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A NOVEL METHOD TO FIND THE EQUATION OF CIRCLES

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Abstract

The concept of the circle has been known to human beings since before the beginning of recorded history. With the advent of the wheel, the study of the circle in detail played an important role in the field of science and technology.

According to the author, there are three types of circles, 1) Countup circle, 2) Countdown circle, and 3) Point circle instead of two types of circles as defined by René Descartes in real plane coordinate geometry and Euler in the complex plane.

The author has been successful to solve the equations of three types of circles in the real plane by using three fundamental recent (2021 – 2022) inventions, 1) Theory of Dynamics of Numbers, 2) Rectangular Bhattacharyya's Co-ordinate System, 3) The novel Concept of Quadratic Equation where the author becomes successful to solve the quadratic equation of $x^2 + 1 = 0$ in real number instead of an imaginary number.

In the present paper, the author solved successfully the problem where radius $r = \sqrt{g^2 + f^2 - c}$ if $g^2 + f^2 < c$, c the constant term of the general form of the equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ by using Bhattacharyya's Coordinate system without any help from the complex plane where Euler solved it by using a complex plane.

According to Bhattacharyya's Co-ordinate System, the equation of the countdown circle is as follows :

$\overleftarrow{(x-a)^2} + \overleftarrow{(x-b)^2} = \overleftarrow{r^2} = -r^2$ where, the coordinates of the moving point P are (x, y) with Centre C (a, b) and radius = – r

The concept of a countdown circle is very much interesting and it exists really in nature. We may consider that the rotational motion of the Earth around the Sun is a countdown rotational motion.

Keywords: Bhattacharyya's Coordinate System, Cartesian Coordinate system, Equation of the circles, Quadratic equation, Theory of Dynamics of Numbers.

I. Introduction

The knowledge and progress in understanding the circle are directly correlated with the progress of human civilization in general. So, from past to present the mathematicians and philosophers of the world showed their keen interest to develop the concepts concerning circles in detail from a different angle. Rene Descartes has given the equation of a circle with the help of a Cartesian coordinate system in a real plane. Euler formulated the equation of a circle in a complex plane.

The author developed a new concept regarding the equation of a circle based on three newly invented concepts by the author in mathematics :

(1) Theory of Dynamics of Numbers [XVIII]
(2) Rectangular Bhattacharyya's Co-ordinate System in Real Plane. [XVII]
(3) Novel Concept of Quadratic Equation [XIX]

According to The Theory of Dynamics of Numbers, 0 (zero) is the starting point of any number. The numbers that move away from the starting point 0 (zero) are the countup numbers and the numbers that move towards 0 (zero) are the countdown numbers. The author framed the laws and rules of the Theory of Dynamics of Numbers.

Four-dimensional Bhattacharyya's coordinate system in the real plane is based on the Theory of Dynamics of Numbers where all the abscissas and ordinates are denoted as positive axes whereas in the Cartesian coordinate system one abscissa is positive and the other abscissa is negative and also one ordinate is positive and the other ordinate is negative.

The Novel Concept of Quadratic Equation is also based on the Theory of Dynamics of Numbers. The author solved the problem of the quadratic equation, $x^2 + 1 = 0$ in real numbers. So, the solution of the equation of a circle of any form can be solved by Bhattacharyya's coordinate system in a real plane without using a complex plane.

Euler solved the problem where the radius of a circle, $r = \sqrt{g^2 + f^2 - c}$ if $g^2 + f^2 < c$, c the constant term of the general form of equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$

by introducing the equation of a circle in a complex plane as

$$|Z - C| = r$$

But the author has invented the method of solution of the quadratic equation of $x^2 + 1 = 0$ in real number.

So, the author becomes successful to solve the problem where radius, $r = \sqrt{g^2 + f^2 - c}$ if $g^2 + f^2 < c$, c the constant term of the general form of equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$. using Bhattacharyya's coordinate system without any help of a complex plane.

According to Bhattacharyya's coordinate system, there are three types of the circle:

(1) Countup circle
(2) Countdown circle
(3) Point circle

The equation of countup circle is

$$\overrightarrow{(x-a)^2} + \overrightarrow{(y-b)^2} = \overrightarrow{r^2} = +r^2$$

where, co-ordinates of P is (x, y) and co-ordinates of center C is (a, b), and radius = $\overrightarrow{r} = +r$

The equation of the countdown circle is

$$\overleftarrow{(x-a)^2} + \overleftarrow{(y-b)^2} = \overleftarrow{r^2} = -r^2$$

where, co-ordinates of P is (x, y) and co-ordinates of center C is (a, b), and radius = $\overleftarrow{r} = -r$

The equation of the point circle is

$$\overrightarrow{(x-a)^2} + \overrightarrow{(y-b)^2} = 0 \text{ or } \overleftarrow{(x-a)^2} + \overleftarrow{(y-b)^2} = 0$$

where, coordinates of P is (x, y) and coordinates of center C is (a, b), and radius = $\overrightarrow{r} = 0$ or $\overleftarrow{r} = 0$.

In Fluid Dynamics we may consider SOURCE as countup circle and SINK as count down circle. . Also, we can find the existence of countdown motion in Mechanics. We may find it in 'Centripetal Force'.

However, similar problems as indicated above, in the case of the method of solution of the equation of a circle have not been investigated by a similar approach.

Finally, the author presented new theories in the equation of the circles introducing three new concepts.

II. Literature Review

The word circle has been derived from Greek whose meaning is hoop or ring [XV]. The concept of a circle as a geometrical figure has touched human life since the advent of the wheel. The Sun, the Moon, and other planets in the sky would have been considered a natural circle. In the ancient period geometry, astrology and astronomy were connected to the divine for most medieval scholars and many of them believed that there was something intrinsically 'divine' or 'perfect' that could be found in the circle [I, XX].

Development in understanding various geometric features of the circle is directly correlated with the progress of human civilization in general. In the Indian context Sulvasutra, Jainna composition, Aryabhata's mathematical astronomy, and works from the Kerala school of mathematics from different periods of history had an important role in the development of the circle. The composition of Sulvasutra was concerned with the construction of Alters (Vedi) and fire platforms (Citi) for the Vedic rituals.

In 1700 BCE, the Rhind Papyrus has given a method to find the area of a circular field. The result corresponds to $\frac{256}{81} = 3.16049$ ….. as an approximate value of π [IV].

According to general consensus, we can determine the period of Sulavasutra was about 800 BCE to 200 BCE, where Boudhyan being the oldest and Katyana being the latest [XXI, XXVI, XXVII]. The English version of the Sanskrit verse of Manava Sulavasutra 11.15 are as follows :

"Divide the square into nine parts, (by) dividing the sides into three (equal) parts. Mark a fifth of the part jutting out (of the square) and cover (the corresponding circle with center at the origin) with loose earth".
For a square of side length 2, the length of the segment between the squares and the outer circle is seen to be $(\frac{\sqrt{17}}{3} - 1)$, so

$$r^2 = \{1 + \frac{1}{5}\,(\frac{\sqrt{17}}{3} - 1\}^2 + \frac{1}{9}$$

For the unit square of this circle to work out to 0.9964………, a substantially more accurate value compared to the earlier one, the value of π in this case works out to 3.1583….

Archimedes (287 BCE – 212 BCE), A Greek mathematician proved that the area enclosed by a circle is equal to that of a triangle whose base area has the length of the circle's circumference and whose height equals the circle's radius [XIII] which comes to π multiplied by the radius squared πr^2.

In 300 BCE, in Book – 3 of Euclid's Elements, he described the properties of circles. Most likely in 353 BC in Plato's seventh letter there is a detailed definition and explanation of the circle. Ploto explained the perfect square and he showed how it is different from any drawing or explanation.

Aryabhata (476 CE) had a deeper knowledge of circles both in terms of geometry and trigonometry. English version of his Sanskrit verse (Ganapada 10, in Aryabhatia) [XXIX] is as follows: "The circumference of a circle with twenty thousand is approximately hundred and four times eight and sixty – two thousand [viz 62832]. This has given the value of π = 3.1416, which is actually equal to the value π truncated at 4 decimal places.

In China, this approximation for π had been given by Chong – Zhi (429 CE – 500 CE). He determined the value of π = 3.1415929…… which is accurate to 6 decimal of π = 3.1415926…… .

In 1880 CE Lindemann proved that π is transcendental, effectively settling the millennia-old problem of squaring the circle [XXVIII].

René Descartes (1596 – 1650) invented Cartesian coordinate system. According to the Cartesian coordinate system, the equation of a circle is

$$(x - a)^2 + (y - b)^2 = r^2$$

where, the coordinates of the moving point P is (x, y) with center C (a, b) and radius r. Using trigonometric function sine and cosine, the equation can be written in a parametric form as x = a + r cos t

y = b + r sin t

where t parametric variable in the range 0 to 2π, interpreted geometrically as the angle that the ray from (a, b) to (x, y) makes with the positive x-axis.

General form of the equation of a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

where, the coordinates of the centre are (– g, – f) and radius = $\sqrt{g^2 + f^2 - c}$ where c is the constant term.

Eules (1707 – 1783) introduced the notation $i = \sqrt{-1} =$ an imaginary number,

A circle with a center at C and having a radius in a complex plane has the equation

I Z – C I = r

The equation can be written in the parametric form as

$Z = re^{it} + c.$

Based on recently (2021 – 2022) invented three concepts by the author:

(1) Theory of Dynamics of Numbers.
(2) Rectangular Bhattacharyya's Co-ordinate System in Real Plane.
(3) Novel Concept of Quadratic Equation.

The author defined three types of circles :

(1) Countup circle
(2) Countdown circle
(3) Point circle

Instead of two types of the circle as defined by the cartesian coordinate system and the equation of a circle in a complex plane as defined by Euler.

III. Formulation of the Problem and its Solution

Definition of Circle

If a point P moves on a plane in such a way that its distance from the fixed-point C (say) on the plane is always the same then the locus of the moving point P is called a Circle. The fixed-point C is called the center of the circle and the constant distance of $\overleftrightarrow{CP}$ is called the radius of the circle (Fig. 1)

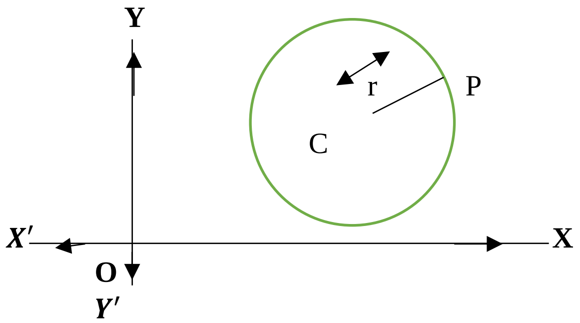

Fig. 1.

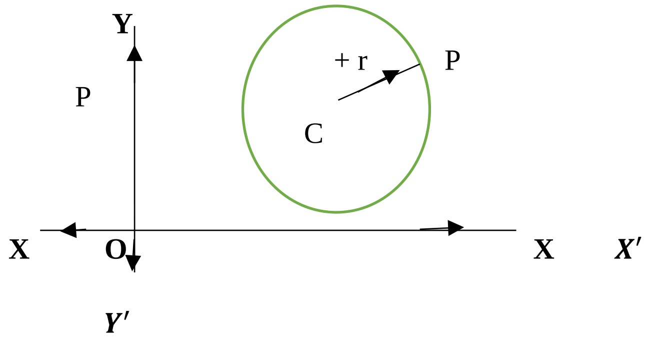

Fig. 2(a)

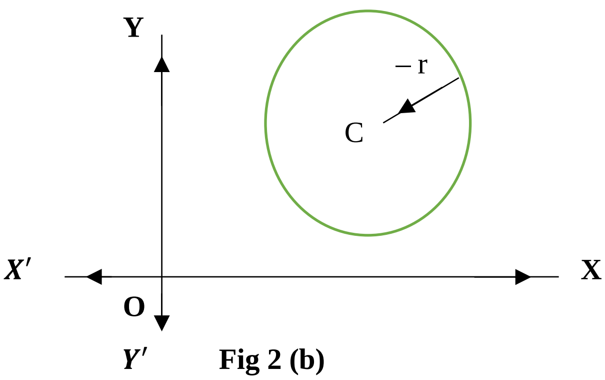

Fig 2 (b)

If a point P on a plane is moving away from the center C, then the point P is called Countup point P. So, the inherent nature of the point P on the plane is countup motion of the point P (Fig. 2.a)

If a point P on a plane is moving towards the center C, then the point P is called Countdown point P. So, the inherent nature of the point P on the plane is countdown motion of the point P (Fig. 2.b)

The same point P on the same plane can be distinguished only depending on the inherent nature of the motion of the point P.

There are three types of circles: 1) Countup circle, 2) Countdown Circle and 3) Point circle.

1) **Countup Circle:** The locus of a moving point P is called a Countup circle when the point P moves in infinite directions away from the center C keeping the fixed countup distance $r = \overrightarrow{r} = +r$ (Fig 2.a).

2) **Countdown Circle:** The locus of a moving point P is called a Countdown circle when the point P moves in infinite directions towards the Centre C of the circle keeping the fixed countdown distance $r = \overleftarrow{r} = -r$ (Fig 2.b).

3) **Point Circle:** If the locus of countup point P or the countdown point P moves on a plane keeping 0 (zero) distance from its center C is called a point circle. In case the radius r = 0, countup point P or countdown point P and the center C are the same points.

To find the Equation of a Circle

If we know the center and radius of a circle, we can easily determine the equation of a circle.

I. To find the equation of a circle whose centre is at the origin O and its radius is r units.

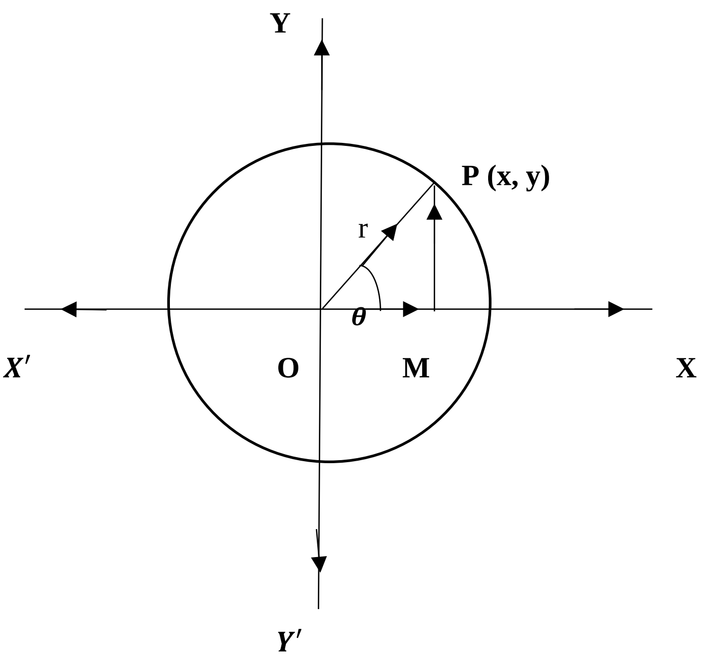

Fig. 3 (a)

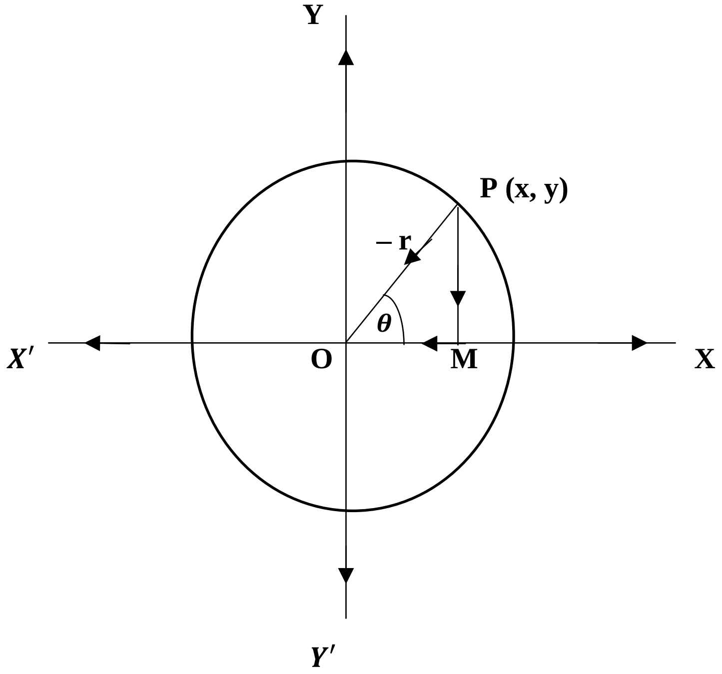

Fig. 3 (b)

Let P (x, y) be any point on the required circle. By definition, for all positions of the moving point P on its path we have from fig 3.a.

$\overrightarrow{OP} = \overrightarrow{r}$ = + r, the radius of the circle.

So, $\overrightarrow{OP^2} = \overrightarrow{r^2}$

Therefore,

$$\overrightarrow{x^2} + \overrightarrow{y^2} = \overrightarrow{r^2} \quad (1)$$

$$\Rightarrow \overrightarrow{x^2} + \overrightarrow{y^2} + \overleftarrow{r^2} = 0 \quad (2)$$

According to the Theory of Dynamics of Numbers the equation (2) takes the form

$$x^2 + y^2 - r^2 = 0 \quad (3)$$

Therefore, equation (3) represents the equation of a countup circle.

From fig. 3 (b), we have

$\overrightarrow{PO} = \overleftarrow{OP} = \overleftarrow{r}$ = – r, the radius of the circle

$\therefore \overleftarrow{OP^2} = \overleftarrow{r^2}$ = – r^2

Therefore,

$$\overleftarrow{x^2 + y^2} = \overleftarrow{r^2} \qquad (4)$$

$$\Rightarrow \overleftarrow{x^2 + y^2} + \overrightarrow{r^2} = 0 \qquad (5)$$

According to the Theory of Dynamics of Numbers the equation (5) takes the form

$$x^2 + y^2 + r^2 = 0 \qquad (6)$$

Therefore, equation (6) represents the countdown circle.

In case the radius, r = 0, the equation takes the form

$$x^2 + y^2 = 0 \qquad (7)$$

$\Rightarrow$ x = 0 and y = 0.

So, the equation (7), $x^2 + y^2 = 0$ represents a point circle.

I (A) <u>Parametric Equation of the Circle</u>

(i) Parametric equation of a countup circle :

Concerning Fig. 3(a), 0 as the pole and $\overrightarrow{OX}$ as the initial line of the polar coordinates of P (r, θ), then

$\overrightarrow{r} = \overrightarrow{OP} = radius\ of\ the\ circle\ and\ \angle POX = \theta$.

From Fig. 3(a), we have, $\overrightarrow{x} = \overrightarrow{r\ cos\theta}\ and\ y = \overrightarrow{r\ sin\theta}$

Or, $x = rcos\theta$ and $y = r\ sin\theta$ represent the parametric equation of the countup circle (3)

(ii) Parametric equation of countdown circle :

concerning Fig. 3(b), 0 as the pole and $\overrightarrow{OX}$ as the initial line of the polar co-ordinates of P (– r, θ), then

$\overleftarrow{r} = \overleftarrow{OP}$ = radius of the circle = – r and ∠POX = θ

Then from Fig. 3(b), we have $\overleftarrow{x} = \overleftarrow{r\ cos\theta}\ and\ \overleftarrow{y} = \overleftarrow{r\ sin\theta}$

Therefore, $\overleftarrow{x} = -r\ cos\theta\ and\ \overleftarrow{y} = -r\ sin\theta$, represents the parametric equation of the circle (6)

I (B). Comparative study between Bhattacharyya's Coordinate System and Cartesian Coordinate System to find the location of a point on a plane.

Bhattacharyya's Co-ordinate System Cartesian Co-ordinate System

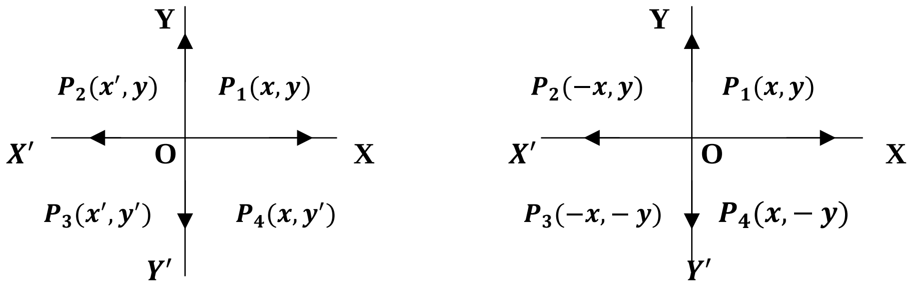

Fig. 4 (a) **Fig. 4 (b)**

According to Bhattacharyya's Co-ordinate System abscissas OX and OX' are both positive and the ordinates OY and OY' are also positive. Co-ordinates of any point in any quadrant are also positive. The author used notation '′' (dash) over co-ordinates as in Fig. 4(a) for identifying the positions of the points in which quadrant on the plane the point lies. Here $x' = +x\ and y' = +y$. There is no existence of negative abscissa or negative ordinate as in the Cartesian Co-ordinate system Fig. 4(b).

Bhattacharyya's Plane Coordinate geometry is four-dimensional. [XVII]

II. (A) To find the equation of a circle and its radius according to Bhattacharyya's Co-ordinate System

(a) When the moving point P and the center C lie in the same quadrant of the plane.

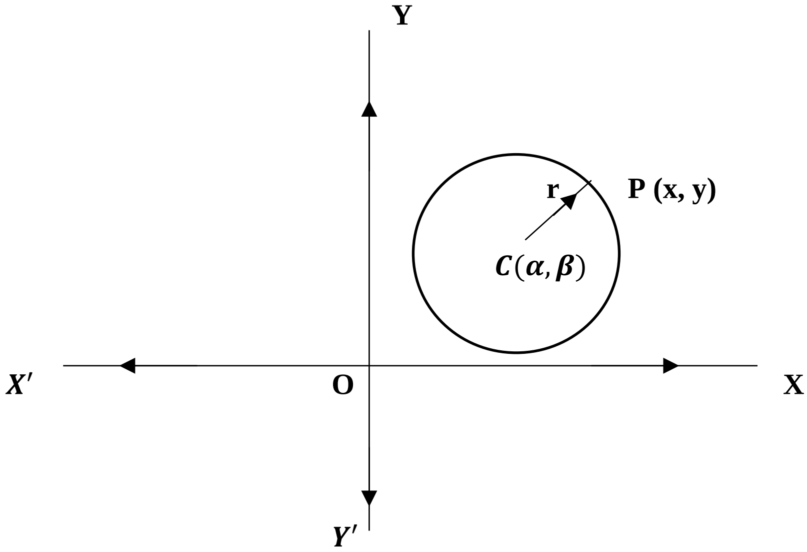

Fig. 5.1

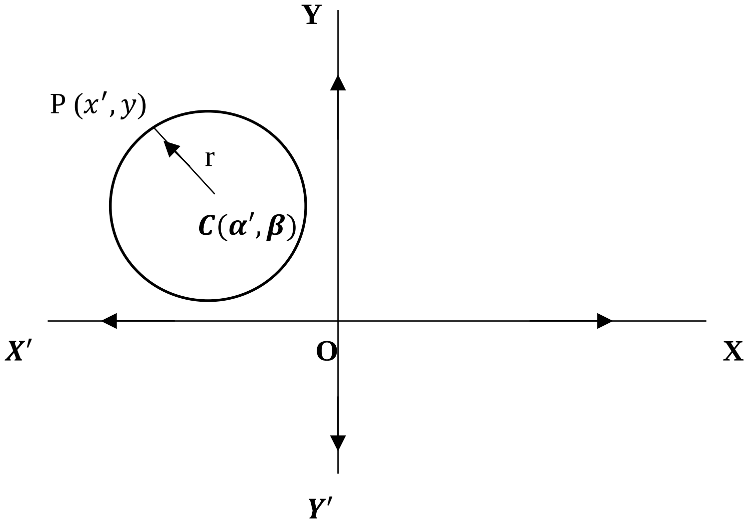

Fig. (5.2)

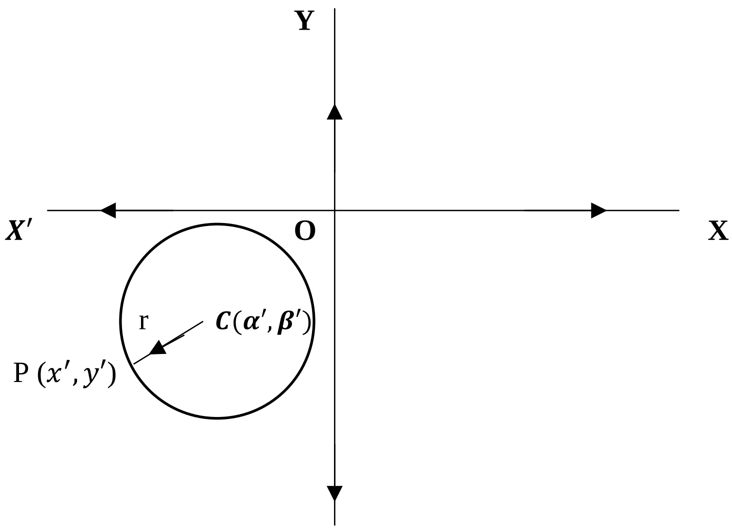

Fig. (5.3)

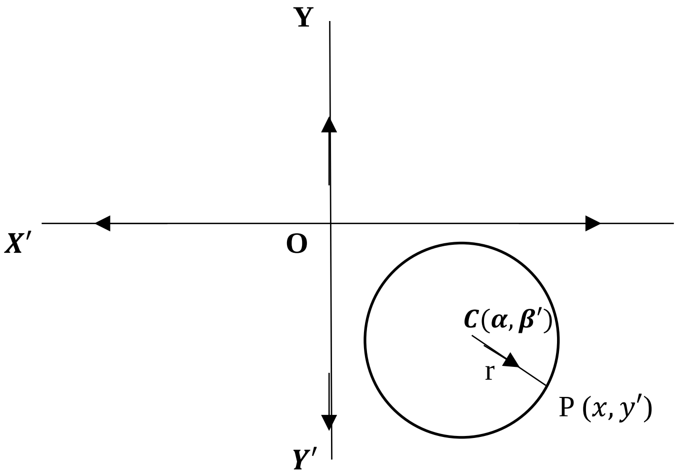

Fig. (5.4)

From Fig. (5.1), Equation of the circle.

$$(x - \boldsymbol{\alpha})^2 + (y - \beta)^2 = r^2 \quad (1)$$

and, radius,

$$r = \sqrt{(x - \boldsymbol{\alpha})^2 + (y - \beta)^2}$$

From Fig 5.2, the Equation of the circle

$$(x' - \alpha')^2 + (y - \beta)^2 = r^2$$

$$\Rightarrow (x - \alpha)^2 + (y - \beta)^2 = r^2 \text{ [Since the numerical value of } x' = x \text{ and } \alpha' = \alpha] \quad (2)$$

and, radius,

$$r = \sqrt{(x - \alpha)^2 + (y - \beta)^2}$$

From Fig 5.3, the Equation of the circle

$$(x' - \alpha')^2 + (y' - \beta')^2 = r^2$$

$$\Rightarrow (x - \alpha)^2 + (y - \beta)^2 = r^2 \text{ [Since the numerical value of } x' = x \text{ and } y' = y \text{ ; } \alpha' = \alpha \text{ and } \beta' = \beta] \quad (3)$$

and radius, $r = \sqrt{(x - \alpha)^2 + (y - \beta)^2}$

From Fig 5.4, the Equation of the circle

$$(x - \alpha)^2 + (y' - \beta')^2 = r^2$$

$$\Rightarrow (x - \alpha)^2 + (y - \beta)^2 = r^2 \text{ [Since } y' = y \text{ and } \beta' = \beta \text{]} \quad (4)$$

and, radius,

$$r = \sqrt{(x - \alpha)^2 + (y - \beta)^2}$$

Therefore, we find that when the moving point P and the Centre C lie in the same quadrant the equation of the circle is

$$(x - \alpha)^2 + (y - \beta)^2 = r^2 \quad (5)$$

and, radius,

$$r = \sqrt{(x - \alpha)^2 + (y - \beta)^2} \quad (6)$$

II. (B) When the point P and the centre C are not lying in the same quadrant of the plane.

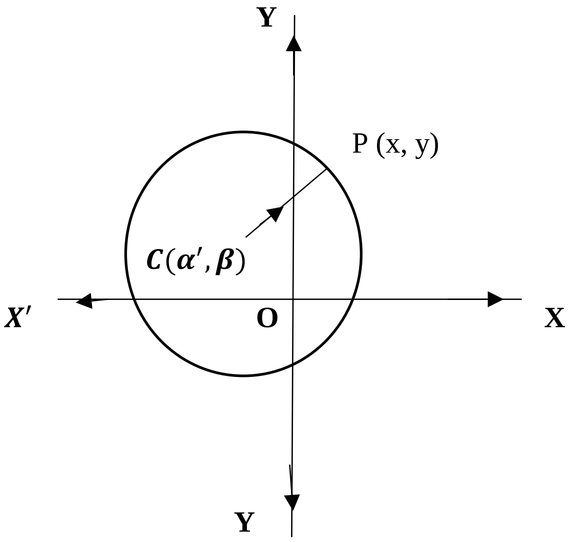

Fig 6.1

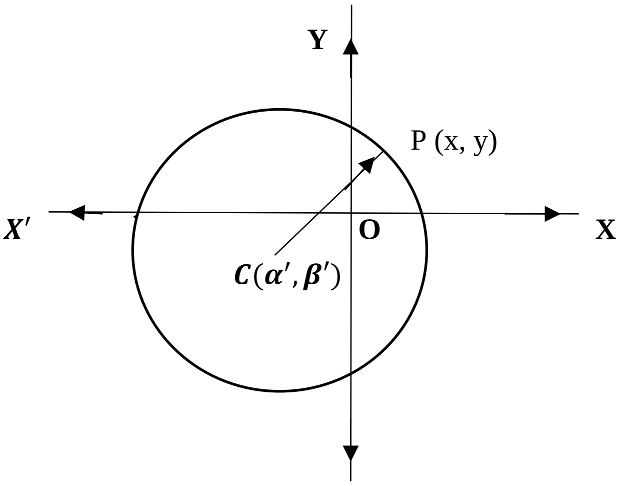

Fig 6.2

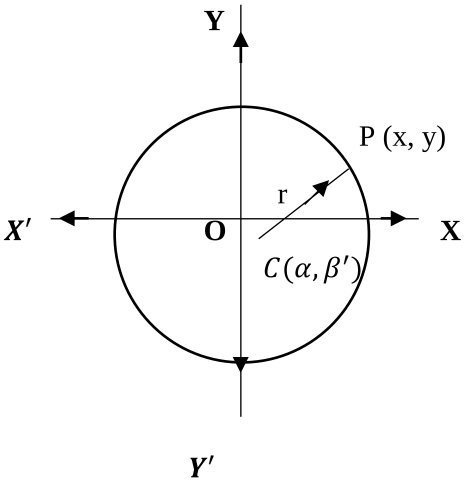

Fig (6.3)

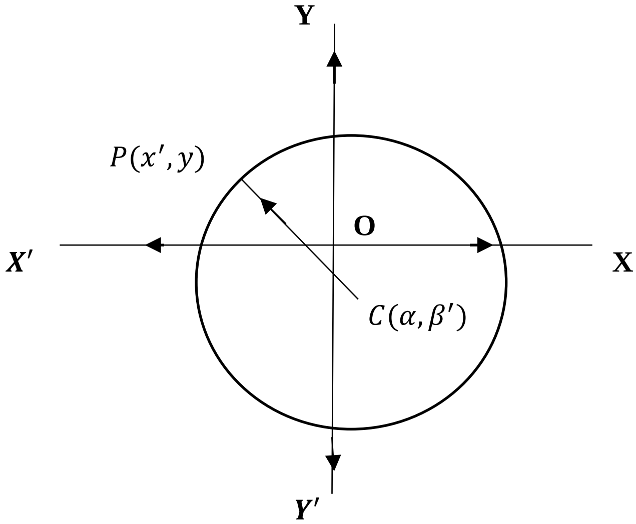

Fig. (6.4)

From Fig. 6.1, Equation of the circle :

$$(x+\ \alpha')^2+(y-\ \beta)^2=\ r^2$$

$$\Rightarrow (x+\ \alpha)^2+(y-\ \beta)^2=\ r^2\ \ [\text{Since }\alpha'=\ \alpha\ ] \tag{7}$$

and, radius,

$$\text{r} = \sqrt{(x\ +\ \alpha)^2+(y-\ \beta)^2} \tag{8}$$

From Fig. 6.2, Equation of the circle :

$$(x+\ \alpha')^2+(y+\ \beta')^2=\ r^2$$

$$\Rightarrow (x+\ \alpha)^2+(y+\ \beta)^2=\ r^2\ [\text{Since }\alpha'=\ \alpha\ \ and\ \ \beta'=\ \beta\ ] \tag{9}$$

and, radius,

$$\text{r} = \sqrt{(x\ +\ \alpha)^2+(y+\ \beta)^2} \tag{10}$$

From Fig. 6.3, Equation of the circle :

$$(x-\ \alpha)^2+(y+\ \beta')^2=\ r^2$$

$$\Rightarrow (x\ -\ \alpha)^2+(y+\ \beta)^2=\ r^2\ [\text{Since }\ \beta'=\ \beta\ ] \tag{11}$$

and, radius,

$$\text{r} = \sqrt{(x-\ \alpha)^2+(y+\ \beta)^2} \tag{12}$$

From Fig. 6.4, Equation of the circle :

$$(x'+\ \alpha)^2+(y+\ \beta')^2=\ r^2$$

$$\Rightarrow (x+\ \alpha)^2+(y+\ \beta)^2=\ r^2\ [\text{Since }\ x'=x\ and\ \beta'=\ \beta\ ] \tag{13}$$

and, radius,

$$r = \sqrt{(x + \alpha)^2 + (y + \beta)^2} \qquad (14)$$

Note :

(i) We know from Bhattacharyya's Co-ordinate System the numerical value of

$x' = x, y' = y, \alpha' = \alpha \text{ and } \beta' = \beta$

The abscissa OX' is vertically opposite to the abscissa OX, the coordinate of the abscissa OX' is denoted as x', i.e., $\overleftarrow{x} = \overrightarrow{x'}$ and $\overleftarrow{x'} = \overrightarrow{x}$. Similarly, $\overleftarrow{y} = \overrightarrow{y'}$ and $\overleftarrow{y'} = \overrightarrow{y}$

and $x, x'; y, y'; \alpha, \alpha'$ and β, β' possess positive numerical value.

'$'$' (dash) notation is used to identify the coordinates in which quadrant the point P and the center C lie.

(ii) To find the distance between two points the following rules are followed :

(a) If the Abscissas and ordinates of the two points are with the same notations

$$\text{Distance} = \sqrt{(Abscissa_1 - Abscissa_2)^2 + (Ordinate_1 - Ordinate_2)^2}$$

(b) If the Abscissas of the two points are with different notations and ordinates are with the same notations

$$\text{Distance} = \sqrt{(Abscissa_1 + Abscissa_2)^2 + (Ordinate_1 - Ordinate_2)^2}$$

(c) If the Abscissas of two points are with the same notation and ordinates are with different notations

$$\text{Distance} = \sqrt{(Abscissa_1 - Abscissa_2)^2 + (Ordinate_1 + Ordinate_2)^2}$$

(d) If the Abscissas and Ordinates of the two points are both with different notations

$$\text{Distance} = \sqrt{(Abscissa_1 + Abscissa_2)^2 + (Ordinate_1 + Ordinate_2)^2}$$

II. (a) General form of the equation of a circle :

An equation in two unknown quantity (say x and y) in the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \ (a \neq 0, \ b \neq 0) \qquad (1)$$

where a, h, b, g, f, c are constants which is called a quadratic equation in two unknown quantity (say x and y)

Now if a = b = 1 and h = 0, the equation (1) will take the general form of the equation of the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (2)$$

This quadratic equation (2) represents the equation of a circle.

II. (b) Conditions for finding the inherent nature of two known quantities (say x and y)

The numerical value of the inherent nature of x and y which satisfies the quadratic equation (2) is called the root of the equation. The inherent nature of x and y can be uniquely determined from the following conditions of the quadratic equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, provided the character of the structure of the second degree expression $x^2 + y^2 + 2gx + 2fy + c$ of the quadratic equation of the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$, must be in second degree.

Let us consider the general form of equation of a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Case – I

If $g^2 + f^2 > c$, the inherent nature of two unknown quantities (say x and y) will be the countup numbers.

The equation (1) will represent a Countup circle.

Case – II

If $g^2 + f^2 < c$, the inherent nature of two unknown quantities (say x and y) will be the countdown numbers. The equation (1) will represent a Countdown circle.

Case – III

If $g^2 + f^2 = c$, the inherent nature of two unknown quantities (say x and y) will be (0,0)

The equation (1) will represent a Point circle, i.e., the circle will be converted to a point whose coordinates will be (g', f')

II. (c) The general form of the equation of countup and countdown circles

We consider the equation of a circle of general form concerning the inherent nature of two unknown coordinates (say x and y). There are two types of form: Form A and Form B.

Form A :

$$x^2 + y^2 + 2gx + 2fy - c = 0 \qquad (1)$$

where g, f, c are constants and $g^2 + f^2 > c$

Since, $g^2 + f^2 > c$, the inherent nature of x and y will be countup numbers.

According to the Theory of Dynamics of Numbers the equation (1) takes the form

$$\overrightarrow{x^2 + y^2 + 2gx + 2fy} + \overleftarrow{c} = 0 \qquad (2)$$

$$\Rightarrow \overrightarrow{(x^2 + 2gx + g^2)} + \overrightarrow{(y^2 + 2fy + f^2} + \overleftarrow{g^2 + f^2} + \overleftarrow{c} = 0$$

$$\Rightarrow \overrightarrow{(x+g)^2} + \overrightarrow{(y+f)^2} = \overrightarrow{g^2 + f^2 + c} \qquad (3)$$

$$\Rightarrow \overrightarrow{(x + g')^2} + \overrightarrow{(y + f')^2} = \overrightarrow{r^2} = +r^2 \text{ [According to the Bhattacharyya's co-ordinate system]} \qquad (4)$$

where

$$r = \sqrt{g'^2 + f'^2 + c} \quad > 0$$

$\Rightarrow r = \sqrt{g^2 + f^2 + c} \quad > 0$ [According to the Bhattacharyya's co-ordinate system]

In this case of equation (4), the point P (x, y)) lies in the 1st quadrant, and the center C must lie in the 3rd quadrant. So the coordinates of the center C will be (g', f') though we know the numerical value of $g' = g$ and $f' = f$ (according to Bhattacharyya's coordinate system)

Here equation (1) represents a countup circle.

Form B :

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad (5)$$

where g, f, c are constants and $g^2 + f^2 < c$

Since, $g^2 + f^2 < c$, the inherent nature of x and y will be countdown numbers.

According to the theory of dynamics of numbers the equations (5) take the form

$$\overleftarrow{x^2 + y^2 + 2gx + 2fy} + \overrightarrow{c} = 0 \quad (6)$$

$$\Rightarrow \overleftarrow{(x^2 + 2gx + g^2)} + \overleftarrow{(y^2 + 2fy + f^2} + \overrightarrow{g^2 + f^2} + \overrightarrow{c} = 0$$

$$\Rightarrow \overleftarrow{(x + g)^2} + \overleftarrow{(y + f)^2} = \overleftarrow{g^2 + f^2 + c}$$

$$\Rightarrow \overleftarrow{(x + g')^2} + \overleftarrow{(y + f')^2} = \overleftarrow{r^2} = -r^2 \text{ (According to the Bhattacharyya's Co-ordinate System)} \quad (7)$$

Where, $\overleftarrow{r} = \overleftarrow{\sqrt{g'^2 + f'^2 + c)}}$

$$\Rightarrow \overleftarrow{r} = \overleftarrow{\sqrt{g^2 + f^2 + c)}}$$

In this case of equation (7) the point P (x, y)) lies in the 1st quadrant and the Centre C must lie in the 3rd quadrant. So the coordinates of the center C will be (g', f') though we know the numerical value of $g' = g$ and $f' = f$ (according to Bhattacharyya's coordinate system).

The equation (7) represents a countdown circle.

Note: When $g^2 + f^2 = c$, then the radius of the circle will be Zeo (0). In this case, the circle reduces to the point (g', f'). The circle represents a Point circle.

Problem – 1 :

Find the location of the centre of the circle and its radius

$x^2 + y^2 – 3x + 2y – 19 = 0$

Solution :

$$x^2 + y^2 - 3x + 2y - 19 = 0 \qquad (1)$$

Here, $g = -\frac{3}{2}$, $f = 1$ and c = – 19

So,

$$g^2 + f^2 = (-\frac{3}{2})^2 + (1)^2 = \frac{9}{4} + 1 = \frac{13}{4}$$

$\frac{13}{4} > -19$, therefore, $g^2 + f^2 > c$

Since, $g^2 + f^2 > c$, the inherent nature of x and y are countup x and countup y in equation (1)

According to the Theory of Dynamics of Numbers, the equation (1) takes the form

$$\overrightarrow{x^2 + y^2 - 3x + 2y} + \overleftarrow{19} = 0 \qquad (2)$$

$$\Rightarrow \overrightarrow{(x - \frac{3}{2})^2} + \overrightarrow{(y + 1)^2} + \overleftarrow{\frac{9}{4}} + \overleftarrow{1} + \overleftarrow{19} = 0$$

$$\Rightarrow \overrightarrow{(x - \frac{3}{2})^2} + \overrightarrow{(y + 1)^2} = \overrightarrow{\frac{9}{4} + 1 + 19}$$

$$\Rightarrow \overrightarrow{(x - \frac{3}{2})^2} + \overrightarrow{(y + 1)^2} = \frac{\overrightarrow{89}}{4} \qquad (3)$$

According to Bhattacharyya's coordinate system, the equation (3) takes the form

$$\overrightarrow{(x - \frac{3}{2})^2} + \overrightarrow{(y + 1')^2} = r^2 \qquad (4)$$

where, $r = \frac{\sqrt{89}}{2}$

It is clear from equation (4) that the coordinates of the center of the circle are $\left(\frac{3}{2}, 1'\right)$

and radius, $r = \frac{\sqrt{89}}{2}$

Here, the point P (x, y) lies in the first quadrant, and the center C of the circle $\left(\frac{3}{2}, 1'\right)$ lies in the fourth quadrant of the plane.

Problem – 2 :

Find the location of the centre of the circle and its radius

$x^2 + y^2 – 3x + 2y + 19 = 0$

Solution :

$$x^2 + y^2 - 3x + 2y + 19 = 0 \quad (1)$$

Here, $g = -\frac{3}{2}$, $f = 1$ and c = 19

$$g^2 + f^2 = (-\frac{3}{2})^2 + (1)^2 = \frac{9}{4} + 1 = \frac{13}{4}$$

$\frac{13}{4} < 19$, therefore, $g^2 + f^2 < c$

Since, , $g^2 + f^2 < c$, the inherent nature of x and y are countdown x and countdown y in equation (1)

According to the Theory of Dynamics of Numbers, the equation (1) takes the form

$$\overleftarrow{x^2 + y^2 - 3x + 2y} + \overrightarrow{19} = 0 \quad (2)$$

$$\Rightarrow \overleftarrow{(x - \frac{3}{2})^2} + \overleftarrow{(y + 1)^2} + \overrightarrow{\frac{9}{4} + 1 + 19} = 0$$

$$\Rightarrow \overleftarrow{(x - \frac{3}{2})^2} + \overleftarrow{(y + 1)^2} = \overleftarrow{\frac{9}{14} + 1 + 19}$$

$$\Rightarrow \overleftarrow{(x - \frac{3}{2})^2} + \overleftarrow{(y + 1)^2} = \frac{\overleftarrow{89}}{4} \quad (3)$$

According to Bhattacharyya's coordinate system, the equation (3) takes the form

$$\overleftarrow{(x - \frac{3}{2})^2} + \overleftarrow{(y + 1')^2} = \overleftarrow{r^2} = -r^2 \quad (4)$$

where, $\overleftarrow{r} = \frac{\overleftarrow{\sqrt{89}}}{2} = -\frac{\sqrt{89}}{2}$

Equation (4) represents the equation of the countdown circle. Here the point P (x, y) lies on the first quadrant, and the coordinates of the center, $C\left(\frac{3}{2}, 1'\right)$ lies on the fourth quadrant of the plane and its radius, $\overleftarrow{r} = -\frac{\sqrt{89}}{2}$.

Now, let us solve the said problem by conventional method

$$x^2 + y^2 - 3x + 2y + 19 = 0 \quad (1)$$

$$\Rightarrow (x - \frac{3}{2})^2 + (y + 1)^2 - \frac{9}{4} - 1 + 19 = 0$$

$$\Rightarrow (x - \frac{3}{2})^2 + (y + 1)^2 = -\frac{63}{4}$$

$$\Rightarrow (x - \frac{3}{2})^2 + (y + 1)^2 = r^2 \quad (2)$$

where, $r = \sqrt{\frac{-63}{4}} = \frac{i\sqrt{63}}{2}$

It is clear from equation (2) that the coordinates of the center of the circle are $\left(\frac{3}{2}, -1\right)$ and radius

$r = \frac{i\sqrt{63}}{2}$. Hence the point P (x, y) lies in the first quadrant, and the circle $\left(\frac{3}{2}, -1\right)$ lies in the fourth quadrant.

Observation :

I. We know that the distance between any two points having two real co-ordinates on a plane cannot be imaginary quantity according to Bhattacharyya's Co-ordinate System whereas we find that the radius, $r = \frac{i\sqrt{63}}{2}$ by conventional method. It is absurd.

II. In case of the same problem if we solve it according to the Bhattacharyya's co-ordinate system which is based on the Theory of Dynamics of Numbers that the numerical value of the radius of the countdown circle, $\overleftarrow{r} = -\frac{\sqrt{89}}{2}$

Problem: 3

Find the equation of the countup circle and countdown circle which are passing through three points, (2,1), (5,2), and $(1,6')$. Find the coordinates of its center with its location on the plane and also find its radius for both the circles.

Solution:

Let us consider the general equation of the required countup circle be

$$x^2 + y^2 + 2gx + 2fy - c = 0 \qquad (1)$$

According to Bhattacharyya's coordinate system, in this case of equation (1) the coordinates of the moving point P (x,y) lie in the 1st quadrant, and the coordinates of the center C (g,f) will lie in the 3rd quadrant. So, the coordinates of the center C will be (g', f') instead of (g,f).

Hence the equation (1) will take the form

$$\overrightarrow{(x+g')^2} + \overrightarrow{(y+f')^2} = \overrightarrow{r^2} \qquad (2)$$

Since the circle passes through the point (2,1), hence

$$(2+g')^2 + (1+f')^2 = {g'}^2 + {f'}^2 + c \qquad (3)$$

$$\Rightarrow 4 + 4g' + g'^2 + 1 + 2f' + f'^2 = g'^2 + f'^2 + c$$

$$\Rightarrow 4g' + 2f' - c = -5 \qquad (4)$$

Now, the circle passes through the point (5,2).

Hence

$$(5+g')^2+(2+f')^2=g'^2+f'^2+c \quad (5)$$

$$\Rightarrow 25+10g'+g'^2+4+4f'+f'^2=g'^2+f'^2+c$$

$$\Rightarrow 10g'+4f'-c=-29 \quad (6)$$

According to Bhattacharyya's coordinate system when the coordinates of the point P is (x,y') and the coordinates of the center C is (g',f') we know that the equation of the circle takes the form

$$(x+g')^2+(y'-f')^2=g'^2+f'^2+c \quad (7)$$

Since the circle passes through the point $(1,6')$.

Hence

$$(1+g')^2+(6'-f')^2=g'^2+f'^2+c \quad (8)$$

$\Rightarrow 1+2g'+g'^2+36-12f'+f'^2=g'^2+f'^2+c$ $[Since, 36'=36\ and\ 6'=6]$

$$\Rightarrow 2g'-12f'-c=-37 \quad (9)$$

Now,

(4) – (9) gives $2g'+14f'=32\ \ or\ \ g'+7f'=16$ (10)

and again (6) – (9) gives $8g'+16f'=8\ \ or\ \ g'+2f'=1$ (11)

Now, solving equation (10) and (11) we get $g'=-5\ \ and\ f'=3$

Again, putting the values of $g'\ and\ f'$ in equation (4) we get c = – 9 .

So, the radius of the countup circle,

$$r=\sqrt{g'^2+f'^2+c}=\sqrt{(-5)^2+(3)^2-9}=\sqrt{25}=5\ units.$$

Now, putting the values of $g'=-5, f'=3\ and\ c=\ -9$ in equation (1) we get

$$x^2+y^2-10x+6y+9=0 \quad (12)$$

Equation (12) is the equation of the required countup circle.

Here, we find that equation (12) satisfies the required condition $g^2+f^2>c$.

Now, the equation (12) can be expressed in the form

$$\overrightarrow{(x-5)^2}+\overrightarrow{(y+3)^2}=\overrightarrow{(5)^2} \quad (13)$$

Therefore, according to Bhattacharyya's Coordinate System, the coordinates of the center C is $(5,3')$ and the center C $(5,3')$ lies in the 4^{th} quadrant of the plane, and its radius $\vec{r}=5\ units$.

Now, to find the equation of the countdown circle, the equation (13) will take the form

$$\overleftarrow{(x-5)^2} + \overleftarrow{(y+3)^2} = -r^2 = \overleftarrow{25} \qquad (14)$$

$$\Rightarrow \overleftarrow{(x-5)^2} + \overleftarrow{(y+3)^2} + \overrightarrow{25} = 0 \qquad (15)$$

Therefore, from the equation (14) we know that the radius of the countdown circle is

$\overleftarrow{r} = \sqrt{\overleftarrow{25}} = \overleftarrow{5} = -5$ units.

Now, according to the Theory of Dynamics of Numbers the equation (15) will take the form

$$(x-5)^2 + (y+3)^2 + 25 = 0 \qquad (16)$$

$$\Rightarrow x^2 + y^2 - 10x + 6y + 59 = 0 \qquad (17)$$

Therefore, the equation (17)

$$x^2 + y^2 - 10x + 6y + 59 = 0$$

will be the required equation of the countdown circle.

Here, the equation (17) satisfies the condition $g^2 + f^2 < c$, when $g = -5$, $f = 3$ and $c = 59$. According to Bhattacharyya's Coordinate System from the equation (16) it is clear that the coordinate of the point P is (x, y) which lie in the 1st quadrant and the coordinate of the centre C $(5,3')$ of countdown circle lie in the 4th quadrant of the plane.
The radius of the countdown circle is $\overleftarrow{r} = -5$ units.

IV. Conclusion

First of all, the author has developed a new mathematical concept that there are three types of circles: 1) Countup circle, 2) Countdown circle and 3) Point circle based on three novel concepts: 1) Theory of Dynamics of Numbers, 2) Rectangular Bhattacharyya's Coordinate System, and 3) Novel Concept of Theory of Quadratic Equation. Applying the new concepts the author becomes successful to determine the inherent nature of two unknown quantities (say x and y) from the equation of the circles keeping the second-degree expression of the quadratic equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ intact.

Also, the author becomes successful to solve the problems of the general form of the equation of the circle: $x^2 + y^2 + 2gx + 2fy + c = 0$ even if $g^2 + f^2 < c$, c the constant term by using Bhattacharyya's Coordinate System without any help of complex plane. The new concept of countdown circle is a great achievement in the field of the equation of a circle which has real existence in nature as well as in Mechanics. For example, in Fluid Dynamics we may consider SOURCE as countup circle and SINK as countdown circle and also the force which is acting towards the center of the circle along the radius = $-r$, is a centripetal force.

Note that the motion of the Earth around the Sun is nothing but the countdown rotational motion of the Earth around the Sun.

This concept of the countdown circle will play a significant role in the field of science and technology.

Now, with these innovative concepts, the author has developed new theories in the equation of the circles. These mathematical concepts regarding the circles are applicable in any branch of mathematics, science, and technology.

It is the author's firm belief that today or tomorrow these innovative concepts related to the circle will be acceptable by the scientific communities of the world.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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