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HETEROGENEOUS TWO SERVER QUEUE WITH BREAKDOWN AND WITH VARIANT REPAIR POLICY

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Abstract

In this paper, we consider a system with two heterogeneous servers Markovian queue. In which the system breakdown occurs when the system is in busy mode. Immediately the system undergoes repair. After completion of the repair, the system either undergoes optional repair mode or becomes busy mode based on a Bernoulli schedule. It is assumed that the number of repairs follows the Poisson process and the repair periods follow an exponential distribution. The model has been solved in steady-state using the matrix analytic method. Some performance measures and numerical results are obtained.

Keywords: Markovian queue, heterogeneous server, breakdown, repair, steady-state solution, matrix-Geometric method

I. Introduction

Multi-server queueing systems with server breakdowns are more flexible and applicable in real-world situations than single server counterparts. However, due to their analytical complexity, there have been only a few studies carried out on a multi-server queueing system with server breakdowns. Mitrany and Avi-Itzhak (1968), studied an M/M/N queue with server breakdowns and ample repair capacity. In their study, the moment generating function of the queue size is obtained by using the transformation method. Vinod (1985), analysed the same model using the matrix-geometric solution method. For N=1, The author imposed some restrictions on the server down-periods (either independent of the queue length or only occurring when the sever is active). Neuts and Lucantoni (1979) and Wartenhosrt (1995), extended the models studied by Mitrany and Avi-Itzhak (1968) and Vinod (1985) by considering a limited repair capacity. Neuts and Lucantoni (1979), considered a single queue of customers, each served by one of N parallel servers. Wartenhosrt (1995) considered N parallel queue single-server, each serving its stream of customers. Wang and Chang (2002) studied an M/M/R/N queue with balking, reneging and server breakdowns from the viewpoint of queueing. They solved the steady-state probability equations iteratively and derived the steady-state probabilities in a matrix form.

Heffer (1969) has analyzed the waiting time distribution of $M/E_K/S$ queue. A Markovian queueing system with balking and two heterogeneous servers has been considered in Singh (1970). The Author determines the capacity of the slower server and obtains the optimal service rates. Singh (1973) discussed a Markovian queue with the number of servers depending upon the queue length. Desmit (1983) presented an approach to identify the distribution of waiting times and queue lengths for the queue GI/H_{2}/S. He reduced the problem to the solution of the Wiener-Hopf-type equations and then used a factorization method to solve the system.

In many research works, the authors assumed that the system parameters were variable, which is coincident with many practical situations. In Chang and Liu (2010) and Schouten and Wartenhorst (1993) considered a model with variable repair rates. Sheng et al (2011) analyzed a multiserver machine repairable model with variable breakdown rates. Wu et al (2014) studied multiserver queues with unreliable servers. In their work, they introduced a controllable repair policy and solve the model using the matrix analytic method.

Kalyanaraman and Senthilkumar (2018a) analyzed a two heterogeneous server Markovian queue with switching of service mode. The same authors (2018b) discussed heterogeneous server Markovian queues with restricted admissibility and with reneging. Kalyanaraman and Senthilkumar (2018c) analyzed two heterogeneous server queues with restricted admissibility. A matrix-geometric method approach is a useful tool for solving various queueing problems in different frameworks. Neuts (1981) explained various Matrix-geometric solutions of stochastic models. Matrix geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristics.

In this paper, we consider a two heterogeneous server Markovian queue. At any time the system may be in any one of the modes either in busy mode or repair mode. The repair mode has two states, namely essential repair state and optional repair state. The model has been defined and analyzed in section 2. Also, some performance measures are given in this section. A numerical study has been carried out in section 3. Section 4 contains a case study related to a production system. Finally, a conclusion has been given in section 5.

II. The Model

We consider an $M/M/2$ queueing model with heterogeneous servers, called Server 1 and Server 2 has been considered. The inter-arrival time of customers follows a negative exponential distribution with mean $\frac{1}{\lambda}$. The service time distributions are negative exponential with the rate μ_1 (server 1) and rate μ_2 (server 2)$\left(\mu = \mu_1 + \mu_2\right)$ respectively. Each customer is served by only one server and the queue discipline is first come first served. If the system is empty, the arriving customer joins any one of the servers. At a time the system is either in busy mode or breakdown mode. During busy mode, the system may break down, and the number of breakdowns follows the Poisson process with rateα. Immediately after the repair process starts, the repair period follows a negative exponential distribution with rateβ. In addition after the completion of the repair period, the system undergoes another repair process, called optional repair with probability p or the system enters into the busy mode with probability$(1-p), 0 \leq p \leq 1$. The optional repair period follows a negative exponential distribution with the rate γ. If an arriving customer finds both the server busy the arriving customer waits in a waiting line of an infinite capacity for the first free server.

Let L (t) be the number of customers in the system at time't' and J (t) be the server state at time't',

Where

$$J(t) = \begin{cases} 0 & \text{if the system is in optional repair state} \\ 1 & \text{if the syatem is in essential repair state} \\ 2 & \text{if the servers are in busy state} \\ (0,2) & \text{if the servers are in idle mode} \end{cases}$$

Let $X(t) = (L(t), J(t))$ then $X(t): t \geq 0$ is a Continuous time Markov chain (CTMC) with state space $S = \{(i,j): i = 0,1,2; j \geq 1\} U \{(0,2)\}$, where j denotes the number of customers in the system and i and (0, 2) denotes the server state.

Using the lexicographical sequence for the states, the infinitesimal generator Q of the Markov chain is given by

Q=C0 C1 B0 A1 A0 A2 A1 A0 A2 A1 A0 ... ...

$$Q = \begin{bmatrix} C_0 & C_1 & & & & & \\ B_0 & A_1 & A_0 & & & & \\ & A_2 & A_1 & A_0 & & & \\ & & A_2 & A_1 & A_0 & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ & & & & & \cdot & \cdot & \cdot \end{bmatrix}$$

where the sub-matricesA_0, A_1, and A_2 are of order 3×3 and are appearing as

$$A_0 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda+\gamma) & 0 & \gamma \\ p\beta & -(\lambda+\beta) & (1-p)\beta \\ 0 & \alpha & -(\lambda+\alpha+\mu) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

and the boundary matrices are defined by

$C_0 = (-\lambda)$ $C_1 = (0 \ \ 0 \ \ \lambda)$

$$B_0 = \begin{bmatrix} 0 \\ 0 \\ \mu \end{bmatrix}$$

II.i The Analysis

The model defined in this section 2 can be studied as a quasi-birth-and-death(QBD) process. For the analysis, the following probability notation has been introduced. At time t, let $P(t) = (p_0(t), p_1(t), p_2(t), \dots)$. In steady state, let $P = (p_0, p_1, p_2, \dots)$ be the stationary probability vector associated with Q, such that $PQ = 0$ and $Pe = 1$, where e is a column vector of 1′s of appropriate dimension.
Let $p_0 = (p_{02})$, $p_i = (p_{i0}, p_{i1}, p_{i2})$ for $i \geq 1$.
If the steady-state condition is satisfied, then the sub vectors p_i are given by the following equations:

$$p_0C_0 + p_1B_0 = 0 \qquad (1)$$

$$p_0C_1 + p_1A_1 + p_2A_2 = 0 \qquad (2)$$

$$p_{i-1}A_0 + p_iA_1 + p_{i+1}A_2 = 0, \quad i \geq 2 \qquad (3)$$

$$p_i = p_1R^{i-1}; \quad i \geq 2 \qquad (4)$$

where R is the rate matrix, and is the minimal non-negative solution of the matrix quadratic equation (see Neuts(1981)).

$$R^2A_2 + RA_1 + A_0 = 0, \qquad (5)$$

Substituting the equation (4) in (2), we have

$$p_0C_1 + p_1(A_1 + RA_2) = 0 \qquad (6)$$

and the normalizing condition is

$$p_0e + p_1(I - R)^{-1}e = 1$$

Theorem: 2.1

The queueing system described in this article is stable if and only if $\rho < 1$,
Where

$$\rho = \frac{\lambda(\alpha\beta p + \alpha\gamma + \beta\gamma)}{\gamma\mu\beta}$$

Proof

Consider the infinitesimal generator $A = \begin{bmatrix} -\gamma & 0 & \gamma \\ p\beta & -\beta & (1-p)\beta \\ 0 & \alpha & -\alpha \end{bmatrix}$, which is a square matrix of order 3, the row vector $\pi = (\pi_0, \pi_1, \pi_2)$ satisfies the condition $\pi A = 0$ and $\pi e = 1$.
That is,
The system is stable if and only if $\frac{\lambda(\alpha\beta p + \alpha\gamma + \beta\gamma)}{\gamma\mu\beta} < 1$

Theorem:2.2

If $\rho < 1$, the matrix equation (5) has the minimal non-negative solution

$$R = -A_0 A_1^{-1} - R^2 A_2 A_1^{-1}$$

Proof

Since A is reducible. The analysis present in Neuts (1978) is not applicable. In Lucantoni (1979), a similar reducible matrix is treated for the case when the elements are probabilities.

Equation (5), can be written as,

$$A_0 A_1^{-1} + R A_1 A_1^{-1} + R^2 A_2 A_1^{-1} = 0 A_1^{-1}$$

Since A_1 is non-singular, A_1^{-1} exists

$R = -A_0 A_1^{-1} - R^2 A_2 A_1^{-1}$ (8)

where

$$A_1^{-1} = \frac{1}{\alpha\beta\gamma p - (\lambda+\gamma)[(\lambda+\beta)(\lambda+\alpha+\mu) - (1-p)\alpha\beta]} \times$$

$$\begin{bmatrix} (\lambda+\beta)(\lambda+\mu+\alpha) - \alpha\beta(1-p) & \alpha\gamma & \gamma(\lambda+\beta) \\ p\beta(\lambda+\alpha+\mu) & (\lambda+\gamma)(\lambda+\gamma+\mu) & (\lambda+\gamma)(1-p)\beta + \beta\gamma p \\ \alpha\beta p & \alpha(\lambda+\gamma) & (\lambda+\gamma)(\lambda+\beta) \end{bmatrix}$$

Using Neuts(1978) and Lucantoni(1979), the matrix R is numerically computed by using the recurrence relation with $R(0) = 0$ in equation (8).

Theorem: 2.3

If $\rho < 1$, the stationary probability vectors p_0 and $p_i = (p_{0i}, p_{1i})$ are

$$p_0 = \left(\frac{\mu_1}{2\lambda}\right)\left[1 + \frac{\left[(\lambda+\beta+\mu_1) - \mu_1\left[\frac{1}{2} + r_0^2 + r_{01}r_{10}\right]\right]}{\left[\frac{\mu_1}{2} + \alpha + \mu_1\left(r10((r0+r1))\right)\right]}\right] p_{10}$$

$$p_{11} = \left[1 + \frac{\left[(\lambda+\beta+\mu_1) - \mu_1\left[\frac{1}{2} + r_0^2 + r_{01}r_{10}\right]\right]}{\left[\frac{\mu_1}{2} + \alpha + \mu_1\left(r10((r0+r1))\right)\right]}\right] p_{10}$$

$$p_{10} = \frac{1}{\frac{\mu_1}{2\lambda} + \left(\frac{\left[(\lambda+\beta+\mu_1) - \mu_1\left[\frac{1}{2} + {r_0}^2 + r_{01}r_{10}\right]\right]}{\left[\frac{\mu_1}{2} + \alpha + \mu_1\left(r_{10}(r_0+r_1)\right)\right]}\right) V_0 + \left(\frac{1-r_1+r_{01}}{(1-r_0)(1-r_1) - r_{10}r_{01}}\right)}$$

and $p_i = p_1 R^{i-1};\ i \geq 2$

where $V_0 = \left(\frac{\mu_1}{2\lambda} + \frac{1-r_0+r_{10}}{(1-r_0)(1-r_1) - r_{10}r_{01}}\right)$

Proof

p_0, p_{10} and p_{11} follows from the equations (1), (6), and (7).

Remark: 2.1

Even though R in Theorem 2.2 has a nice structure, it may not be easy to Cary out the computation required to calculate the p_i' s and the performance measures. Hence, we explore the possibility of algorithmic computation of R. The computation of R can be carried out using a number of well-known methods. We use Theorem 1 of Latouche and Neuts (1980). The matrix R is computed by successive substitutions in the recurrence relation:

$$R(0) = 0 \qquad (9)$$

$$R(n+1) = -A_0A_1^{-1} - [R(n)]^2A_2A_1^{-1} \text{ for } n \geq 0 \qquad (10)$$

and is the limit of the monotonically increasing sequence of matrices $\{R(n), n \geq 0\}$.

2.2 Performance Measures

Using straightforward calculations the following performance measures have been obtained:

(i) Probability that the servers are in idle mode $= p_{02}$

(ii) Probability that both the servers be busy $P_B = \sum_{n=1}^{\infty} p_{n2}$

(iii) Probability that the system is in essential repair mode $P_{ES} = \sum_{n=1}^{\infty} p_{n1}$

(iv) Probability that the system is in optional repair mode $P_{OS} = \sum_{n=1}^{\infty} p_{n0}$

(v) Expected number of customers in the system $E(L) = \sum_{n=0}^{\infty} p_{ne}$

(vi) Expected number of customers in the system when the system is optional

repair mode $E(N) = \sum_{n=1}^{\infty} p_{n0}$

(vii) Expected number of customers in the system when the system is essential

repair mode $E(K) = \sum_{n=1}^{\infty} p_{n0}$

(viii) Expected number of customers in the system when the servers are busy

mode $E(R) = \sum_{n=1}^{\infty} p_{n2}$

(ix) Expected number of customers served $E(M) = \mu \sum_{n=1}^{\infty} p_{n2}\, e$.

(x) Expected waiting time of a customer in the system, according to Little's law, is $E(W) = \frac{E(N)}{\lambda}$

III. Numerical Study

In this section, some examples are given to show the effect of the parameters λ, μ, α, β, γ on the Probability that the servers are in idle mode, the Probability that both the servers be busy, the Probability that the system is in essential repair mode, Probability that the system is in optional repair mode, Expected number of customers in the system, Expected number of customers in the system when the system is in optional repair mode, Expected number of customers in the system when the system is in essential repair mode, Expected number of customers in the system when the servers are in busy mode, Expected number of customers served, and Expected waiting time of a customer in the system for the model analyzed in this paper. The corresponding results are presented in graphs and tables.

In the graphs, the probabilities concerning various states are presented for different values of the probability p (0.01, 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0). We fix the parameters $\lambda = 0.4, \mu = 3.0, \alpha = 0.5, \beta = 0.8$ and$\gamma = 0.4$. First, we find the R matrix, and using equations (1), (7), and (4) we obtain the probabilities $p_{02}, p_i : i \geq 1$. From these probabilities, we calculated the performance measures in subsection 2.2. In Figure 1, the probabilities corresponding to the optional repair state are presented, in figure 2, the probabilities corresponding to the essential repair state are shown, in figure 3, the probabilities corresponding to the servers' busy state are given and in graph 4, the probabilities of a number of customer in the system are depicted.

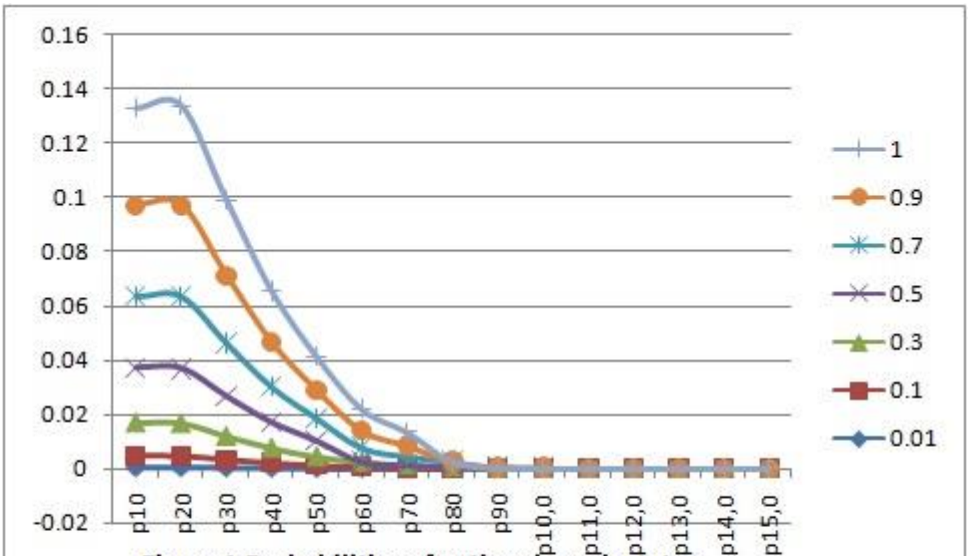

Figure 1:Probabilities of optional repair state

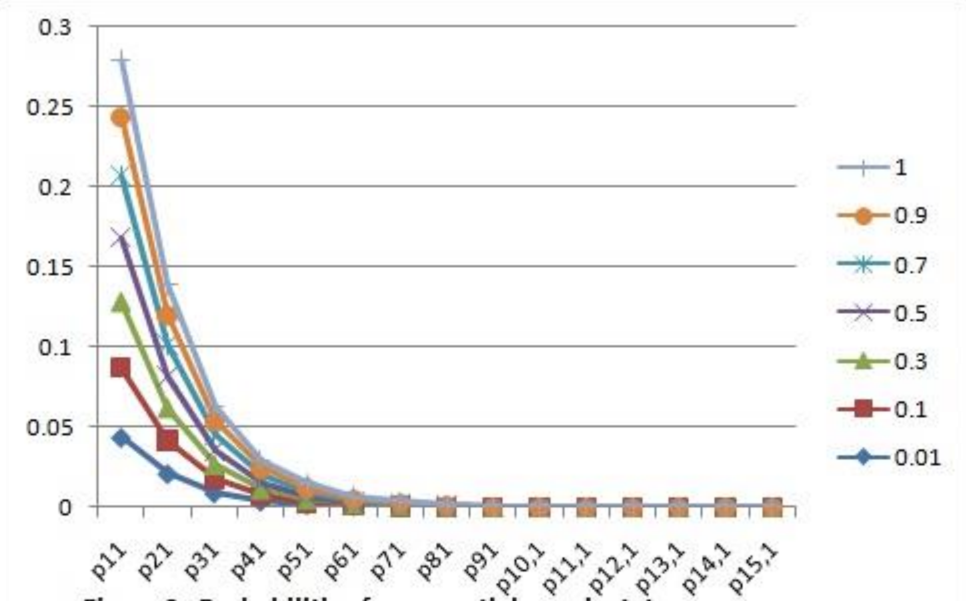

Figure 2: Probabilities for essential repair state

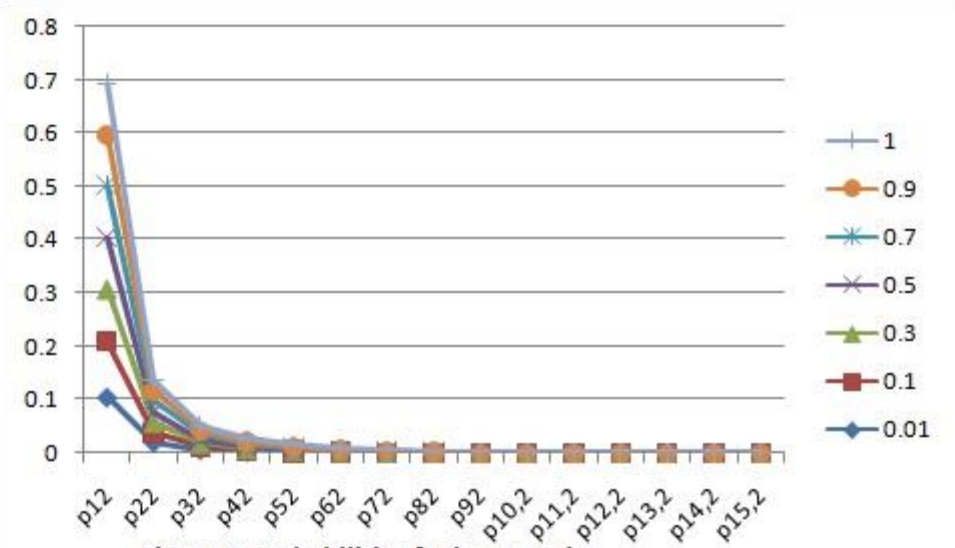

Figure 3: Probabilities for busy mode

From all the graphs it is clear that as the number of customers in the system is too large, the corresponding probabilities are too small. In all the figures, it is seen that as the values of p increases, the values of probabilities of several customers in the system concerning different system state decreases.

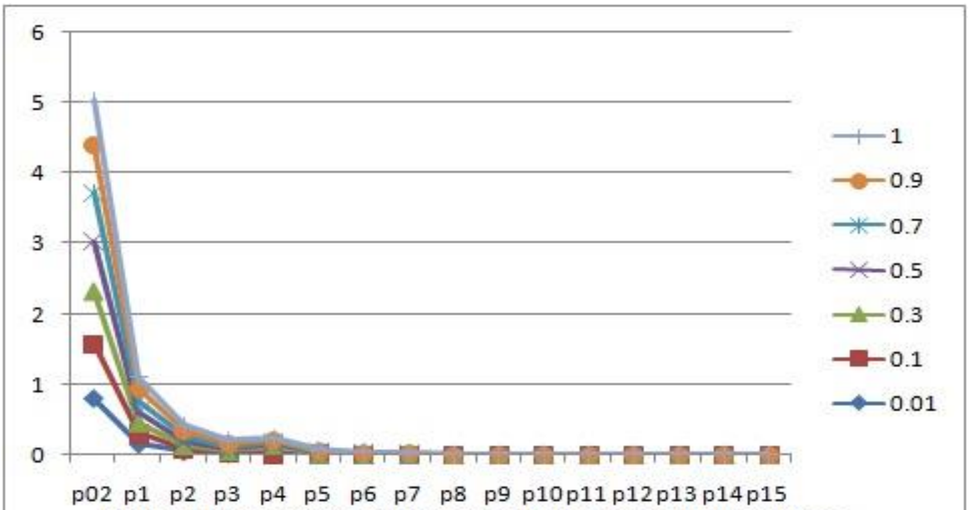

Figure 4:Probabilities for number of customers in the system

Table1: Idle Probability

p	0.01	01	0.2	0.3	0.4	0.5	0. 6	0.7	0.8	0.9	1.0
p_{02}	0.787 4	0.779 9	0.758 7	0.743 7	0.725 3	0.721 4	0. 7	0.685 6	0.671 5	0.667 1	0.643 7

Table 2: Probabilities P_B, P_{ES}, P_{os}

p	0.01	01	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
p_B	0.13 19	0.13 08	0.13 15	0.13 14	0.12 64	0.12 92	0.130 56	0.13 51	0.13 01	0.13 26	0.13 57
P_{ES}	0.07 91	0.07 65	0.07 85	0.07 81	0.08 19	0.07 56	0.077 16	0.07 64	0.07 62	0.07 35	0.07 48
P_{OS}	0.00 16	0.01 34	0.03 13	0.04 68	0.06 06	0.06 98	0.092 28	0.10 50	0.12 19	0.12 84	0.14 44

In Table 1, the probability that the servers are in idle mode is shown. From the table, it is clear that as p increases the probability value decreases. This conclusion coincides with our guss.

Table 3: Expected Number of Customers in the System

p	E(N)	E(K)	E(R)	E(M)	E(L)
0.001	0.0043	0.1183	0.1616	0.3917	0.3141
0.1	0.0292	0.1258	0.1673	0.3924	0.3222
0.2	0.0882	0.1436	0.1834	0.3945	0.4152
0.3	0.1346	0.1466	0.1903	0.3942	0.4714
0.4	0.1598	0.1569	0.1767	0.7473	0.4934
0.5	0.1702	0.1352	0.1838	0.3876	0.4893
0.6	0.2787	0.1554	0.2104	0.3917	0.6446
0.7	0.3018	0.1526	0.2149	0.4053	0.6693
0.8	0.3803	0.1616	0.2245	0.3903	0.7663
0.9	0.3292	0.1387	0.1993	0.3972	0.6672
1.0	0.3774	0.1466	0.2206	0.4071	0.7446

Table 4: Expected Waiting time

p	0.01	01	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
E (W)	0.7853	0.8085	1.0379	1.1786	1.2335	1.2233	1.6115	1.6733	1.9158	1.668	1.8615

In Table 2, we present the probabilities related to the servers being busyP_B, the system is in essential repair modeP_{Es}, the system is in optional repair modeP_{OS}. Table 3 shows the Expected number of customers in different system states. From the table, it is clear that as p increases the expected number of customers in the system also increases. Expected waiting times in the system are given in table 4. This table shows that as p increases the mean waiting time also increases.

IV.a Case Study

A production system has been characterized by its constituent elements and their relations as well as the relations of transformation of input raw material into output resources. It is a purposefully designed and organized material, energetic, and information system used for the production of products (goods, services) to satisfy the consumers' needs (Durilk (2005)). As a case study, we consider the production process considered in the article by Witold and Juraj (2018). In their paper they analyzed a production process of a two-part toothed rim; three machines are used in the production process of the two-part toothed rim: plate-typed milling and boring machine, a turning lathe, and a toothed rim machining tool. In their paper, a failure frequency analysis has been carried out for the turning lathe.

In this paper, we fit this production system to our queueing model. The arrival of raw materials follows the Poisson process and the service system has three machines (plate-typed milling and boring machine, a turning lathe, and a toothed rim machining tool).

In each machine the processing time follows negative exponential distributions with rates μ_1(plate-typed milling and boring machine) and μ_2(turning lathe and a toothed rim machining tool) with$\mu = \mu_1 + \mu_2$. The total processing time (called service time) is negative exponential with rateμ. The system failure occurs based on a Poisson process with rateα, the system is immediately sent for repair. At this point, we have considered the model with repair (called essential repair) and optional repair (The model considered in this paper). The repair period (essential) and optional repair period follow different exponential distributions with rates β(essential repair period), γ (optional repair period). After the repair has been carried out the system is reinstalled. Using Table 3.2, we calculate the number of services, the number of essential repairs, and the number of optional repairs by assuming the total number of events as 10000. The results are presented in Table 4. From the table, it is clear that the optional repair period has a great impact on the model.

Table 5: Number of Events

p	Number of services	Number of essential repairs	Number of optional repairs
0.01	1316	788	16
0.1	1308	765	134
0.2	1315	785	313
0.3	1314	781	468
0.4	1284	829	626
0.5	1292	776	718
0.6	1306	772	922
0.7	1351	764	1050
0.8	1301	762	1216
0.9	1326	735	1284
1.0	1371	748	1444

In this case study, we fit the production system to our model, which leads to an equivalent to a theoretical model. This model can be manipulated for any real situation by taking suitable parameters.

V. Conclusion

In this paper, we have solved heterogeneous two servers' Markovian queues with two different breakdowns. Once breakdown occurs immediately repair takes place in two modes namely, essential mode and optional mode. This model can be generalized by taking C heterogeneous servers$(C > 2)$. But the solution of this general model is not easy, since it has a mathematically complicated structure. In another direction, the same model can be generalized by taking service time distributions that are general. Solving this model in the existing setup is also not easy.

Conflict of Interest: There are no conflicts of interest regarding the publication of this research article.

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