (A, s_A^*, t_A^*) is a pairwise $\alpha - T_0(iii)$. Similarly, we can prove (a), (b) and (d).

Theorem 3.4: Suppose $\{\,(X_i, s_i^*, t_i^*), i \in \Lambda\}$ is a family of supra fuzzy bitopological spaces and $(\prod X_i, \prod s_i^*, \prod t_i^*) = (X, s^*, t^*)$ be the product topological space on X, then

(a) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_0(i)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$.

(b) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_0(ii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_0(ii)$.

(c) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_0(iii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_0(iii)$.

(d) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_0(iv)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_0(iv)$.

Proof: Suppose $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_0(i)$. Let $x, y \in X$ with $x \neq y$, then $x_i \neq y_i$, for some $i \in \Lambda$. Since (X_i, s_i^*, t_i^*) is α pairwise $- T_0(i)$, for $\alpha \in I_1$, there exists $u_i \in s_i^*$, $i \in \Lambda$ such that $u_i(x_i) = 1, u_i(y_i) \leq \alpha$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $u_i(\pi_i(x)) = 1$ and $u_i(\pi_i(y)) \leq \alpha$ i.e. $(u_i o \pi_i)(x) = 1, (u_i o \pi_i)(y) \leq \alpha$ or there exists $v_i \in t_i^*$, $i \in \Lambda$ such that $v_i(x_i) \leq \alpha\ and\ v_i(y_i) = 1$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $v_i(\pi_i(x)) \leq \alpha$ and $v_i(\pi_i(y)) = 1$ i.e., $(v_i o \pi_i)(x) \leq \alpha$, $(v_i o \pi_i)(y) = 1$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$

Conversely, suppose that (X, s^*, t^*) is a pair wise $\alpha - T_0(i)$. We have to show that (X_i, s_i^*, t_i^*) , $i \in \Lambda$ is$a$ pairwise $\alpha - T_0(i)$. Let a_i be a fixed point in X_i and $A_i = \left\{x \in X = \prod_{i \in \Lambda} X_i : x_j = a_{i\ ,}\ for\ some\ i \neq j\right\}$. Thus A_i is a subset of X and hence $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a subspace of (X, s^*, t^*). Since (X, s^*, t^*) is a pairwise $\alpha - T_0(i)$, $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a pairwise $\alpha - T_0(i)$. Now we have A_i is a homeomorphic image of X_i. Thus (X_i, s_i^*, t_i^*) , $i \in \Lambda$ is a pairwise $\alpha - T_0(i)$.
Similarly (b), (c) and (d) can be proved.

Theorem 3.5: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) be two supra fuzzy bitopological spaces. $f: X \to Y$ be one-one, onto and open map, then

(a) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(i)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(i)$.
(b) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(ii)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(ii)$.
(c) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(iii)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(iii)$.
(d) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(iv)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(iv)$.

Proof:(b) Suppose (X, s_1^*, t_1^*) is a pair wise $\alpha - T_0(ii)$. Then for $y_1, y_2 \in Y$ with $y_1 \neq y_2$, there exist $x_1, x_2 \in X$ with $f(x_1) = y_1, f(x_2) = y_2$, since f is onto and $x_1 \neq x_2$ as f is one-one. Again since (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(ii)$, $\alpha \in I_1$, there exists $u \in s_1^*$ such that $u(x_1) = 0, u(y_1) > \alpha$ or there exists $v \in t_1^*$ such that $v(x_1) > \alpha, v(y_1) = 0$.

Now $f(u)(y_1) = \{\sup u(x_1) : f(x_1) = y_1\}$
$=0$; otherwise
and $f(u)(y_2) = \{\sup u(x_2) : f(x_2) = y_2\}$
$> \alpha$; otherwise
Now $f(v)(y_1) = \{\sup v(x_1) : f(x_1) = y_1\}$
$>\alpha$; otherwise
and $f(v)(y_2) = \{\sup v(x_2) : f(x_2) = y_2\}$
$= 0$; otherwise

Since f is open, $f(u) \in s_2^*$ as $u \in s_1^*$. We observe that there exists $f(u) \in s_2^*$ such that $f(u)(y_1) = 0, f(u)(y_2) > \alpha$ or there exists $f(v) \in t_2^*$, as $v \in t_1^*$ and f is open such that $f(v)(y_1) > \alpha, f(v)(y_2) = 0$. Hence by definition (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(ii)$. Similarly (a), (c), (d) can be proved.

Theorem 3.6: Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) be two supra fuzzy bitopological spaces. $f: X \to Y$ be a continuous and one-one map, then

(a) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(i)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(i)$.
(b) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(ii)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(ii)$.
(c) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(iii)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(iii)$.
(d) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_0(iv)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_0(iv)$.

Proof: (c) Suppose (Y, s_2^*, t_2^*) is pairwise $\alpha - T_0(iii)$. Then for $x_1, x_2 \in X$ and $x_1 \neq x_2$, we have $f(x_1) \neq f(x_2)$, since f is one-one. Also since (Y, s_2^*, t_2^*) is pairwise $\alpha - T_0(iii)$, for $\alpha \in I_1$ there exists $u \in s_2^*$ such that $0 \leq u(f(x_1) \leq \alpha < u(f(x_2) \leq 1$.This implies that $0 \leq f^{-1}(u)(x_1) \leq \alpha < f^{-1}(u)(x_2) \leq 1$.Since $u \in$

s_2^* and f is continuous, $f^{-1}(u) \in s_1^*$. Thus there exists $f^{-1}(u) \in s_1^*$ such that $0 \leq f^{-1}(u)(x_1) \leq \alpha < f^{-1}(u)(x_2) \leq 1$ or there exists $v \in t_2^*$ such that $0 \leq v(f(x_2) \leq \alpha < v(f(x_1) \leq 1$. Since $v \in t_2^*$ and f is continuous, $f^{-1}(v) \in t_1^*$. Thus there exists an, $f^{-1}(v) \in t_1^*$ such that $0 \leq f^{-1}(v)(x_2) \leq \alpha < f^{-1}(v)(x_1) \leq 1$. Hence by definition (X, s_1^*, t_1^*) is pairwise $\alpha - T_0(iii)$. Similarly (a), (b) and (d) can be proved.

Conflict of Interest:

There is no conflict of interest regarding this article

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