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THE FLOW OF DUSTY VISCO-ELASTIC FLUID BETWEEN TWO PARALLEL FLAT PLATES.

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Abstract

The flow of dusty visco-elastic fluid between two parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane is considered because of its growing importance in various technical problems in modern applied science.

In this paper, we would like to consider the laminar flow of visco-elastic fluid containing uniformly small solid particles between two infinitely extended parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane. The analytical expressions for velocity fields of fluid and dust particles are obtained which are in elegant forms. The effects of elastic elements in the fluid, the mass concentration, and the relaxation time of dust particles on the velocity profiles are studied in detail. The skin friction at the lower plate wall and the total volume flow in between the plates are also obtained.

Keywords: Dusty fluid, visco-elastic fluid, laminar flow, elastic element, harmonic oscillation.

I. Introduction

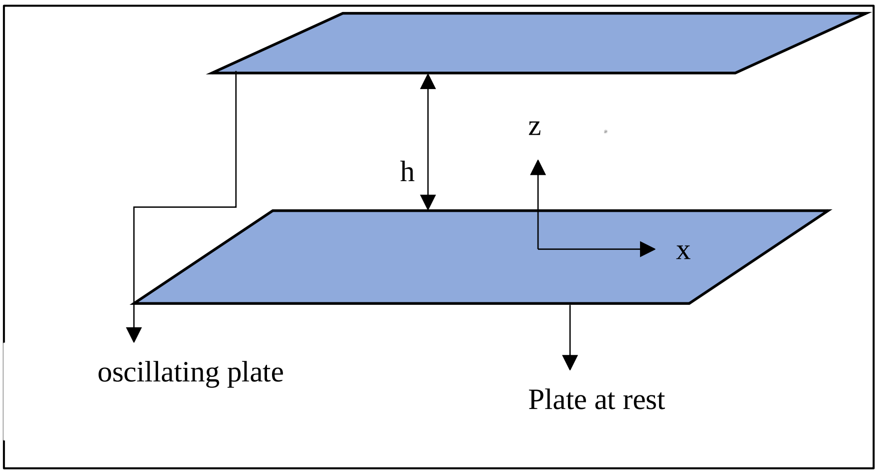
In recent years, we have lot of interest in problems of flows of a dusty gas that is a mixed system of fluid and particles. A model equation describing the motion of such a mixed system has been given by P. G. Saffman (IX). Based on Saffman's model, a large number of famous authors investigated several dusty gas flow problems in various scientific areas. In recent years, the study of non-Newtonian fluids has received special attraction under a wide range of geometrical, rheological, and dynamical conditions. A few examples are the flow of nuclear fuel slurries, the flow of liquid metals and alloys such as the flow of gallium metal (melting point 302.5K) at ordinary temperatures (303K), the flow of plasma, the flow of mercury amalgams, handling of biological fluids, the flow of blood, a Bingham fluid with some thixotropic behaviours, coating of paper, petroleum production, plastic extrusion, molten paper pulp, emulsion, paints, lubrication with heavy oils and greases, aqueous solutions of polyacrylamide and polyisobutylene, etc, as important raw materials and chemical products in a large variety of industrial processes. The subject of Rheology is of great technological importance as in many branches of modern industry, the problem arises of designing apparatus to transport or process substances that cannot be governed by the classical stress-strain velocity relations. Visco-Elastic fluids are particular cases of non-Newtonian fluids that exhibit appreciable elastic behaviour and stress-strain velocity relations are time-dependent. Many common liquids such as oils, certain paints, polymer solutions, some organic liquids, and many new materials of industrial importance exhibit both viscous and elastic properties. Though the above fluids, called visco-elastic fluids, are also being studied extensively.

Saffman has expressed model equations describing the influence of dust particles on the motion of fluids. Several authors using the equations of Saffman have investigated several dusty gas flow problems in different situations. Kapur(V) investigated the problem of two immiscible viscous liquids between two fixed parallel plates under a certain pressure gradient. The flow of visco-elastic Maxwell liquid down an inclined plane was investigated by Bagchi and Maiti(I). The unsteady flow of two immiscible visco-elastic conducting liquids between two inclined parallel plates has been studied by Lahiri and Ganguly(VI). Mandal et al (VII) have considered unsteady flow of dusty visco-elastic (kuvshiniski type) liquid between two oscillating plates. Ghosh and Debnath (II) focused on unsteady hydromagnetic flows of a dusty viscous fluid between two oscillating plates. Johari et al (IV) have studied the MHD flow of a dusty Visco-elastic (kuvshiniski type) liquid past in an inclined plane. Ghosh, Debnath (III) investigated the hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates. Singh et al (X) have studied the MHD flow of a dusty visco-elastic liquid past on an inclined plane. Prakash, Kumar, and Dwivedi (VIII) discussed the MHD free convection flow of a viscoelastic (Kuvshiniski type) dusty gas through a semi-infinite plate moving with velocity decreasing exponentially with time.

II. Mathematical Formulation of the Problem and its Solution:

Let us suppose that the dusty visco-elastic fluid fills the region between two horizontal infinite parallel flat plates at a distance h apart. The lower plate is kept at rest and the upper one begins to perform harmonic oscillations with a frequency in its own plane. The physical model is shown in figure-1. The present analysis takes a coordinate system such that the x-axis coincides with the lower fixed plate and the z-axis is perpendicular to it.

Figure-1



The dust particles are assumed to be spherical in shape and uniform in size and the number density of dust particles is taken as constant throughout the flow and be ρ_0. Since the plates are infinite, the velocity will depend on z and time t mainly. For the constitutive equation, we adopt Kuvshiniski type fluid, given by

$$P_{ij} = -p\delta_{ij} + p'_{ij} \quad (1)$$

$$\left(1 + \lambda_0 \frac{D}{Dt}\right) p'_{ij} = 2\mu\, e_{ij} \quad (2)$$

$$\frac{D}{Dt} p'_{ij} = \frac{\partial p'_{ij}}{\partial t} + u_m \frac{\partial p'_{ij}}{\partial x_m} \quad (3)$$

$$2e_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \quad (4)$$

Where P_{ij} is stress tensor and p'_{ij} the deviatoric stress tensor, $\frac{D}{Dt}$ is the convective time derivative following a fluid element and u_i is the velocity of the fluid particle. Here λ_0 and μ denote the elastic coefficient and viscosity of the fluid. Using the above equations, we get the equation of motion of a dusty visco-elastic fluid (dropping dashes)

$$\left(1 + \alpha \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = R \frac{\partial^2 u}{\partial z^2} + \frac{f}{\tau}\left(1 + \alpha \frac{\partial}{\partial t}\right)(v - u) \quad (5)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u - v) \quad (6)$$

Here $u' = \frac{u}{h\omega}$; $v' = \frac{v}{h\omega}$; $t' = \omega t$; $z' = \frac{z}{h}$; $\alpha = \lambda_0 \omega$; $R = \frac{\gamma}{h^2 \omega}$

Where v is the velocity of a dusty particle.

f = mass concentration = $\frac{mB_0}{\rho}$

τ = relaxation time = $\frac{m\omega}{K}$

The relevant initial and boundary conditions in non-dimensional form are

$$t \le 0$$

$$u = \frac{\partial u}{\partial t} = 0 \ \ for\ all\ \ z \tag{7}$$

$$t > 0$$

$$u = a\ Sint \quad at\ \ z = 1$$

$$u = 0 \quad at\ \ z = 0 \tag{8}$$

Taking $u = u_z(z)e^{-bt}$, $v = v_z(z)e^{-bt}$, $b > 0$ (9)

Equations (5) and (6) takes the form

$$R\frac{d^2u_z}{dz^2} + b(1 - b\alpha)u_z + \frac{f}{\tau}(1 - b\alpha)(v_z - u_z) = 0 \tag{10}$$

and

$$v_z = \frac{1}{1-b\tau}u_z \tag{11}$$

Boundary conditions are (t > 0)

$$u_z = ae^{bt}\sin t \qquad at\ \ z = 1 \tag{12}$$

$$u_z = 0 \quad at \quad z = 0$$

By substituting equation (11) into equation (10) we can get

$$\frac{d^2u_z}{dz^2} - M^2u_z = 0 \tag{13}$$

Here

$$M^2 = \frac{(b\alpha-1)bF}{R(1-b\tau)} \quad \text{and} \quad F = 1 - b\tau + bf$$

Thus, the solution to the equation (13) is

$$u_z = A\cos hMz + B\sin hMz \tag{14}$$

Using boundary conditions (12) we can get

$$u_z = \frac{ae^{bt}\sin t}{\sin hM}\sin hMz \qquad for\ all\ z \tag{15}$$

$$v_z = \frac{ae^{bt}\sin t}{(1-b\tau)\sin hM}\sin hMz \tag{16}$$

Therefore, velocity profile of dusty fluid,

$$u = \frac{a\sin t}{\sin hM}\sin hMz\ , \quad t > 0 \tag{17}$$

Velocity profile of dust particles

$$v = \frac{a\sin t}{(1-b\tau)\sin hM}\sin hMz \quad , \quad t > 0 \tag{18}$$

The dimensionless shearing stress i.e., skin friction (τ_p) at the lower plate due to the dusty visco-elastic fluid is

$$\tau_p = \left[\left(1 - \alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial z}\right] \quad z = 0$$

Or, $\tau_p = \frac{aM}{\sin hM}(\sin t - \alpha\cos t)$

Or, $\tau_p = \frac{a(\sqrt{k^2+1})M}{\sin hM}\sin(t - \theta_k)$, $\tan\theta_k = k$, *here* $(k = 1,2,3,\ldots.)$

The volume flow (ϕ) of dusty Visco-elastic fluid discharged per unit breadth of the plate is given by

$$\phi = 2\int_0^1 udz = \frac{2a\sin t}{M\sin hM}(\cos hM - 1)$$

For very small harmonic oscillations, velocity profile of the dusty visco-elastic fluid maintains the inequalities

$$\sum_{k=1}^{\infty}\frac{(Mz)^{2k+1}}{(2k+1)!} \geq 0 \qquad or \qquad \sum_{k=1}^{\infty}\frac{(M)^{2k+1}}{(2k+1)!} \leq 0$$

For very large harmonic oscillations, velocity profile of the dusty fluid maintains the inequalities

$$\sum_{k=1}^{\infty}\left(\frac{(Mz)^{2k+1}}{(2k+1)!} + \frac{(M)^{2k+1}}{(2k+1)!}\right) \geq 0 \qquad or \qquad \sum_{k=1}^{\infty}\left(\frac{(Mz)^{2k+1}}{(2k+1)!} - \frac{(M)^{2k+1}}{(2k+1)!}\right) \leq 0$$

III. Results and Discussion

The present analysis reveals that the solution of the given problem contains four pertinent non-dimensional parameters viz α (elastic parameter of the fluid particles), τ (relaxation time of dust particles), f (mass concentration of dust particles), and t (t is represented by ωt, harmonic oscillation). Behaviours of these parameters yield a physical insight into the problem. Numerical computation is made to observe the effects of these parameters on the velocity profile. Also, obtain the expression of skin friction at the lower plates and volume flow in between the plates.

Figure -1 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when τ and f are fixed at $\alpha = 1$.

Figure -2 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when τ and f are fixed at $\alpha = 2$.

Figure -3 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when τ and f are fixed at $\alpha = 3$.

Figure -4 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when τ and f are fixed at $\alpha = 4$.

IV. The Conclusions of the Study

(i) The velocity of the dusty fluid increases as t increases and after attaining a maximum value near the plate, it increases as z increases.

(ii) The velocity of dusty particles decreases rapidly as t increases and after attaining a minimum value near the plate, it decreases as z increases.

(iii) Lastly it is also observed that from the graphs the magnitude of the velocity profile of dusty fluid and the magnitude of the velocity of dusty particles are identical.

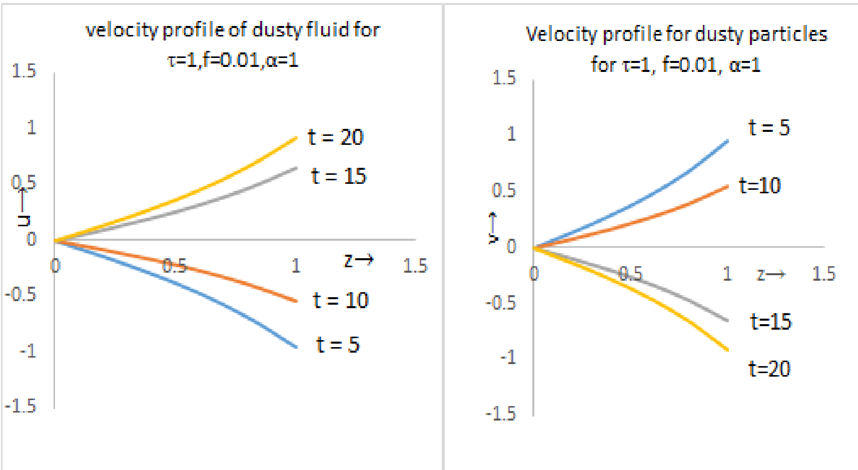


Fig. 1

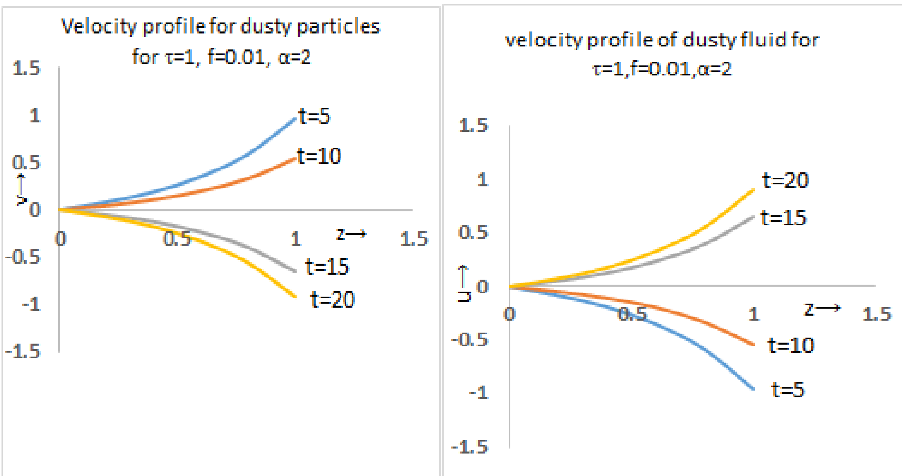


Fig. 2

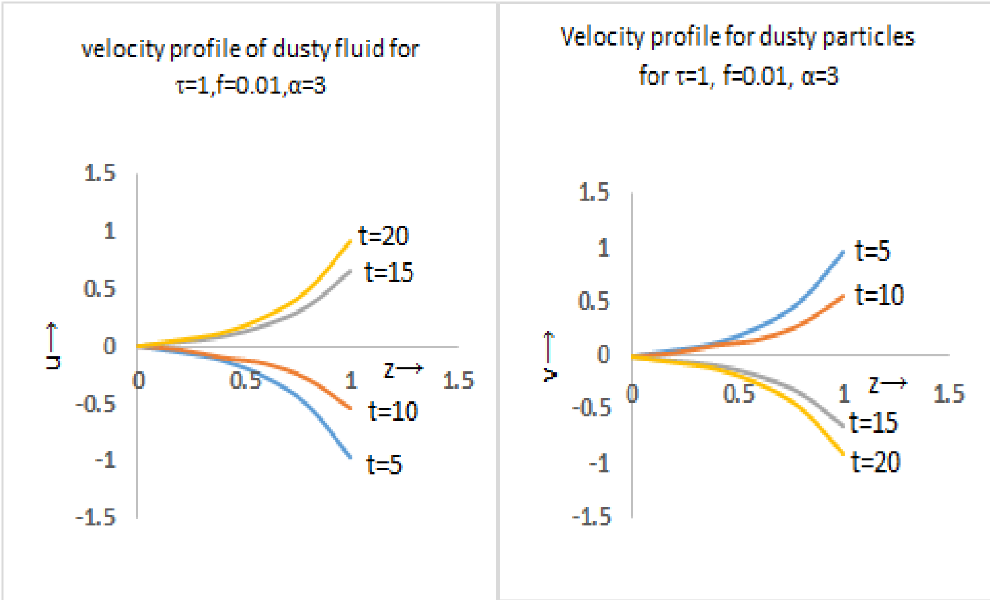


Fig. 3

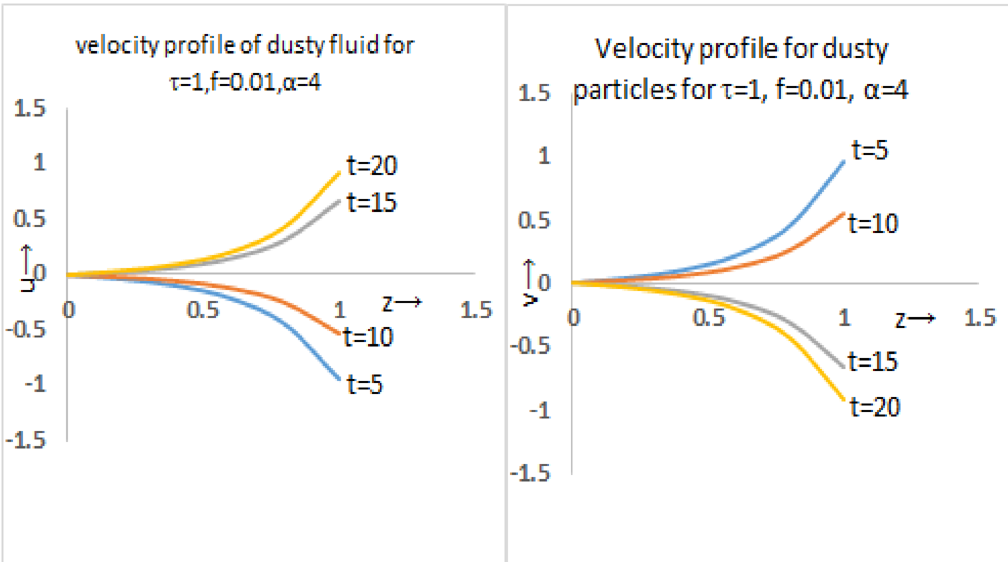


Fig. 4

Conflicts of Interest:

There is no conflict of interest regarding the paper.

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