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A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION

Prabir Chandra Bhattacharyya

Professor, Department of Mathematics and Director of Research and Development, Institute of Mechanics of Continua and Mathematical Sciences, India

P-9/1, LIC Township, Madhyamgram, Kolkata - 700129, West Bengal, India.

E-mail : prabirbhattacharyya@yahoo.com

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Abstract

The basic idea of a quadratic equation is one of the most important topics in algebra. The mathematical concept for the method of solution of a quadratic equation is dependent on the advancement of the theory of numbers. The author developed a new concept regarding the method of solution of the quadratic equation based on "Theory of Dynamics of Numbers". The author determined the inherent nature of one unknown quantity (say x) from the quadratic expression $ax^2 + bx + c$ *of the quadratic equation* $ax^2 + bx + c = 0$ *by keeping the structure of the second-degree expression intact and then finding the solution of the quadratic equation using the novel concept of the Theory of Dynamics of Numbers.*

The author solved any quadratic equation in one unknown number (say x) of the quadratic equation in the form of $ax^2 + bx + c = 0$*, whether the numerical value of the discriminant is* $b^2 - 4ac \geq 0$ *or* $b^2 - 4ac < 0$*, is real numbers only without using any imaginary numbers. With these new inventive concepts, the author developed new theories in the theory of quadratic equation.*

Keywords: Bhattacharyya's Co-ordinate System, Cartesian Co-ordinate System, Quadratic Equation, Theory of Dynamics of Numbers, Theory of Numbers

I. Introduction

The invention of a method of solution of the quadratic equation from past k./to present there were four basic methods, namely (1) The square root method (2) Completing Square (3) Quadratic formula (4) Factorization. The object of this present paper is to develop a new method of solution of the quadratic equation based on the newly invented mathematical concept of the 'Theory of Dynamics of Numbers'[Bhattacharyya P. C., January 2022, pp 37-53].

According to the theory of dynamics of numbers, 0 (zero) is the starting point of any number. There are infinite numbers of directions through which the numbers can move from the starting point 0 (zero). The numbers which are moving away from the origin 0 are called the Countup numbers while the numbers moving toward the origin 0 are called Countdown numbers.

Countup numbers are denoted as $\overrightarrow{x}$, $\overrightarrow{y}$, $\overrightarrow{z}$ ……

The numerical value of Countup $x = \overrightarrow{x} = +x$

Countdown numbers are denoted as $\overleftarrow{x}$, $\overleftarrow{y}$, $\overleftarrow{x}$ ……

The numerical value of Countdown $x = \overleftarrow{x} = -x$

There are three laws of the theory of dynamics of numbers [Bhattacharyya P. C., January 2022, pp 37-53] :

(1) 0 (zero) is defined as starting point of any number. There is an infinite number of directions through which the numbers can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers.

(2) The Countup numbers are always greater than or equal to the countdown numbers. The Countup numbers can move independently but the motion of the countdown numbers is dependent on the motion of Countup numbers. The motion of the countdown numbers exists if and only if there are motions of the Countup numbers.

(3) For every equation, the Countup numbers are always equal to the countdown numbers.

Another basic mathematical concept to find the root of the quadratic equation depends on the inherent nature of one unknown quantity (say x) in the quadratic equation.

For Example :

$$x^2 = (+x) \times (+x) \tag{1}$$

$$x^2 = (-x) \times (-x) \tag{2}$$

From equation (1) and (2), numerical values of x^2 are one and same but inherent nature of unknown quantity x is + x of x^2 of equation (1) and the inherent nature of unknown quantity x is – x of x^2 in equation (2). Though the numerical value of x^2 of equation (1) and equation (2) are one and same but according to the inherent nature of one unknown quantity, they are different. Therefore, the solution of equation (1) will be Countup $x = \overrightarrow{x} = +x$ only, but not $\pm x$. Similarly, the solution of the equation (2) will be Countdown $x = \overleftarrow{x} = -x$ only, but not $\pm x$.

However, similar problems as indicated above, in the case of the method of solution of the quadratic equation have not been investigated by a similar approach.

The object of the present paper is to investigate the inherent nature of one unknown quantity x of the quadratic equation and to find the method of solution of any quadratic equation in real numbers only without using imaginary numbers, even if the discriminant $b^2 - 4ac < 0$ of the quadratic equation $ax^2 + bx + c = 0$.

Finally, the author presented new theories in quadratic equations introducing the new concept of theory of dynamics of numbers.

II. Literature Review

The origin of the quadratic equation was based on the concept of the rectangular area which had length and breadth. The unknown quantities such as length, breadth, or area can be calculated depending on the known quantities. A solution of the quadratic equation was found for the first time in Berlin papyrus (CA 2160 – 1700 BC) in Egypt [Smith, 1953, P. 443]. Indus civilization (3000 BC – 500 AD) had the famous centers of culture at Harappan, Mohenjo-Daro, Lothal, Dholavira, Kalibagan. Though the Indus civilization had not had any written evidence of development in the field of mathematics its presence was found in the construction of the cities, construction of buildings followed standardized measurements of bricks in the ratio of 4:2:1. It became difficult to establish connections since the script of the Harappans has not yet been deciphered. Standard weights and length measurement scales were known to Harappan civilization in the decimal system [Thapar, R., (2000)]. Without a strong mathematical base and background in precision, it was not possible to have such an advanced civilization. Even in absence of evidence, the effect of Indian mathematics on Babylonians can not be ruled out.

From Babylonian clay tablets (2000 – 1700 BC) Babylonian Mathematicians constructed a set of quadratic problems and solved the quadratic equations. The basic method of solving this problem is completing the square (Katz, 2007). There were eight types of Babylonian equations. Out of eight equations, the first six equations were regarded as Diophantine type because they were solved by the Diophantus method. The last two types of equations were denoted as the Arabic type because the Arabic algebra of Al-Khwarizmi first made them known and gave the solution for these. The Babylonian types are as follows (Gandz, 1937, P. 405).

I. $x + y = a;\ xy = b \quad \left.\begin{matrix} x \\ y \end{matrix}\right\} = \frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - b}$

II. $x - y = a;\ xy = b \quad \left.\begin{matrix} x \\ y \end{matrix}\right\} = \sqrt{\left(\frac{a}{2}\right)^2 - b} \pm \frac{a}{2}$

III. $x + y = a;\ x^2 + y^2 = b \quad \left.\begin{matrix} x \\ y \end{matrix}\right\} = \frac{a}{2} \pm \sqrt{\frac{b}{2} - \left(\frac{a}{2}\right)^2}$

IV. $x - y = a;\ x^2 + y^2 = b \quad \left.\begin{matrix} x \\ y \end{matrix}\right\} = \sqrt{\frac{b}{2} - \left(\frac{a}{2}\right)^2} \pm \frac{a}{2}$

V. $x + y = a;\ x^2 - y^2 = b \quad \left.\begin{matrix} x \\ y \end{matrix}\right\} = \frac{a}{2} \pm \frac{b}{2a} = \frac{1}{2} . \frac{a^2 \pm b}{a}$

VI. $x - y = a;\ x^2 - y^2 = b \quad \left.\begin{matrix} x \\ y \end{matrix}\right\} = \frac{b}{2a} \pm \frac{a}{2} = \frac{1}{2} . \frac{b \pm a^2}{a}$

VII. $x^2 + ax = b$ (A.I)

VIII. $x^2 - ax = b$ (A.I)

Euclid (325 – 270), a Greek Mathematician used the Babylonian problem and their solution method by operating differently on geometrical figures to solve quadratic equations (Gandz, 1937). In the 9th century, Al-Khwarizmi tried to solve the same problems in Babylonian mathematics by using the Arabic type as different from their solution method. He worked on the abstract problem and applied the algorithm to solve the quadratic equation algebraically. There were a good number of a mathematician who contributed to the development of quadratic equations. Al – Karki, Savadorsa, Ibn Erza, Immanuel Bonfils contributed their studies from Babylonian and Egyptian tradition (Gandz, 1937). The mathematicians of Babylon, Egypt, Greek, Arabic countries had no perception of negative numbers in the quadratic equations at that time. Indian mathematician, Bhaskara – II (1114 – 1185 AD) introduced the negative roots and showed that the solution of the quadratic equation could be both real and imaginary [Katz, 1998. Pp. 226-227].

Formulation of a quadratic equation as $x^2 + (b - a)x = \frac{1}{2}[c^2 - (b - a)^2]$ was found in the text available in Chinese Mathematics which is in "Nine Chapters on Mathematical Art" (≅100 BC) written by Jiu Shang Suanchu. At that ancient period, the Chinese used geometrical models, using arithmetic of counting rods, to express their algebraic notations [Ling, W. & Needham, J., 1955].

Aryabhata (476 – 550 AD) can be regarded as the inventor of Algebra in India. Aryabhata has used the quadratic equations but has not given a solution anywhere. Sutra [10, Sutra 25] states the rule on how to calculate the interest on the principal. Original Sutra was written in Sanskrit verse. English version of the sutra states that "Multiply Amount (A), Time (t) and Principal (p) and add half of the principal square. Take the square root and subtract half of the principal" [Dutta, B.B. 1929].

In general,

$$\sqrt{Apt + \left(\frac{p}{2}\right)^2} - \frac{p}{2}$$

Divide it by time (t), the result is interest (I) for one month

$$I = \frac{\sqrt{Apt + \left(\frac{p}{2}\right)^2} - \frac{p}{2}}{t}$$, which is the solution of the quadratic equation

$$tx^2 + px - Ap = 0$$

if we consider I = x (unknown)

Sridhara Acharya (870 – 930) a Bengali, Hindu Pandit, and Mathematician of India was the first person who had given an algorithm for solving Quadratic Equations in Sanskrit Verse. [http://en.wikipedia.org/wiki/Shridhara]. His formula for solving quadratic equations had become available from the quotation of Bhaskara II and others [Dutta, B.B. 1929]. B. B. Dutta has translated the Sanskrit verse into English which is as follows. :

"Multiple both the sides (of an equation) by a known quantity equal to four times the coefficient of the square of the unknown, add both the sides the known quantity equal to the square of the (original) coefficient of the unknown; then extract the root."

Now, the Sanskrit verse takes the form

$$ax^2 + bx = c$$

Multiply both side by 4a to get.

$$4a.ax^2 + 4a.bx = 4a.c$$

$$\Rightarrow 4a^2x^2 + 4abx = 4ac$$

Then adding b^2 to both sides

$$4a^2x^2 + 4abx + b^2 = b^2 + 4ac$$

$$\Rightarrow (2ax + b)^2 = b^2 + 4ac$$

$$\Rightarrow 2ax + b = \sqrt{b^2 + 4ac}$$

$$\Rightarrow 2ax = -b + \sqrt{b^2 + 4ac}$$

$$\Rightarrow x = \frac{-b + \sqrt{b^2+4ac}}{2a}$$

Sridhara did not consider the negative sign. Maintaining the same approach Bhaskar – II considered two values as the solution of the quadratic equation and he had considered ± sign with square root (Smith 1951, P. 159). Brahmagupta (628 AD) considered one root (Smith 1951, p – 159; Smith 1953, P 445 – 446) [ ]. Cardano had given complex solutions for the quadratic equation. Viete invented the relations between roots of a quadratic equation and the coefficient and a constant of the equation (Smith 1953, P 445 – 446). In the 17th century, René Descartes discovered a complete solution for both positive and negative imaginary roots of the quadratic equation (Katz 1998, p 448). In 1637, Descartes published a book "La Geometric" which contained special cases of the quadratic formula in the form known today (Cooley, 1993, P. 95-96).

P. C. Bhattacharyya published a paper "An Introduction to Theory of Dynamics of Numbers: A New Concept" in January 2022. In the light of this theory, there shall be one and only one root of any quadratic equation and that one root may be positive or negative in real number only based on unknown quantity (say x) whether it is 'Countup' number or 'Countdown' number even if the discriminant, $b^2 - 4ac < 0$, of the quadratic equation $ax^2 + bx + c = 0$. According to Bhattacharyya, there is no existence of the solution of the quadratic equation in complex numbers.

III. Formulation of the problem and its solution :

An equation in one unknown quantity (say x) in the form $ax + b = 0$ is called a linear equation in one degree. Here $a \neq 0$ is the coefficient of x and b is the constant term.

Let us consider a problem of linear equation in one unknown quantity (say x) in one degree.

Problem – 1

A man started his morning walk from his home to a park and returned from the park to home in the same path. While returning home he noticed from k.m. stone on the roadside that he has covered a 5-kilometer distance to reach home. Formulate the problem and solve it.

Solution :

We can formulate the problem in a linear equation in one degree :

$$x - 5 = 0 \tag{1}$$

Thus, the inherent nature of unknown quantity x is a Countup motion in x from home to park and the inherent nature of returning from the park to home covering a 5-kilometer distance is a Countdown motion in 5.

So, equation (1) takes the form

$$\overrightarrow{x} + \overleftarrow{5} = 0 \tag{2}$$

According to the 3rd law of Theory of Dynamics of Numbers: For every equation, the Countup number is always equal to the Countdown number.

Therefore,

$$x = 5 \tag{3}$$

Thus, inherent nature of x is Countup x

$\therefore$ The numerical value of $\overrightarrow{x} = + x = + 5$

$\therefore$ x = + 5, is the solution of the equation (1).

Problem – 2

Solve :

$$x + 5 = 0$$

Solution :

$$x + 5 = 0 \tag{1}$$

Here, + 5 > 0, so + 5 is Countup number in 5

$\therefore$ The equation (1) takes the form

$$\overleftarrow{x} + \overrightarrow{5} = 0 \tag{2}$$

According to the 3rd law of Theory of Dynamics of Numbers

$$x = 5 \tag{3}$$

Because the inherent nature of x is Countdown x and the numerical value of

$$\overleftarrow{x} = -x$$

$$\overleftarrow{x} = -x = -5$$

is the solution of the equation (1)

Definition of a Quadratic Equation in one unknown quantity (say x):

An equation in one unknown quantity (say x) in the form $ax^2 + bx + c = 0$ is called a quadratic equation or an equation in the second degree. Here, $a \neq 0$ is called the coefficient of x^2, b the coefficient of x, and c the constant term.

The numerical value of the inherent nature of x which satisfies the quadratic equation is called the root of the equation. The inherent nature of x can be determined uniquely from the nature of the constant term of quadratic equation $ax^2 + bx + c = 0$ provided the character of the structure of the second-degree expression, $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ must be in the second degree.

How to determine the inherent nature of one unknown quantity (say x):

Let us consider the quadratic equation

$$ax^2 + bx + c = 0$$

Case – I

If $c > 0$, the inherent nature of unknown quantity x will be the Countdown number.

Case – II

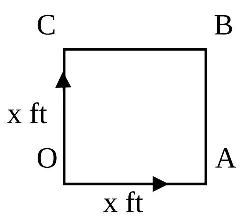
If $c < 0$, the inherent nature of unknown quantity x will be the Countup number.

Case – III

If $c = 0$, the inherent nature of unknown quantity x will be 0 (zero)

Problem – 3

Find the length and breadth of a square land whose area is 49 square ft.



Solution :

Let us construct a square land with a length and breadth of x ft. each and the area of square land is 49 sqft.

Let us consider square OABC whose length OA = x ft and breadth OC = x ft.

Therefore, the area of the square land is x^2 sqft.

According to the problem

$$x^2 = 49 \tag{1}$$

$$\Rightarrow x^2 - 49 = 0 \tag{2}$$

Now, we solve the quadratic equation (2) by factorization method

$$x^2 - 49 = 0$$

$$\Rightarrow (x + 7)(x - 7) = 0 \tag{3}$$

$\therefore$ x + 7 = 0 and x – 7 = 0

Therefore, x = – 7 and x = + 7

$\therefore$ Length OA = + 7 ft and breadth OC = + 7 ft

And also

Length OA = – 7 ft and breadth OC = – 7 ft

Now, the question arises whether we can construct a square land with length = – 7 ft and breadth = – 7 ft.

The answer will be 'No' but computationally it is true.

Why it has happened?

Because we have degenerated the structure of the quadratic expression of the quadratic equation $x^2 - 49 = 0$ by factorization into two linear one-degree expressions as (x + 7) and (x – 7) of the quadratic equation $x^2 - 49 = 0$.

Now, let us solve $x^2 - 49 = 0$ by using the theory of dynamics of numbers.

$$x^2 - 49 = 0 \tag{1}$$

Since – 49 < 0, the inherent nature of x is Countup x

$\therefore$ The equation (1) takes the form

$$\overrightarrow{x^2} + \overleftarrow{49} = 0 \tag{2}$$

According to 3 rd law of theory of dynamics of numbers

$$x^2 = 49 \tag{3}$$

$\Rightarrow$ x = 7

Here, the inherent nature of x is Countup $x = \overrightarrow{x} = +x$

$\therefore$ x = + 7 only, is the solution of the equation (1)

$\therefore$ Length and breadth of the square = + 7 ft.

Observation :

I. The inherent nature of unknown quantity x depends on the structural characteristics of the second-degree expression of the quadratic equation. But by factorization, the structural character of the second-degree expression has been converted into the one-degree structural character of two linear expressions in one degree. These two linear equations may conceive two different unknowns in x though the numerical values of x^2 are one and the same. This is a gross violation of the definition of quadratic equation in one unknown quantity (say x)

II. If we solve the quadratic equation by using theory of dynamics of numbers the question of solution of the quadratic equation $x^2 - 49 = 0$ as $x = -7$ does not arise.

III. The solution of any quadratic equation by the conventional method is incorrect until they consider the inherent nature of the unknown quantity x.

Geometrical representation for the formulation of the quadratic equation and its algebraic solution:

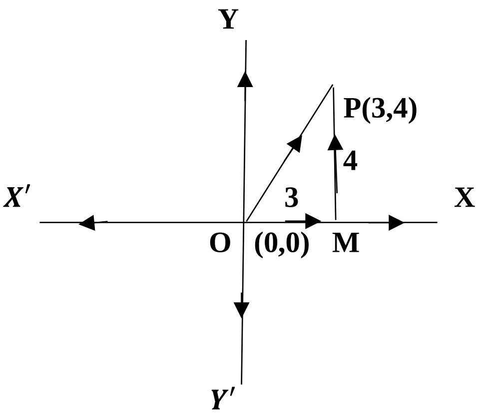


Fig. 1 (a)

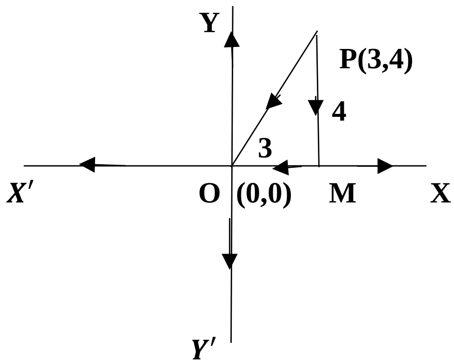


Fig. 1 (b)

From fig. 1 (a)

$$\overrightarrow{OP^2} = \overrightarrow{OM^2} + \overrightarrow{MP^2}$$

$$\Rightarrow \overrightarrow{OP^2} = \overrightarrow{3^2} + \overrightarrow{4^2}$$

$$\Rightarrow \overrightarrow{OP^2} = \overrightarrow{9} + \overrightarrow{16}$$

$$\Rightarrow \overrightarrow{OP^2} = \overrightarrow{25}$$

$$\Rightarrow \overrightarrow{OP^2} = \overrightarrow{5^2}$$

$$\Rightarrow \overrightarrow{OP} = \overrightarrow{5}$$

Numerical value of $\overrightarrow{5} = +5$ (According to theory of dynamics of numbers)

From fig. 1 (b)

$$\overrightarrow{PO^2} = \overrightarrow{PM^2} + \overrightarrow{MO^2}$$

$$\overleftarrow{OP^2} = \overleftarrow{MP^2} + \overleftarrow{OM^2}$$

$$\overleftarrow{OP^2} = \overleftarrow{4^2} + \overleftarrow{3^2}$$

$$\overleftarrow{OP^2} = \overleftarrow{16} + \overleftarrow{9}$$

$$\overleftarrow{OP^2} = \overleftarrow{25}$$

$$\overleftarrow{OP} = \overleftarrow{\sqrt{25}}$$

$$\overleftarrow{OP} = \overleftarrow{5}$$

Numerical value of $\overleftarrow{5} = -5$ (According to theory of dynamics of numbers)

through the distance $\overline{OP} = \overline{PO}$

Observations :

I. Countup OP + Countdown $OP = \overrightarrow{OP} + \overleftarrow{OP} = \overrightarrow{5} + \overleftarrow{5} = +5 - 5 = 0$. Thus, satisfies the 3 rd law of the theory of dynamics of numbers.

II. For solving the said problem by the Cartesian co-ordinate system we can find that $\overline{OP} = \pm 5$ which means $\overline{OP}\ +\ 5$ and $\overline{OP} = -5$. Though $\overline{OP} = -5$ has been discarded with the argument that distance can not be negative. But if we solve the problem by Bhattacharyya's co-ordinate system [Bhattacharyya P. C., November 2021, pp 76-86] which is based on the theory of dynamics of numbers, the question of $\overleftarrow{OP} = -5$ does not arise.

From fig.1 (a) we can construct a Quadratic Equation in the form :

Let us consider $\overline{OP} = x$.

$$x^2 - 25 = 0 \tag{1}$$

Since $-25 < 0$, equation (1) takes the form

$$\overrightarrow{x^2} + \overleftarrow{25} = 0 \tag{2}$$

According to 3 rd law of theory of dynamics of numbers

$$x^2 = 25 \tag{3}$$

$$\Rightarrow x = 5 \tag{4}$$

Therefore, the inherent nature of unknown quantity of x = 5 will be in Countup $x = \overrightarrow{x}$

So, $\overrightarrow{x} = \overrightarrow{5} = +5$ is the solution of equation (1)

Now, let us examine whether the numerical value of $\overrightarrow{x} = \overrightarrow{5} = +5$, satisfy the equation (1) or not.

According to the theory of dynamics of numbers of equation (1)

$$x^2 - 25 = 0$$

will take the form

$$\overrightarrow{x^2} + \overleftarrow{25} = 0$$

Now,

$$\overrightarrow{x^2} + \overleftarrow{25}$$

$$\Rightarrow +(5)^2 - 25$$

$$\Rightarrow +25 - 25$$

$$\Rightarrow 0$$

Thus, the numerical value of $\overrightarrow{x} = +5$, satisfies the equation (1)

Similarly, from fig 1 (b) we can construct a Quadratic Equation in the form

$$x^2 + 25 = 0 \qquad (6)$$

Since, + 25 > 0 , equation (6) takes the form

$$\overleftarrow{x^2} + \overrightarrow{25} = 0 \qquad (7)$$

According to 3 rd law of theory of dynamics of numbers

$$x^2 = 25 \qquad (8)$$

$$\Rightarrow x = 5 \qquad (9)$$

$\therefore$ The unknown quantity x = 5, will be in Countdown motion of x

So, $\overleftarrow{x} = -5$ is the solution of the quadratic equation (6)

Now, let us examine whether the numerical value of $\overleftarrow{x} = \overleftarrow{5} = -5$, satisfies the equation (6) or not.

According to the theory of dynamics of numbers the equation (6)

$$x^2 + 25 = 0$$

will take the form

$$\overleftarrow{x^2} + \overrightarrow{25} = 0$$

According to 3rd law of theory of dynamics of numbers

$$x^2 = 25$$

$$\Rightarrow x = 5$$

Now,

$$\overleftarrow{x^2} + \overrightarrow{25}$$

$$\Rightarrow -x^2 + 25$$

$$\Rightarrow -(5)^2 + (+25)$$

$$\Rightarrow -25 + 25$$

$$\Rightarrow 0$$

Thus, the numerical value of $\overleftarrow{x} = -5$ satisfies the equation (6)

It shows that solution of any Quadratic Equation is dependent on inherent nature of unknown quantity (say x)

Problem 4 : Solve

$$x^2 - 1 = 0 \qquad (1)$$

and

$$x^2 + 1 = 0 \qquad (2)$$

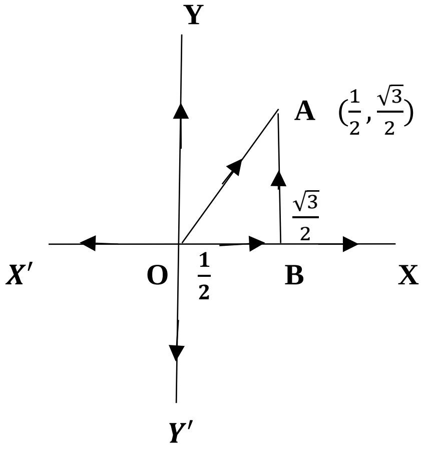


Fig. 2 (a)

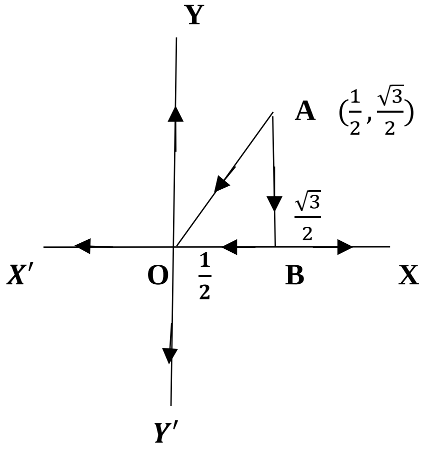


Fig. 2 (b)

From fig. 2 (a)

$$\overrightarrow{OA^2} = \overrightarrow{OB^2} + \overrightarrow{BA^2}$$

$$\Rightarrow \overrightarrow{OA^2} = \overrightarrow{(\frac{1}{2})^2} + \overrightarrow{(\frac{\sqrt{3}}{2})^2} = \overrightarrow{\frac{1}{4}} + \overrightarrow{\frac{3}{4}} = \overrightarrow{1}$$

$$\Rightarrow \overrightarrow{OA} = \overrightarrow{\sqrt{1}} = \overrightarrow{1}$$

Therefore, the numerical value of $\overrightarrow{1}$ = + 1 (According to the theory of dynamics of numbers)

From fig. 2 (b)

$$\overrightarrow{AO^2} = \overrightarrow{AB^2} + \overrightarrow{BO^2}$$

$$\Rightarrow \overleftarrow{OA^2} = \overleftarrow{BA^2} + \overleftarrow{OB^2} = \overleftarrow{(\frac{\sqrt{3}}{2})^2} + \overleftarrow{(\frac{1}{2})^2} = \overleftarrow{\frac{3}{4}} + \overleftarrow{\frac{1}{4}} = \overleftarrow{1}$$

$$\Rightarrow \overleftarrow{OA} = \overleftarrow{\sqrt{1}} = \overleftarrow{1}$$

Numerical value of $\overleftarrow{1}$ = – 1 (According to the theory of dynamics of numbers)

From fig. 2 (a) we can construct a quadratic equation in the form :

Let us consider $\overline{OA} = x$

$$x^2 - 1 = 0 \tag{1}$$

Since – 1 < 0, we can write the equation (1) in the form

$$\overrightarrow{x^2} + \overleftarrow{1} = 0 \tag{2}$$

According to 3 rd law of theory of dynamics of numbers

$$x^2 = 1 \tag{3}$$

$$\Rightarrow x = \sqrt{1} = 1 \tag{4}$$

$$\therefore \overrightarrow{x} = \overrightarrow{1} = +1 \tag{5}$$

Therefore, The inherent nature of unknown quantity x = 1 will be in Countup motion of x

So, $\overrightarrow{x} = +1$ is the solution of equation (1)

Again, from fig. 2 (b) we can construct a quadratic equation in the form :

Let us consider $\overline{AO} = x$

$$x^2 + 1 = 0 \tag{6}$$

Since, 1 > 0, we can write the equation (6) in the form

$$\overleftarrow{x^2} + \overrightarrow{1} = 0 \tag{7}$$

According to 3 rd law of theory of dynamics of numbers

$$x^2 = 1 \tag{8}$$

$$\Rightarrow x = \sqrt{1}$$

$$\Rightarrow x = 1 \tag{9}$$

$$\therefore \overleftarrow{x} = \overleftarrow{1} = -1 \tag{10}$$

Therefore, The inherent nature of unknown quantity x = 1 will be in Countdown motion of x

So, $\overleftarrow{x} = -1$ is the solution of equation (6)

Note : If we solve this equation (6) by conventional method

$$x^2 + 1 = 0$$

$$\Rightarrow x^2 = -1$$

$$\Rightarrow x = \pm\sqrt{-1}$$

Or, x = ± i, where $\sqrt{-1} = i$ (imaginary quantity)

Observation :

I. We cannot find the solution of equation (6) since we do not know the numerical value of $\sqrt{-1}$ or i.

II. We can find the solution of equation (6) in numerical value with the help of the theory of dynamics of numbers, i. e. $\overleftarrow{x} = -1$

III. Thus, $\overleftarrow{x} = -1 \neq \sqrt{-1}$

IV. Therefore, we can find the solution of equation (6) without using imaginary numbers.

Theorem – 1. A quadratic equation in one unknown quantity (say x) has one and only one root.

We consider the quadratic equation of the general form concerning the inherent nature of one unknown quantity. There are two types of form: Form A and Form B.

<u>Form A</u> :

$$ax^2 + bx - c = 0 \; [a \neq 0]$$

Proof :

$$ax^2 + bx - c = 0 \tag{1}$$

Since, $-c < 0$, according to the theory of dynamics of numbers the equation (1) takes the form

$$\overrightarrow{ax^2 + bx} + \overleftarrow{c} = 0 \tag{2}$$

According to 3 rd law of theory of dynamics of numbers

$$ax^2 + bx = c \tag{3}$$

$$\Rightarrow x^2 + \frac{b}{a}x = \frac{c}{a}$$

$$\Rightarrow x^2 + 2.\frac{b}{2a}.x + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 = \frac{c}{a}$$

$$\Rightarrow (x + \frac{b}{2a})^2 = \frac{b^2}{4a^2} + \frac{c}{a}$$

$$\Rightarrow (x + \frac{b}{2a})^2 = \frac{b^2+4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow x = \frac{-b+\sqrt{b^2+4ac}}{2a} \tag{4}$$

From equation (2) we know the inherent nature of unknown quantity

$x = \frac{-b+\sqrt{b^2+4ac}}{2a}$ will be Countup x

Therefore, the value of x will be $\overrightarrow{x}$ and the numerical value of $\overrightarrow{x} = +x$

$\therefore \overrightarrow{x} = + \frac{-b+\sqrt{b^2+4ac}}{2a}$ will be the solution of equation (1)

<u>Form B</u> : $ax^2 + bx + c = 0 \; [a \neq 0]$

Proof : $ax^2 + bx + c = 0$ (5)

Since $c > 0$, according to the theory of dynamics of numbers the equation (5) takes the form

$$\overleftarrow{ax^2 + bx} + \overrightarrow{c} = 0 \tag{6}$$

According to 3 rd law of dynamics of numbers

$$ax^2 + bx = c \quad (7)$$

$$\Rightarrow x^2 + \frac{b}{a}x = \frac{c}{a}$$

$$\Rightarrow x^2 + 2.\frac{b}{2a}.x + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 = \frac{c}{a}$$

$$\Rightarrow (x + \frac{b}{2a})^2 = \frac{b^2}{4a^2} + \frac{c}{a}$$

$$\Rightarrow (x + \frac{b}{2a})^2 = \frac{b^2+4ac}{4a^2}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right) = \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow x = -\frac{b}{2a} + \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow x = \frac{-b+\sqrt{b^2+4ac}}{2a}$$

From equation (6) we know the inherent nature of unknown quantity $x = \frac{-b+\sqrt{b^2+4ac}}{2a}$ will be Countdown x.

Therefore, the value of x will be $\overleftarrow{x}$ and the numerical value of $\overleftarrow{x} = -x$

So, $\overleftarrow{x} = -\frac{-b+\sqrt{b^2+4ac}}{2a}$ will be the solution of equation (5)

Since the equation (1), $ax^2 + bx - c = 0$ and equation (5), $ax^2 + bx + c = 0$ is satisfied by one and only one numerical value of $x = +\frac{-b+\sqrt{b^2+4ac}}{2a}$ and $x = -\frac{-b+\sqrt{b^2+4ac}}{2a}$ respectively and there is no other value of x. It is evidence that equation (1) and equation (5) has one and only one root.

Fundamental Theorem of Algebra according to the Theory of Dynamics of Numbers:

Statement: Every algebraic equation with real coefficients has one and only one real root.

Theorem 2: A quadratic equation with one unknown quantity cannot have more than one distinct real root.

Proof : Consider the quadratic equation of the general form :

$$ax^2 + bx + c = 0 \ (a \neq 0) \quad (1)$$

And if possible, let us assume that equation (1) has two distinct roots

α *and* β. Then α *and* β will satisfy the equation (1).

$$\therefore a\alpha^2 + b\alpha + c = 0 \quad (2)$$

and

$$a\beta^2 + b\beta + c = 0 \tag{3}$$

Now, in equation (2), c > 0

So, according to the theory of dynamics of numbers, equation (2) takes the form

$$\overleftarrow{a\alpha^2 + b\,\alpha} + \vec{c} = 0 \tag{4}$$

According to the 3 rd law of the theory of dynamics of numbers we have

$$a\alpha^2 + b\alpha \;= c \tag{5}$$

$$\Rightarrow \alpha^2 + \frac{b}{a}\alpha = \frac{c}{a}$$

$$\Rightarrow \alpha^2 + 2.\frac{b}{2a}.\alpha \;+ (\frac{b}{2a})^2 - \;(\frac{b}{2a})^2 = \frac{c}{a}$$

$$\Rightarrow (\alpha + \frac{b}{2a})^2 = \frac{b^2}{4a^2} + \frac{c}{a}$$

$$\Rightarrow (\alpha + \frac{b}{2a})^2 = \frac{b^2+4ac}{4a^2}$$

$$\Rightarrow \alpha + \frac{b}{2a} = \sqrt{\frac{b^2+4ac}{4a^2}}$$

$$\Rightarrow \alpha + \frac{b}{2a} = \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow \alpha \;= -\frac{b}{2a} + \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow \alpha \;= \frac{-b+\sqrt{b^2+4ac}}{2a}$$

From equation (4) we know that the inherent nature of the unknown quantity $\alpha \;= \frac{-b+\sqrt{b^2+4ac}}{2a}$ will be Countdown α

The numerical value of $\overleftarrow{\alpha} \;= \;-\alpha$

So, $\overleftarrow{\alpha} \;= \;-\;\frac{-b+\sqrt{b^2+4ac}}{2a}$

Again, let us consider equation (3)

$$a\beta^2 + b\beta + c = 0 \tag{3}$$

Now, in equation (3), c > 0

So, according to the theory of dynamics of numbers, equation (3) takes the form

$$\overleftarrow{a\beta^2 + b\beta} + \vec{c} = 0 \tag{6}$$

According 3rd law of theory of dynamics of numbers we have

$$a\beta^2 + b\beta = c \qquad (7)$$

$$\Rightarrow \beta^2 + \frac{b}{a}\beta = \frac{c}{a}$$

$$\Rightarrow \beta^2 + 2.\frac{b}{2a}.\beta + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 = \frac{c}{a}$$

$$\Rightarrow (\beta + \frac{b}{2a})^2 = \frac{b^2}{4a^2} + \frac{c}{a}$$

$$\Rightarrow (\beta + \frac{b}{2a})^2 = \frac{b^2+4ac}{4a^2}$$

$$\Rightarrow (\beta + \frac{b}{2a}) = \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow \beta = -\frac{b}{2a} + \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow \beta = \frac{-b + \sqrt{b^2+4ac}}{2a}$$

Now, from equation (6) we know that the inherent nature of the unknown quantity $\beta = \frac{-b + \sqrt{b^2+4ac}}{2a}$ will be Countdown β

The numerical value of $\overleftarrow{\beta} = -\beta$

So, $\overleftarrow{\beta} = -\frac{-b + \sqrt{b^2+4ac}}{2a}$

Therefore, $\overleftarrow{\alpha} = \overleftarrow{\beta} = -\frac{-b + \sqrt{b^2+4ac}}{2a}$

The solution of equation (2) and equation (3) are one and the same. So, there cannot exist more than one distinct real root in any quadratic equation.

Formulation of Quadratic Equation whose root is given:

Problem 5: Find the quadratic equation whose root is $\frac{\sqrt{5}-1}{2}$

Solution: Let us consider

$$x = \frac{\sqrt{5}-1}{2} \qquad (1)$$

Since, the numerical value of root $x = \frac{\sqrt{5}-1}{2} > 0$, the inherent nature of x is Countup x. So, the root is $\overrightarrow{x} = +x$

Now,

$$x = \frac{\sqrt{5}-1}{2}$$

$$\Rightarrow 2x = \sqrt{5} - 1$$

$$\Rightarrow 2x + 1 = \sqrt{5}$$

$$\Rightarrow (2x+1)^2 = 5$$
$$\Rightarrow 4x^2 + 4x + 1 = 5$$
$$\Rightarrow 4x^2 + 4x = 5 - 1$$
$$\Rightarrow 4x^2 + 4x = 4$$
$$\Rightarrow 4(x^2 + x) = 4$$
$$\Rightarrow x^2 + x = 1 \qquad (2)$$

According to the 3rd law of dynamics of number, equation (2) takes the form

$$\overrightarrow{x^2 + x} + \overleftarrow{1} = 0 \qquad (3)$$

So, the required quadratic equation is

$$x^2 + x - 1 = 0 \qquad (4)$$

Observation :

We have considered

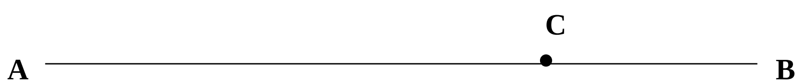
$$x = \frac{\sqrt{5}-1}{2}$$

Then the quadratic equation is

$$x^2 + x - 1 = 0$$
$$\Rightarrow x^2 = 1 - x$$

Geometrically, if AB = 1, AC = x, then

$AC^2 = AB.CB$



and AB is divided 'in golden section' by C. These relations are fundamental in construction of a regular pentagon inscribed in a circle (Euclid IV.II) [Hardy G. H. and Wright E. M, P. 52]

Formation of the Quadratic Equation whose root is given:

Problem 6 : Let us consider the root of the quadratic equation

$$x = -\frac{\sqrt{5}-1}{2} \qquad (1)$$

Since numerical value of $x = -\frac{\sqrt{5}-1}{2} < 0$

The inherent nature of x is Countdown x, that is $\overleftarrow{x} = -x$

So,

$$-x = -\frac{\sqrt{5}-1}{2} \qquad (2)$$
$$\Rightarrow \; x = \frac{\sqrt{5}-1}{2}$$
$$\Rightarrow \; 2x = \sqrt{5} - 1$$
$$\Rightarrow \; 2x + 1 = \sqrt{5}$$
$$\Rightarrow \; 4x^2 + 4x + 1 = 5$$

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$\Rightarrow\ 4x^2+4x=5-1$

$\Rightarrow\ 4x^2+4x=4$

$\Rightarrow\ x^2+x=1$ (3)

According to the third law of the theory of dynamics of numbers, we have Countdown number = Countup number. The equation takes the form

$$\overleftarrow{x^2+x}+\overrightarrow{1}=0 \quad (4)$$

So, the required equation is

$$x^2+x+1=0 \quad (5)$$

Observation: We know that the discriminant of quadratic equation,

$ax^2+bx+c=0$ is b^2-4ac.

Here, the discriminant of the quadratic equation $x^2+x+1=0$ is

$(1)^2-4.1.1=-3<0$

Though the discriminant of the quadratic equation $x^2+x+1=0,$ is less than 0 (zero) we can find the numerical value of x in real numbers by using the Theory of Dynamics of Numbers. But if we solve this quadratic equation $x^2+x+1=0$ by a conventional method.

The value of $x=\frac{-1\pm\sqrt{-3}}{2}$

Since we do not know the numerical value of $\sqrt{-3}$. We can not find the actual numerical solution of that quadratic equation.

Problem: 7

Solve : $x^2-x-6=0$

Solution : $x^2-x-6=0$ (1)

Because, $-6<0$, the inherent nature of x is Countup x.

According to the Theory of Dynamics of Numbers the equation (1) takes the form

$$\overrightarrow{x^2-x}+\overleftarrow{6}=0 \quad (2)$$

According to 3 rd law of the Theory of Dynamics of Numbers

$$x^2-x=6 \quad (3)$$

$\Rightarrow x^2-2.\frac{1}{2}x+(\frac{1}{2})^2-(\frac{1}{2})^2=6$

$\Rightarrow (x-\frac{1}{2})^2=6+(\frac{1}{2})^2$

$\Rightarrow (x-\frac{1}{2})^2=6+\frac{1}{4}$

$\Rightarrow (x-\frac{1}{2})^2=\frac{25}{4}$

$\Rightarrow (x-\frac{1}{2})=\sqrt{\frac{25}{4}}$

$\Rightarrow x-\frac{1}{2}=\frac{5}{2}$

$$\Rightarrow x = \frac{5}{2} + \frac{1}{2}$$

$$\Rightarrow x = 3$$

Since the inherent nature of x is Countup $x = \overrightarrow{x}$

Numerical value of $\overrightarrow{x} = +x$ is the solution of the equation (1)

Therefore, x = + 3

Now let us consider the solution of the quadratic equation by factorization method

$$x^2 - x - 6 = 0 \quad (1)$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x(x-3) + 2(x-3) = 0$$

$$\Rightarrow (x-3)(x+2) = 0 \quad (2)$$

$$\therefore\ x - 3 = 0 \text{ and } x + 2 = 0$$

$$\therefore\ x = +3 \text{ and } x = -2$$

Observation :

I. By factorization method, the basic structure of the second-degree equation in one unknown quantity (say x) has been converted into two linear equations of one degree in two unknown quantities (say x) ignoring the basic concept of the character of the structure of the quadratic equation.

II. The inherent nature of one unknown quantity (say x) of the quadratic equation has not been taken into account.

III. So, x = – 2, can not be the solution of the quadratic equation $x^2 - x - 6 = 0$, which is a gross violation of the definition of quadratic equation in one unknown quantity (say x).

Problem: 8

Solve :

$$x^2 + x + 3 = 0$$

Solution : $x^2 + x + 3 = 0$ (1)

Because, 3 > 0, the inherent nature of one unknown quantity x of the equation (1) is Countdown x.

Therefore, according to the theory of dynamics of numbers the equation (1) takes the form

$$\overleftarrow{x^2 + x} + \overrightarrow{3} = 0 \quad (2)$$

according to the 3 rd law of dynamics of numbers

$$x^2 + x = 3 \quad (3)$$

$$\Rightarrow x^2 + 2.\frac{1}{2}x + (\frac{1}{2})^2 - (\frac{1}{2})^2 = 3$$

$$\Rightarrow (x + \frac{1}{2})^2 = 3 + (\frac{1}{2})^2 = 3 + \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4}$$

$$\Rightarrow \left(x + \frac{1}{2}\right) = \frac{\sqrt{13}}{2}$$

$$\Rightarrow x = \frac{\sqrt{13}}{2} - \frac{1}{2}$$

$$\Rightarrow x = \frac{\sqrt{13}-1}{2}$$

According to the inherent nature of x, x is Countdown $x = \overleftarrow{x} = -x$.

So, the solution equation (1) will be

$$\overleftarrow{x} = -x = -\frac{\sqrt{13}-1}{2}$$

Now, let us solve equation (1) by a conventional method

$$x^2 + x + 3 = 0$$

$$\therefore x = \frac{-1 \pm \sqrt{(1)^2 - 4.1.3}}{2} = \frac{-1 \pm \sqrt{-11}}{2} = \frac{-1 \pm i\sqrt{11}}{2}$$

Observation :

I. We can find the solution of the equation (1) in numerical value which is $x = -\frac{\sqrt{13}-1}{2}$ by using the Theory of Dynamics of Numbers.

II. We can not find the solution of the equation (1) in the numerical value of x if we solve it by conventional method because we do not know the numerical value of $i = \sqrt{-1}$

IV. Conclusion

First of all, the author developed a new mathematical concept for the solution of the quadratic equation based on the novel concept of the Theory of Dynamics of Numbers. Applying this concept successfully the author determined the inherent nature of the unknown quality (say x) from the quadratic equation $ax^2 + bx + c = 0$ by keeping the structure of the second-degree expression of the quadratic equation intact. Introducing the concept of the inherent nature of one unknown quantity (say x) of the quadratic equation $ax^2 + bx + c = 0$, the author becomes successful to find the solution of any form of quadratic equation in one unknown (say x).

The author solved the quadratic equation in one unknown quantity (say x) in the form $ax^2 + bx + c = 0$ even if the numerical value of the discriminant $b^2 - 4ac < 0$ in real numbers only without considering imaginary numbers though the solution cannot be obtained without considering imaginary numbers by the conventional method till today.

Now, with these innovative concepts, the author developed new theories in the theory of quadratic equations. These mathematical concepts are applicable in any branch of mathematics, science, engineering, and technology.

Today or tomorrow this novel concept in the quadratic equation will be accepted by all scientific communities of the world.

V. Statements and declarations :

The paper is neither published nor sent for publication elsewhere.

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