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A PIONEERING AND COMPREHENSIVE DATABASE OF BALANCED AND UNBALANCED TRANSPORTATION PROBLEMS FOR READY PERFORMANCE EVALUATION OF EXISTING AND NEW METHODS

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Abstract

In this paper, we present a comprehensive database of the data tables of some important transportation problems from literature, and experience with the proposition of new initial basic feasible (IBF) solution methods for the transportation problems. The paper contains a comprehensive database of 140 transportation problems, of which 103 are balanced, 25 are unbalanced and 12 are from research papers. The detailed description of the varying-nature test problems is described, and the optimal solutions to the 140 problems have been obtained by using the TORA software with the modified distribution (MODI) method. The algorithms of three methods: North-West-Corner (NWCM), Least cost (LCM) and Vogel's approximation (VAM) have been used for IBF solutions. The final optimal results are also quoted for the ready reference of researchers and practitioners. The database of problems and their optimal solutions will be a great aid to researchers and practitioners working with existing and new methods for solving transportation problems. A pioneering investigation of the performance evaluation of NWCM, LCM and VAM has also been conducted as a benchmark for the ailar assessment of other existing and forthcoming IBF and /or optimal solution methods for the transportation problems.

Keywords: Transportation problem, optimal solution, MODI method, TORA software, Minimum cost, performance evaluation.

I. Introduction

Optimization methods and their refinements [XIII], [XXV] are important in mathematical research because many applied problems demand maximization or minimization to improve the output of the systems arising in various fields of science and engineering [II], [XXIX]. Generally, a transportation problem attempts to minimize the value of objective function – which represents the cost of transporting goods from sources to destinations – subject to linear constraints describing the nature of decision variables, the capacity of sources and the demand of destinations. The number of sources and their capacity, number of destinations and their demands, and the cost of transporting the goods/material from each of the sources to each of the destinations should be known. Such data enables the definition of the mathematical model of a transportation problem, which is also known as a linear programming problem [XXIX]. The values of decision variables are found in a way that minimizes the total transportation cost. The model is considered balanced if total supply and demand are equal, otherwise unbalanced. A demanding task of the present time is to assure maximum achievement with minimum cost and in minimum time. The methods for solving transportation problems attempt to minimize the cost of transporting goods or material from sources to destinations [XXV], [XXVIII], [XX].

Due to advancements in mathematical research, several new methods have been proposed by researchers for solving transportation problems, for example, [XI], [VII], [VI], [XVII], [XXIII], [XXII], [IV], [VIII], [XXIV], [I], [XXVII], [XXI], [XVIII], [XVII], [XIV], [XV], [V], [IX], [X], [XXIX]. The performance of the ancient methods [XXV] and the new methods must be compared using a set of standard transportation problems. Such a database of transportation problems can be considered as a test database for forthcoming studies in the field of initial basic feasible and optimal solution methods for transportation problems.

In this research work, we provide a test database of transportation problems. The comprehensive database of cost tables of 140 transportation problems to support the findings of this article can be accessed at [XIX]. In addition to the database, the optimal solutions of these 140 problems have been obtained using the standard modified distribution (MODI) method for the help of researchers and practitioners. The algorithmic details of the procedure and the software implementation are also discussed.

The worked-out database of transportation problems presented with this article along with the optimal solutions will serve to be a benchmark for future researches on new methods for solving the transportation problems. This study is going to be a direct and useful aid in assessing the accuracy of existing and new methods for finding initial basic feasible (IBF) or optimal solution of transportation problems. The optimal solutions presented here can save the time of researchers to solve the problems again and again in real-time to make comparisons.

II. Material and Methods

The general form of the transportation problem (T.P) table is described in Table 1 where S_i, D_j, A_i, B_j and C_{ij} represent sources, destinations, supplies from sources, demands at the destinations, and cost coefficients from transporting material from S_i to D_j, respectively and $i = 1,2,\ldots, m$; $j = 1,2,\ldots,n$.

The objective is to transport a certain amount of units of products/goods from sources to destinations in such an efficient way that the source capacities and destination demands are exhausted with the minimum possible cost of transportation.

Table 1: General form of transportation problem array

Destinations / Sources	"n" destinations D_1 to D_n	Supply of sources
"m" sources S_1 to S_m	Cost matrix $C_{mn} = [C_{ij}]$ i=1,2,…,m j=1,2,…,n	A_1 to A_m
Demand of destinations	B_1 to B_n	Balanced model if total supply = total demand otherwise, unbalanced model.

The optimal solution to the T.P described in Table 1 can be attained using the modified distribution (MODI) method iteratively by using the IBF solutions as starting guesses. IBF solutions are obtained by North-West-Corner method (NWCM), Least cost method (LCM) and Vogel's approximation method (VAM) traditionally [XXV], [XXVIII], [XX]. For the detailed performance evaluation of the traditional methods and the forthcoming methods, the algorithmic details of the method are provided before the executions and results.

Algorithm of NWCM

The NWCM provides a solution that is not close to the optimal solution. Though, it gives a quick solution. The steps NWCM 1-5 are defined as:

NWCM 1. Allocate minimum units from supply and demand to the upper left corner cell.

NWCM 2. Subtract allocated units to the upper left corner cell from supply or demand.

NWCM 3. If the requirement of the upper left corner cell is fulfilled, then continue the first row.

NWCM 4. If the first row is satisfied, go to the first cell of the 2nd row.

NWCM 5. If supply equals demand, the allocation may be done subsequently row or column.

Algorithm of LCM

LCM 1. Select the cell with the least unit transportation cost.

LCM 2. Allocate minimum units of product from supply and demand.

LCM 3. Delete satisfied row or column. If both the row and column are fulfilled, remove row or column, assign zero units to supply or demand.

LCM 4. Continue, till supply and demand conditions are satisfied.

Algorithm of VAM

The VAM determines the IBF solution for the T.P by considering the penalty of two smaller numbers in each row and column.

VAM 1. Balance the transportation model.

VAM 2. Find row and column penalty. Write penalty before each row and column.

VAM 3. Select the row or column with the maximum penalty. Select any one of them if a tie occurs.

VAM 4. Allocate minimum units to that cell having minimum cost in that row or column.

VAM 5. If row or column is satisfied, delete any one of them, and assign zero units to supply and demand.

VAM 6. Continue, till demand and supply rules are obeyed.

VAM 7. Calculate the minimum transportation cost.

Algorithm of optimal MODI

MODI 1. Find an initial solution of a T.P by an IBF method: NWCM, LCM, VAM or any other.

MODI 2. If the given result is optimal, stop the process otherwise go to the next step.

MODI 3. Compute u_i and v_j values for occupied cells using $x_{i\,j} = u_i + v_j - c_{ij}$ and also compute the cost change C_{ij} for empty cells using , $x_{ij} : c_{ij} = c_{ij} - (u_i + v_j)$

MODI 4. Allocate maximum possible units to the non-basic variable that will decrease in the result.

MODI 5. Repeat steps 2 to 4 until all $c_{ij} \geq 0$

The algorithmic details in this section are directly related to the forthcoming results in section IV on the datasets described in section III, and main contributions of the present research for providing a ready and accurate worked-out database of test T. Ps and optimal solutions for practitioners and future studies pertaining to this field of research. The dataset of 140 test T. Ps is described in section III.

III. Dataset Description

The comprehensive dataset comprising of 140 transportation problems is given [XIX]. The structure of problems is consistent with the general form given in Table 1 here. Of the included problems in the database, Problems 1-103 is balanced, Problems 104-128 are unbalanced and Problems 129-140 are from research papers.

The size and nature of the transportation array of Problems 1-103 (balanced), Problems 104-128 (unbalanced) and Problems 129-140 (from research papers) are described in Tables 2-4, respectively. We used most of the problems in Tables 2 and 3 in current projects to propose new and efficient methods for IBF solution of transportation problems [X], and some forthcoming. However, Problems 19 and 68 are taken from [XXI], Problems 10, 17 and 52 from [XII], Problems 41, 80 and 127 from [XXVIII], and Problems 98 from [XXIII]. The references for problems 129-140 are also given in Table 4.

IV. Results and Discussion

The NWCM, LCM and VAM have been used according to the algorithms through the software TORA on 140 T.Ps to get the IBF solutions, and then the optimal solutions are attained through the MODI method. Using the software environment, the optimal solutions attained for the 140 test problems T.P 1-140 described in Tables 2-4, are summarized in Table 5.

Table 2: Test Problems 1-103 (balanced) with size

T.P	Sources	Destinations	T.P	Sources	Destinations	T.P	Sources	Destinations
1.	3	3	36.	3	4	70.	3	3
2.	3	3	37.	3	4	71.	4	5
3.	3	3	38.	3	4	72.	3	3
4.	3	3	39.	3	4	73.	3	4
5.	3	3	40.	3	4	74.	4	4
6.	3	3	41.	3	4	75.	4	4
7.	3	3	42.	3	4	76.	5	5
8.	3	3	43.	3	4	77.	3	4
9.	3	3	44.	3	4	78.	3	4
10.	3	3	45.	3	4	79.	3	4
11.	3	3	46.	3	4	80.	3	4
12.	3	3	47.	3	4	81.	5	4
13.	3	3	48.	3	4	82.	3	4
14.	3	3	49.	3	3	83.	3	3
15.	3	3	50.	3	4	84.	4	3
16.	3	3	51.	3	4	85.	3	3
17.	3	4	52.	3	4	86.	3	3
18.	3	4	53.	3	4	87.	3	3
19.	3	4	54.	3	4	88.	3	3
20.	3	4	55.	4	3	89.	3	4
21.	3	4	56.	3	5	90.	4	5
22.	3	4	57.	4	5	91.	3	3
23.	3	4	58.	4	5	92.	3	3
24.	3	4	59.	4	3	93.	4	5
25.	3	5	60.	4	4	94.	4	3
26.	3	4	61.	4	5	95.	4	3
27.	3	4	62.	4	5	96.	4	6
28.	3	4	63.	3	3	97.	3	3
29.	3	4	64.	6	6	98.	4	6
30.	3	4	65.	3	4	99.	3	4
31.	3	4	66.	3	4	100.	5	3
32.	3	4	67.	3	3	101.	4	4
33.	3	4	68.	3	3	102.	5	4
34.	3	4	69.	3	3	103.	10	10
35.	3	4						

Table 3: Test Problems 104-128 (unbalanced) with size

T.P	Sources	Destinations	T.P	Sources	Destinations	T.P	Sources	Destinations
104.	3	4	113	3	4	121	4	3
105	3	4	114	3	5	122	4	4
106	3	4	115	3	6	123	4	3
107	3	4	116	3	5	124	4	5
108	3	4	117	3	5	125	4	3
109	3	4	118	4	4	126	3	4
110	3	4	119	4	3	127	4	5
111	3	5	120	4	5	128	4	3
112	3	5						

Using the exhaustive database [XVII] of test T.Ps for the performance evaluation of traditional IBF solution methods NWCM, LCM and VAM, and any other existing and forthcoming methods the results of this study play a vital role since ready solutions are available with this study.

Table 4: Test Problems 129-140 (research papers) with size

T.P	Sources	Destinations	Reference	T.P	Sources	Destinations	Reference
129	3	4	[XVI]	135	4	3	[XVIII]
130	3	4	[V]	136	3	4	[III]
131	5	5	[XI]	137	3	4	[III]
132	3	5	[III]	138	3	3	[III]
133	3	4	[XXIV]	139	4	4	[III]
134	3	4	[XXIV]	140	3	5	[III]

The performance of any IBF solution method in literature and forthcoming studies can therefore be instantly checked through the main results of this research. Here, we focus the interpretation on the traditional and conventional methods: NWCM, LCM and VAM only for brevity. Observing the IBF solutions acquired by the NWCM, LCM and VAM versus the optimal MODI method solutions to T. Ps 1-140, it appears that only for a very few problems, i.e. 6 for NWCM, 26 for LCM and 62 for VAM out of the 140 in total the results were optimal. In Fig. 1 the percentage of optimal and non-optimal cases of solutions observed by the conventional methods have been compared. From Fig. 1, it is clear that the conventional VAM although leads among the traditional methods, but it in many instances couldn't obtain the optimal solution. Although these methods are not at all optimal solution methods, the recent works are devoted to propose new methods resulting in direct optimal or very close approximations to the optimal solution [IX], [X], etc. For example, recently in [X], the minimum demand method (MDM) was suggested to be a better and efficient alternative to the VAM method. The absolute error in the problems where the results have not been optimal, are positive and are shown in Fig.2 for the NWCM, LCM and VAM. It can be seen that the VAM absolute errors are comparatively not too large than the other two methods for the corresponding non-optimal cases.

Like the example, performance evaluation of the conventional methods: NWCM, LCM and VAM through the exhaustive and worked-out database available with this article for all 140 problems in [XIX], the other existing and new methods for IBF solution or direct optimal solution of the transportation problems can be readily examined with the contributions of this work. In the future, the focus may be given to the extension of existing software environments for the transportation problems to include other new and efficient IBF solution methods whose ascending performance is well-established in the recent literature of transportation problems.

Table 5: Optimal solutions of test Problems 1-140 by MODI method

T.P	Optimal solution	T.P	Optimal solution	T.P	Optimal solution	T.P	Optimal solution
1.	24	36.	920	71.	390	106	980
2.	35	37.	730	72.	830	107	4450
3.	20	38.	235	73.	47050	108	1140
4.	62	39.	44100	74.	2484	109	920
5.	4525	40.	102	75.	410	110	735
6.	505	41.	89	76.	59356	111	1620
7.	2350	42.	7350	77.	12100	112	68
8.	15650	43.	3400	78.	776	113	80
9.	380	44.	772	79.	510	114	740
10.	1200	45.	2595	80.	995	115	17300
11.	1130	46.	2221	81.	2544	116	4840
12.	1350	47.	799	82.	105	117	630
13.	14150	48.	47	83.	20550	118	1160
14.	143	49.	173	84.	74	119	515
15.	143	50.	559	85.	146	120	13650
16.	173	51.	2365	86.	21	121	790
17.	743	52.	412	87.	18	122	43476
18.	610	53.	960	88.	130	123	4525
19.	3460	54.	674	89.	90	124	9200
20.	506	55.	76	90.	327	125	2750
21.	886	56.	172	91.	228	126	1650
22.	118	57.	1680	92.	1854	127	6445
23.	2100	58.	420	93.	54	128	13695
24.	10730	59.	5950	94.	760	129	412
25.	11500	60.	184	95.	76	130	743
26.	140	61.	267	96.	270	131	59356
27.	10680	62.	695	97.	745	132	80
28.	796	63.	490	98.	120	133	610
29.	635	64.	6400	99.	261	134	3460
30.	3100	65.	2850	100.	377	135	76
31.	56	66.	3320	101.	968	136	506
32.	12220	67.	1390	102.	381	137	200
33.	7350	68.	555	103.	62500	138	148
34.	5300	69.	625	104.	2424	139	180
35.	900	70.	590	105	3300	140	172

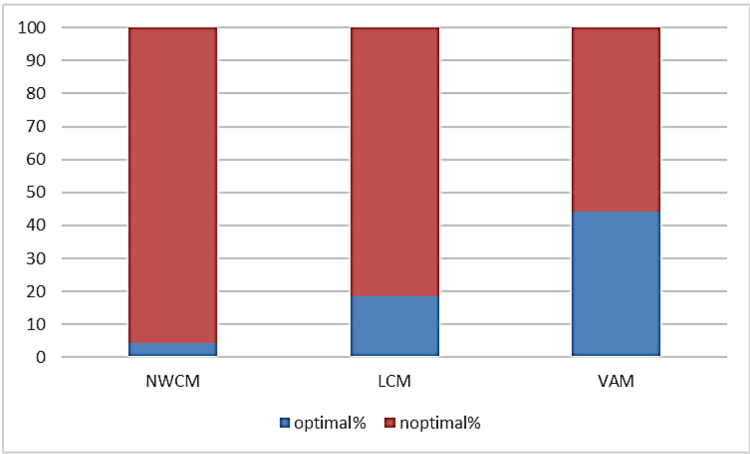

Figure 1. Optimal and non-optimal % comparison of IBF solution methods versus MODI optimal solutions

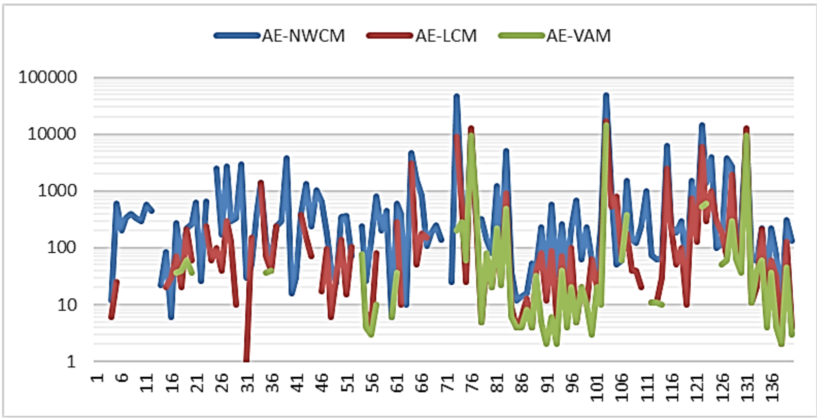

Figure 2. Absolute errors in logarithmic scale between IBF solutions and MODI for non-optimal T.Ps

V. Conclusion

This work focuses on the performance evaluation of the solution methods for transportation problems. In this regard, a pioneering and worked-out attempt has been made to create luxuries in future research projects in the field by presenting a ready and comprehensive database of 140 T.Ps from our recent projects and literature. The 140 test problems and their features have been discussed. The IBF solutions of all problems have been obtained through NWCM, LCM and VAM, whereas the optimal ones using the MODI method. TORA software with the discussed algorithmic details of the methods has been used for the results. On one hand, the optimal solutions of the test database of 140 balanced, as well as unbalanced T.Ps, serves to be a ready means of testing the performance of any existing and forthcoming IBF or optimal solutions methods. On the other hand, the presented performance evaluation procedure has been successfully carried out on the conventional methods, and the procedure may be used in future studies as the reference protocol for the performance evaluation of new methods in the field of transportation research.

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Conflict of Interest:

There is no conflict of interest regarding this article

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