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NEW CONCEPTS OF T_2 SEPARATION AXIOMS IN SUPRA FUZZY TOPOLOGICAL SPACE USING QUASI COINCIDENCE SENSE

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Abstract

Sometimes we need to minimize the conditions of topology for different reasons such as obtaining more convenient structures to describe some real-life problems or constructing some counterexamples which show the interrelations between certain topological concepts or preserving some properties under fewer conditions of those on topology. To contribute to this research area, in this paper, we establish some notions of T_2 separation axioms in supra fuzzy topological space in a quasi-coincidence sense. Also, we investigate some of its properties and establish certain relationships among them and other such concepts. Moreover, some of their basic properties are examined. The concept of separation axioms is one of the most important parts of fuzzy mathematics, mainly modern topological mathematics, which plays an important role in modern networking systems.

Keywords: Fuzzy Set, Fuzzy Topology, Supra Fuzzy Topology, Quasi-coincidence, Initial and Final Supra Fuzzy Topology.

I. Introduction

The fundamental concept of a fuzzy set was introduced by Zadeh [XIV] in (1965) to provide a foundation for the development of many areas of knowledge. Chang [II] (1968) and Lowen [VII] (1976) developed the theory of fuzzy topological spaces using fuzzy sets. Separation axioms in fuzzy mathematics have been studied by several researchers from the early eighties. Wuyts and Lowen [XIII] have been studied separation properties of fuzzy topological spaces, fuzzy neighborhood, and fuzzy uniform space. Ahmd [IV] has introduced some fuzzy separation axioms to study their hereditary and productive properties. Since then extensive work on fuzzy topological spaces has been carried out by many researchers like Gouguen [V], Wong

[XII], Lowen [VII] are created by many authors like P. Wuyts and R. Lowen [XIII], D. M. Ali [VI], Srivastava et al. [X], J. Shen [IX], M. S. Hossain [VI] and others. We consider the study of supra separation axioms in fuzzy topology as a new field of research using quasi-coincidence sense. Supra topological spaces are one of the most important developments of general topology in recent years.

The purpose of this paper is to further contribute to the development of supra fuzzy topological spaces specially on supra fuzzy T_2 topological spaces. In this paper, we have introduced three new notions of supra fuzzy T_2 topological space using quasi-coincidence sense. Moreover, it is proved that our concepts satisfy hereditary, productive, and projective properties. Also, we have observed that these notions are preserved under one-one, onto, supra fuzzy open, and supra fuzzy continuous mappings.

II. Preliminaries

For the purpose of the main results, we need to introduce some definitions and notations.

Definition: [XIV] For a set X, a function $u: X \longrightarrow [0,1]$ is called a fuzzy set in X. For every $x \in X$, $u(x)$ represents the grade of membership of x in the fuzzy set u. Some authors say that u is a fuzzy subset of X.

Definition: [II] Let X and Y be two sets and $f: X \longrightarrow Y$ be a function. For a fuzzy subset v of Y, the inverse image of v under f is the fuzzy subset $f^{-1}(v) = v_0 f$ in X and is defined by $f^{-1}(v)(x) = v\big(f(x)\big)$, for $x \in X$.

Definition: [II] Let X be a nonempty set and t be the collection of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1, 0 \in t$.

(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i\in\Lambda} u_i \in t$.

(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space (fts, in short), and members of t are called t-open (or simply open) fuzzy sets. If u is an open fuzzy set, then the fuzzy sets of the form 1-u are called t-closed (or simply closed) fuzzy sets.

Definition: [III] Let X be a nonempty set. A subfamily t^* of I^X is said to be a supra fuzzy topology on X if and only if

(i) $1, 0 \in t$.

(ii)If $u_i \in t^*$ for each $i \in \Lambda$, then $\cup_{i\in\Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological space. The elements of t^* are called supra open sets in (X, t^*) and complement of supra open set is called supra closed set.

Definition: [III] Let $(X,\ t^*)$ and $(Y,\ s^*)$ be two topological spaces. Let t^* and s^* are associated supra topological with t and s respectively and $f: (X, t^*) \to (Y, s^*)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each i.e., if for any $v \in s^*$, $f^{-1}(v) \in t^*$. The function f is called supra fuzzy homeomorphic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous.

Definition: [III] The function $f: (X, t^*) \to (Y, s^*)$ is called supra fuzzy open if and only if for each supra open fuzzy set u in (X, t^*), $f(u)$ is supra open fuzzy set in (Y, s^*).

Definition: [III] The function $f: (X, t^*) \to (Y, s^*)$ is called supra fuzzy closed if and only if for each supra closed fuzzy set u in (X, t^*), $f(u)$ is supra closed fuzzy set in (Y, s^*).

Definition: [I] Let (X, t^*) and (Y, s^*) be two supra topological spaces. If u_1 and u_2 are two supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1 \times u_2$ is a supra fuzzy subset of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min(u_1(x), u_2(y))$, for each pair $(x, y) \in X \times Y$.

Definition: [XII] Suppose $\{X_i, i \in \Lambda\}$, be any collection of sets and X denoted the Cartesian product of these sets, i.e., $X = \Pi_{i \in \Lambda} X_i$. Here X consists of all points $p =< a_i, i \in \Lambda >$, where $a_i \in X_i$. For each $j_0 \in \Lambda$, we define the projection $\pi_{j_0}: X \longrightarrow X_{j_0}$ by $\pi_{j_0}(< a_i, i \in \Lambda >) = a_{j_0}$. These projections are used to define the product supra topology.

Definition: [I] Let $(X,\ t^*)$ be a topological space and t^* be associated supra topology with T . The function $f: X \to I$ is a lower semi-continuous if and only if $\{x \in X: f(x) > \alpha\}$ is open for all $\alpha \in I$.

Definition: [III] Let (X, t^*) be a supra fuzzy topological space and t^* be associated supra topology with t. Then the lower semicontinuous topology on X associated with t^* is $\omega(t^*) = \{\mu: X \to [1, 0], \mu\ is\ supra\ lsc\}$.

Definition: [XIV] A fuzzy set $u \in X$ is called fuzzy singleton if and only if $u(x) = r, 0 < r \leq 1$ for a certain $x \in X$ and $u(y) = 0$ for all points y of X except x. The fuzzy singleton is denoted by x_r and x is its support. The class of all fuzzy singleton in x is denoted by $S(X)$. If $u \in I^X$ and $x_r \in S(X)$ then we say that $x_r \in u$ if and only if $r \leq u(x)$.

Definition: [XIV] A fuzzy singleton x_r is said to be quasi-coincidence with a fuzzy set u denoted by $x_r q u$ if and only if $u(x) + r > 1$. If x_r is not quasi-coincidence with a fuzzy set u we write $x_r \bar{q} u$ and defined as $u(x) + r \leq 1$.

III. The Main Results

We define our notions in supra fuzzy T_2 topological spaces and show relations among ours and such notions.

Definition: A supra fuzzy topological space (X, t^*) is called

(a) $SFT_2(i)$ if and only if for any pair $x_r,\ y_s \in S(X)$ for distinct x and y, there exists $u_1,\ u_2 \in t^*$ with $x_r q u_1$, $y_s q u_2$ and $u_1 \cap u_2 = 0$.

(b) $SFT_2(ii)$ if and only if for any pair $x_r,\ y_s \in S(X)$ for distinct x and y, there exists $u_1,\ u_2 \in t^*$ with $x_r \in u_1$, $y_s \in u_2$ and $u_1 \bar{q} u_2$.

(c) $SFT_2(iii)$ if and only if for any pair $x_r,\ y_s \in S(X)$ for distinct x and y, there exists $u_1,\ u_2 \in t^*$ with $x_r q u_1$, $y_s q u_2$ and $u_1 \bar{q} u_2$.

(d) $SFT_2(iv)$ if and only if for any pair $x,\ y \in S(X)$ for distinct x and y, there exists $u_1,\ u_2 \in t^*$ such that $u_1(x) = 1$, $u_2(y) = 1$ and $u_1 \cap u_2 = 0$.

Lemma: For a supra fuzzy topological space (X, t^*) the following implications are true:

$SFT_2(i) \Rightarrow SFT_2(iii)$, $SFT_2(iv) \Rightarrow SFT_2(i)$, $SFT_2(iv) \Rightarrow SFT_2(iii)$,

But in general, the converse is not true.

Proof: $SFT_2(i) \Rightarrow SFT_2(iii)$: Let (X, t^*) be a supra fuzzy topological space and (X, t^*) is $SFT_2(i)$. We have to prove that (X, t^*) is $SFT_2(iii)$. Let x_r, y_s be fuzzy points in X for distinct x and y. Since (X, t^*) is $SFT_2(i)$ fuzzy topological space, we have, there exists $u_1, u_2 \in t^*$ such that $x_r q u_1$, $y_s \bar{q} u_2$ and $u_1 \cap u_2 = 0$.

To prove (X, t^*) $SFT_2(iii)$, it is only needed to prove that $u_1 \bar{q} u_2$.

Now, $u_1 \cap u_2 = 0 \Rightarrow (u_1 \cap u_2)(x) = 0$

$\Rightarrow \min\big(u_1(x), u_2(x)\big) = 0 \Rightarrow u_1(x) = 0 \ or\ u_2(x) = 0$

$\Rightarrow u_1(x) + u_2(x) \leq 1 \Rightarrow u_1 \bar{q} u_2$

If follows that there exists $u_1, u_2 \in t^*$ such that $x_r q u_1$, $y_s \bar{q} u_2$ and $u_1 \bar{q} u_2$.

Hence it is clear that (X, t^*) is $SFT_2(iii)$.

To show (X, t^*) is $SFT_2(iii) \nRightarrow (X, t^*)$ is $SFT_2(i)$, we give a counterexample.

Counterexample: Let $X = \{x, y\}$ and $u_1, u_2 \in I^X$ be given by $u_1(x) = 1 - \varepsilon$, $u_1(y) = 1 - \frac{\varepsilon}{3}$ and $u_2(y) = 1 - \varepsilon$, $u_2(x) = \frac{\varepsilon}{3}$, where $\varepsilon = \frac{r}{3}$ for $r \in (0,1]$.

Consider the supra fuzzy topology t^* on X generated by $\{0, u_1, u_2, 1\}$. Then,

$u_1(x) = 1 - \frac{r}{3} \Rightarrow u_1(x) + \frac{r}{3} = 1$

$\Rightarrow u_1(x) + r > 1 \Rightarrow x_r q u_1$

Also, $u_2(y) = 1 - \frac{r}{3} \Rightarrow u_2(y) + \frac{r}{3} = 1$

$\Rightarrow u_2(y) + r > 1 \Rightarrow y_s q u_2$

And, $u_1(x) + u_2(x) = 1 - \varepsilon + \frac{\varepsilon}{3}$

$\Rightarrow u_1(x) + u_2(x) = 1 - \frac{\varepsilon}{3} \leq 1$

$\Longrightarrow u_1(x) + u_2(x) \leq 1 \Longrightarrow u_1 \bar{q} u_2$

Hence, (X, t^*) is $SFT_2(iii)$. But

$\min\,(u_1(x), u_2(x)) \neq 0 \Longrightarrow u_1 \cap u_2 \neq 0$

Thus, (X, t^*) is $SFT_2(i)$. Similarly, we can prove others.

IV. Good Extension Property

In this section, we shall show that our notions satisfy good extension property.

Theorem: Let (X, t^*) be a supra fuzzy topological space. Consider the following statements:

(1) (X, t^*) be a T_2 Topological Space.
(2) $(X, \omega(t^*))$ be a $SFT_2(i)$ Space.
(3) $(X, \omega(t^*))$ be a $SFT_2(ii)$ Space.
(4) $(X, \omega(t^*))$ be a $SFT_2(iii)$ Space.

The following implications are true:

(1) ⇔ (2), (1) ⇔ (3), (1) ⇔ (4)

Proof of $(\mathbf{1}) \Longrightarrow (\mathbf{2})$: Let (X, t^*) be a supra topological space and (X, t^*) is T_2. We have to prove that $(X, \omega(t^*))$ is $SFT_2(i)$. Let x_r, y_s be fuzzy points in X with distinct x and y. Since (X, t^*) is T_2 supra topological space we have, there exists $u_1, u_2 \in t^*$ such that $x_r \in u_1$, $y_s \in u_2$ and $u_1 \cap u_2 = 0$. From the definition of lower semi-continuous we $1_{u_1}, 1_{u_2} \in \omega(t^*)$ and $1_{u_1}(x) = 1$, $1_{u_2}(y) = 1$.

Then $1_{u_1}(x) + r > 1 \Longrightarrow x_r q 1_{u_1}$

Similarly, $\Longrightarrow y_s \bar{q} 1_{u_2}$.

Also, $1_{u_1} \cap 1_{u_2} = 0$. If $1_{u_1} \cap 1_{u_2} \neq 0$, then there exists $z \in X$ such that $\left(1_{u_1} \cap 1_{u_2}\right)(z) \neq 0 \Longrightarrow 1_{u_1}(z) \neq 0, 1_{u_2}(z) \neq 0$.

$\Longrightarrow u_1(z) = 1, u_2(z) = 1 \Longrightarrow z \in u_1, z \in u_2$

$\Longrightarrow z \in u_1 \cap u_2 \Longrightarrow u_1 \cap u_2 \neq \emptyset$ a contradiction.

So, $1_{u_1} \cap 1_{u_2} = 0$. Hence $(X, \omega(t^*))$ is $SFT_2(i)$. It follows that there exists $1_{u_1}, 1_{u_2} \in \omega(t^*)$ such that $x_r q 1_{u_1}$, $y_s \bar{q} 1_{u_2}$ and $1_{u_1} \cap 1_{u_2} = 0$. Hence $(X, \omega(t^*))$ is $SFT_2(i)$. Thus (1) ⟹ (2) holds.

Proof of $(\mathbf{2}) \Longrightarrow (\mathbf{1})$: Let $(X, \omega(t^*))$ is $SFT_2(i)$. We have to prove that (X, t^*) is T_2. Let x_r, y_s be points in X for distinct x and y. Since $(X, \omega(t^*))$ is $SFT_2(i)$, we have, for any fuzzy points x_r, y_s in X, $\exists\ u_1, u_2 \in \omega(t^*)$ such that $x_r q u_1$, $y_s q u_2$ and $u_1 \cap u_2 = 0$.

Now, $x_r q u_1 \Longrightarrow u_1(x) + r > 1$

$\Longrightarrow u_1(x) > 1 - r = \alpha$

$\Longrightarrow x \in u_1^{-1}(\alpha, 1]$

Similarly, $y \in u_2^{-1}(\alpha, 1]$. Also, $u_1^{-1}(\alpha, 1], u_2^{-1}(\alpha, 1] \in t^*$. Now,
$u_1 \cap u_2 = 0 \Longrightarrow (u_1 \cap u_2)(z) = 0$

$\Longrightarrow \min(u_1(z), u_2(z)) = 0$

We claim that $u_1^{-1}(\alpha, 1] \cap u_2^{-1}(\alpha, 1] = \emptyset$. For, if $z \in u_1^{-1}(\alpha, 1] \cap u_2^{-1}(\alpha, 1]$, then $z \in u_1^{-1}(\alpha, 1]$ and $z \in u_2^{-1}(\alpha, 1]$

$\Longrightarrow u_1(z) > \alpha$ and $u_2(z) > \alpha$

$\Longrightarrow \min(u_1(z), u_2(z)) > \alpha$, a contradiction. Then $u_1^{-1}(\alpha, 1] \cap u_2^{-1}(\alpha, 1] = \emptyset$.

It follows that $\exists\, u_1^{-1}(\alpha, 1], u_2^{-1}(\alpha, 1] \in t^*$ such that $x \in u_1^{-1}(\alpha, 1]$, $y \in u_2^{-1}(\alpha, 1]$ and $u_1^{-1}(\alpha, 1] \cap u_2^{-1}(\alpha, 1] = \emptyset$. Thus (2) $\Longrightarrow$ (1) holds.

Similarly, we can prove others.

V. Hereditary Property in Supra Fuzzy T_2 Topological Spaces

In this section, we shall show that our notion satisfies the hereditary property.

Theorem: Let (X, t^*) be a supra fuzzy topological space, $A \subseteq X$, $t_A^* = \{\frac{u_1}{A} : u_1 \in t^*\}$, then (X, t^*) is $SFT_2(j) \Longrightarrow (A, T_A^*)$ is $SFT_2(j)$; where $j = i, ii, iii$.

Proof: Let (X, t^*) be a supra fuzzy topological space and (X, t^*) is $SFT_2(j)$. We have to prove that (A, t_A^*) is $SFT_2(j)$. Let x_r, y_s be fuzzy points in A for distinct x and y. Since $A \subseteq X$, these fuzzy points are also fuzzy points in X. Also since (X, t^*) is $SFT_2(j)$ supra fuzzy topological space we have, there exists $u_1, u_2 \in t^*$ such that $x_r q u_1$, $y_s q u_2$ and $u_1 \cap u_2 = 0$. For $A \subseteq X$, we have u_1/A, $u_2/A \in t_A^*$.

Now, $x_r q u_1 \Longrightarrow u_1(x) + r > 1, x \in X$

$\Longrightarrow \left(\frac{u_1}{A}\right)(x) + r > 1, x \in A \subseteq X$

$\Longrightarrow x_r q(u_1/A)$

And, $y_s q u_2 \Longrightarrow u_2(y) + s > 1, y \in X$

$\Longrightarrow \left(\frac{u_2}{A}\right)(y) + s > 1, y \in A \subseteq X$

$\Longrightarrow y_s q(u_2/A)$

Further, $u_1 \cap u_2 = 0 \Longrightarrow (u_1 \cap u_2)(x) = 0, x \in X$

$\Longrightarrow \min(u_1(x), u_2(x)) = 0, x \in X$

$\Longrightarrow \min\left(\frac{u_1}{A}(x), \frac{u_2}{A}(x)\right) = 0, x \in A \subseteq X$

$\Longrightarrow \left(\frac{u_1}{A}(x) \cap \frac{u_2}{A}(x)\right) = 0$

$\Longrightarrow \left(\frac{u_1}{A}\right) \cap \left(\frac{u_2}{A}\right) = 0$

It follows that there exists u_1/A, $u_2/A \in t_A^*$ such that $x_r q(u_1/A)$, $y_s q(u_2/A)$ and $(u_1/A) \cap (u_2/A) = 0$. Hence, (A, t_A^*) is $SFT_2(j)$.

VI. Productivity and Projectivity in Fuzzy T_2 Topological Spaces

In this section, we shall show that our notions satisfy productive and projective properties.

Theorem: Let $(X_i, t_i^*), i \in \Lambda$ be a supra fuzzy topological space and $X = \prod_{i\in\Lambda} X_i$ and t_i^* be the product topology on X, then for all $i \in \Lambda$, (X_i, t_i^*) is $SFT_2(j)$ if and only if (X, t^*) is $SFT_2(j)$; where $j = i, ii, iii$.

Proof: Let for all $i \in \Lambda$, (X_i, t_i^*) is $SFT_2(j)$ space. We have to prove that (X, t^*) is $SFT_2(j)$. Let x_r, y_s be fuzzy points in X for distinct x and y. Then $(x_i)_r$, $(y_i)_s$ are fuzzy points for distinct x_i and y_i for some $i \in \Lambda$. Since (X_i, t_i^*) is $SFT_2(j)$, there exists u_{1_i}, $u_{2_i} \in t_i^*$ such that $(x_i)_r q u_{1_i}$, $(y_i)_s q u_{2_i}$ and $u_{1_i} \cap u_{2_i} = 0$. Now, $(x_i)_r q u_{1_i}$, $(y_i)_s \bar{q} u_{2_i}$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$.

Now, $(x_i)_r q u_{1_i} \Rightarrow u_{1_i}(x_i) + r > 1, x \in X$

$\Rightarrow u_{1_i}(\pi_i(x)) + r > 1$

$\Rightarrow (u_{1_i} o \pi_i)(x) + r > 1$

$\Rightarrow x_r q (u_{1_i} o \pi_i)$

And, $(y_i)_s \bar{q} u_{2_i} \Rightarrow u_{2_i}(y_i) + s \le 1, y \in X$

$\Rightarrow u_{2_i}(\pi_i(y)) + s \le 1, y \in X$

$\Rightarrow (u_{2_i} o \pi_i)(y) + s \le 1$

$\Rightarrow y_s \bar{q} (u_{2_i} o \pi_i)$

Similarly, we can prove that $x_r \bar{q} (u_{1_i} o \pi_i)$, $y_s q (u_{2_i} o \pi_i)$.

Now, $(u_{1_i} \cap u_{2_i})(x) = 0$

$\Rightarrow \min\left(u_{1_i}(x_i), u_{2_i}(x_i)\right) = 0$

$\Rightarrow \min\left(u_{1_i}(\pi_i(x)), u_{2_i}(\pi_i(x))\right) = 0$

$\Rightarrow \min\left((u_{1_i} o \pi_i)(x), (u_{2_i} o \pi_i)(x)\right) = 0$

$\Rightarrow \left((u_{1_i} o \pi_i) \cap (u_{2_i} o \pi_i)\right)(x) = 0$

Hence, $\left((u_{1_i} o \pi_i) \cap (u_{2_i} o \pi_i)\right) = 0$

It follows that there exists $(u_{1_i} o \pi_i)$, $(u_{2_i} o \pi_i) \in t_i^*$ such that $x_r q (u_{1_i} o \pi_i)$, $y_s \bar{q} (u_{2_i} o \pi_i)$ and $y_s q (u_{2_i} o \pi_i)$, $x_r \bar{q} (u_{1_i} o \pi_i)$ and $\left((u_{1_i} o \pi_i) \cap (u_{2_i} o \pi_i)\right) = 0$. Hence, (X, t^*) is $SFT_2(j)$.

Conversely, Let (X, t^*) be a supra fuzzy topological space and (X, t^*) is $SFT_2(j)$. We have to prove that $(X_i, t_i^*), i \in \Lambda$ is $SFT_2(j)$. Let a_i be a fixed element in X_i. Let

$A_i = \{x \in X = \prod_{i\in\Lambda} X_i : x_j = a_i \ for \ some \ i \neq j\}$.

Then A_i is a subset of X, and hence $(A_i, t^*_{A_i})$ is a subspace of (X, t^*). Since (X, t^*) is $SFT_2(j)$, so $(A_i, t^*_{A_i})$ is $SFT_2(j)$. Now we have A_i is a homeomorphic image of X_i. Hence it is clear that for all $i \in \Lambda$, (X_i, t^*_i) is $SFT_2(j)$ space.

VII. Mapping in Supra Fuzzy T_2 Topological Spaces

In this section, we shall show that our notions satisfy the order-preserving property.
Theorem: Let (X, t^*) and (Y, s^*) be two supra fuzzy topological space and $f: X \longrightarrow Y$ be a one-one, onto and fuzzy open map then, (X, t^*) is $SFT_2(j) \Longrightarrow (Y, s^*)$ is $SFT_2(j)$; where $j = i, ii, iii$.

Proof: Let (X, t^*) be a supra fuzzy topological space and (X, t^*) is $SFT_2(j)$. We have to prove that (Y, s^*) is $SFT_2(j)$. Let $\acute{x}_r$, $\acute{y}_s$ be fuzzy points in Y distinct $\acute{x}$ and $\acute{y}$. Since f is onto then there exists $x, y \in X$ with $f(x) = \acute{x}$, $f(y) = \acute{y}$ and x_r, y_s are fuzzy points in X with $x \neq y$ as f is one-one. Again since (X, t^*) is $SFT_2(j)$ space, there exists $u_1, u_2 \in t^*$ such that $x_r q u_1$, $y_s q u_2$ and $u_1 \cap u_2 = 0$.

Now, $x_r q u_1 \Longrightarrow u_1(x) + r > 1$

And $y_s q u_2 \Longrightarrow u_2(y) + s > 1$

Now, $f(u_1)(\acute{x}) = \{\sup u_1(x) : f(x) = \acute{x}\}$

$\Longrightarrow f(u_1)(\acute{x}) = u_1(x)$, for some x

And $f(u_1)(\acute{y}) = \{\sup u_2(y) : f(y) = \acute{y}\}$

$\Longrightarrow f(u_2)(\acute{y}) = u_2(y)$, for some y

Also since f is an open map then $f(u_1), f(u_2) \in s^*$ as $u_1, u_2 \in t^*$.

Again, $\Longrightarrow u_1(x) + r > 1$

$\Longrightarrow f(u_1)(\acute{x}) + r > 1$

$\Longrightarrow \acute{x}_r q f(u_1)$

And, $u_2(y) + s > 1$

$\Longrightarrow f(u_2)(\acute{y}) + s > 1$

$\Longrightarrow \acute{y}_s q f(u_2)$

Further, $u_1 \cap u_2 = 0$

Here, $f(u_1 \cap u_2)(\acute{x}) = \{\sup(u_1 \cap u_2)(x) : f(x) = \acute{x}\}$

$\Longrightarrow f(u_1 \cap u_2)(\acute{x}) = 0$

And $f(u_1 \cap u_2)(\acute{y}) = \{\sup(u_1 \cap u_2)(y) : f(y) = \acute{y}\}$

Therefore $f(u_1 \cap u_2) = 0 \Longrightarrow f(u_1) \cap f(u_2) = 0$

It follows that there exists $f(u_1), f(u_2) \in s^*$ such that $\acute{x}_r q f(u_1)$, $\acute{y}_s q f(u_2)$ and $f(u_1) \cap f(u_2) = 0$. Hence it is clear that (Y, s^*) is $SFT_2(j)$ space.

Theorem: Let (X, t^*) and (Y, s^*) be two supra fuzzy topological space and $f: X \longrightarrow Y$ be a one-one and fuzzy continuous map then, (Y, s^*) is $SFT_2(j) \implies (X, t^*)$ is $SFT_2(j)$; where $j = i, ii, iii$.

Proof: Let (Y, s^*) be a supra fuzzy topological space and (Y, s^*) is $SFT_2(j)$. We have to prove that (X, t^*) is $SFT_2(j)$. Let x_r, y_s be fuzzy points in X for distinct x and y. Then $(f(x))_r$, $(f(y))_s$ are fuzzy points in Y with $f(x) \neq f(y)$ as f is one-one. Again since, (Y, s^*) is $SFT_2(j)$ space, there exists $u_1, u_2 \in s^*$ such that $(f(x))_r q u_1$, $(f(y))_s q u_2$ and $u_1 \cap u_2 = 0$.

Now, $(f(x))_r q u_1 \Rightarrow u_1\big(f(x)\big) + r > 1$

$\Rightarrow f^{-1}\big(u_1(x)\big) + r > 1 \Rightarrow \big(f^{-1}(u_1)\big)(x) + r > 1$

$\Rightarrow x_r q f^{-1}(u_1)$

And, $(f(y))_s \bar{q} u_2 \Rightarrow u_2\big(f(y)\big) + s > 1$

$\Rightarrow f^{-1}\big(u_2(y)\big) + s > 1 \Rightarrow \big(f^{-1}(u_2)\big)(y) + s > 1$

$\Rightarrow y_s q f^{-1}(u_2)$

Also, $u_1 \cap u_2 = 0$. Then $(u_1 \cap u_2)\big(f(x)\big) = 0$

$\Rightarrow \min\Big(u_1\big(f(x)\big), u_2\big(f(y)\big)\Big) = 0$

$\Rightarrow \min\big(f^{-1}(u_1)(x), f^{-1}(u_2)(x)\big) = 0$

$\Rightarrow \min\, ((f^{-1}(u_1)(x), \big(f^{-1}(u_2)(x)\big) = 0$

$\Rightarrow \big(f^{-1}(u_1) \cap f^{-1}(u_2)\big)(x) = 0$

$\Rightarrow f^{-1}(u_1) \cap f^{-1}(u_2) = 0$

Now since f is a continuous map and $u_1, u_2 \in s^*$ then $f^{-1}(u_1), f^{-1}(u_2) \in t^*$. It follows that there exists $f^{-1}(u_1), f^{-1}(u_2) \in t^*$ such that $x_r q f^{-1}(u_1)$, $y_s q f^{-1}(u_2)$ and $f^{-1}(u_1) \cap f^{-1}(u_2) = 0$. Hence, (X, t^*) is $SFT_2(j)$ space.

VIII. Initial and Final Supra Fuzzy T_2 Topological Spaces

Theorem: If $\{(X_i, t_i^*)_i\} \in \Lambda$ is a family of $SFT_2(iv)$ fuzzy topological space and $\{f_i: X \longrightarrow (X_i, t_i^*)\}_i \in \Lambda$, a family of one-one and fuzzy continuous functions, then the initial supra fuzzy topology on X for the family $\{f_i\}_i \in \Lambda$ is $SFT_2(iv)$.

Proof: Let t^* be the initial supra fuzzy topology on X for the family $\{f_i\}_i \in \Lambda$. Let x_r, y_s be fuzzy points in X for distinct x and y. Then $f_i(x), f_i(y) \in X_i$ and $f_i(x) \neq f_i(y)$ as f_i is one-one. Since (X_i, t_i^*) is $SFT_2(iv)$, then for every two distinct fuzzy points $(f_i(x))_r$, $(f_i(y))_s$ in X_i, there exists fuzzy sets $u_{1_i}, u_{2_i} \in t_i^*$ such that $(f_i(x))_r q u_{1_i}$, $(f_i(y))_s q u_{2_i}$ and $u_{1_i} \bar{q} u_{2_i}$.

Now, $(f_i(x))_r q u_{1_i}$ and $(f_i(y))_s \bar{q} u_{2_i}$

That is $u_{1_i}\big(f_i(x)\big) + r > 1$ and $u_{2_i}\big(f_i(y)\big) + s > 1$

$\Rightarrow f_i^{-1}(u_{1_i})(x) + r > 1$ and $f_i^{-1}(u_{2_i})(y) + s > 1$

Also, $u_{1_i}\bar{q}u_{2_i} \Rightarrow u_{1_i}(f_i(x)) + u_{2_i}(f_i(x)) \leq 1$

$\Rightarrow f_i^{-1}(u_{1_i})(x) + f_i^{-1}(u_{2_i})(x) \leq 1$

This is true for every $i \in \Lambda$. So,

$\inf f_i^{-1}(u_{1_i})(x) + r > 1$ and $\inf f_i^{-1}(u_{2_i})(y) + s > 1$ and $\inf f_i^{-1}(u_{1_i})(x) + \inf f_i^{-1}(u_{2_i})(x) \leq 1$

Let $u_1 = \inf f_i^{-1}(u_{1_i})$ and $u_2 = \inf f_i^{-1}(u_{2_i})$. Then $u_1, u_2 \in t^*$ as f_i is fuzzy continuous. So, $u_1(x) + r > 1$ and $u_2(y) + s > 1$ and $u_1(x) + u_2(x) \leq 1$.

Hence, $x_r q u_1$, $y_s q u_2$ and $u_1 \bar{q} u_2$. Therefore, (X, t^*) is must $SFT_2(iv)$.

Theorem: If $\{(X_i, t_i^*)_i\} \in \Lambda$ is a family of $SFT_2(iii)$ fuzzy topological space and $\{f_i: X \longrightarrow (X_i, t_i^*)\}_i \in \Lambda$, a family of open and bijective function, then the final supra fuzzy topology on X for the family $\{f_i\}_i \in \Lambda$ is $SFT_2(iii)$.

Proof: Let t^* be the final supra topological on X for the family $\{f_i\}_i \in \Lambda$. Let x_r, y_s be fuzzy points in X for distinct x and y. Then $f_i^{-1}(x), f_i^{-1}(y) \in X_i$ and $f_i^{-1}(x) \neq f_i^{-1}(y)$ as f_i is bijective. Since (X_i, t_i^*) is $SFT_2(iii)$, then for every two distinct fuzzy points $(f_i^{-1}(x))_r$, $(f_i^{-1}(y))_s$ in X_i, there exists fuzzy sets $u_{1_i}, u_{2_i} \in t_i^*$ such that $(f_i^{-1}(x))_r q u_{1_i}$, $(f_i^{-1}(y))_s q u_{2_i}$ and $u_{1_i}\bar{q}\, u_{2_i}$.

Now, $(f_i^{-1}(x))_r q u_{1_i}$ and $(f_i^{-1}(y))_s q u_{2_i}$

That is, $u_{1_i}\left(f_i^{-1}(x)\right) + r > 1$ and $u_{2_i}\left(f_i^{-1}(y)\right) + s > 1$

$\Rightarrow f_i(u_{1_i})(x) + r > 1$ and $f_i(u_{2_i})(y) + s > 1$

Also, $u_{1_i}\bar{q}\, u_{2_i} \Rightarrow u_{1_i}\left(f_i^{-1}(x)\right) + u_{2_i}(f_i^{-1}(x)) \leq 1$

$\Rightarrow (f_i\left(u_{1_i}(x)\right) + f_i\left(u_{2_i}(x)\right)) \leq 1$

This is true for every $i \in \Lambda$. So,

$\inf f_i(u_{1_i})(x) + r > 1$ and $\inf f_i(u_{2_i})(y) + s > 1$ and $\inf f_i(u_{1_i})(x) + \inf f_i(u_{2_i})(x) \leq 1$

Let $u_1 = \inf f_i(u_{1_i})$ and $u_2 = \inf f_i(u_{2_i})$. Then $u_1, u_2 \in t^*$ as f_i is fuzzy open. So, $u_1(x) + r > 1$, $u_2(y) + s > 1$ and $u_1(x) + u_2(y) \leq 1$.

Hence, $x_r q u_1$, $y_s q u_1$ and $u_1 \bar{q} u_2$. Therefore, (X, t^*) is must $SFT_2(iii)$.

IX. Conclusion

The main result of this paper is introducing some new concepts of supra fuzzy T_2 topological spaces. We discuss some features of these concepts and present the good extensions, hereditary, productive, and projective properties. Also, we have observed that these notions are preserved under one-one, onto, supra fuzzy open, and supra fuzzy continuous mappings. We think that this research work will contribute to the development of the field of modern mathematics.

Conflicts of Interest:

The authors declare that they have no conflicts of interest to report regarding the present study.

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