



MULTI-WAVE COVID-19 PANDEMIC DYNAMICS IN ICELAND IN TERMS OF DOUBLE SIGMOIDAL BOLTZMANN EQUATION (DSBE)

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Abstract

The world is facing multi-wave transmission of COVID-19 pandemics, and investigations are rigorously carried out on modeling the dynamics of the pandemic. Multi-wave transmission during infectious disease epidemics is a big challenge to public health. Here we introduce a simple mathematical model, the double sigmoidal-Boltzmann equation (DSBE), for analyzing the multi-wave Covid-19 spread in Iceland in terms of the number of cumulative cases. Simulation results and the main parameters that characterize multi waves are derived, yielding important information about the behavior of the multi-wave pandemics over time. The result of the current examination reveals the effectiveness and efficacy of DSBE for exploring the Covid 19 dynamics in Iceland and can be employed to examine the pandemic situation in different countries undergoing multi-waves.

Keywords: Cumulative Case, Daily Infection Rate, Double Sigmoidal Boltzmann Equation, Multi-wave Covid-19 Pandemic, Simulation.

I. Introduction

The continuous worldwide pandemic of Covid-19 is brought about by the extremely intense respiratory condition Covid 2 (SARS-CoV-2). The original infection was detected in Wuhan, China, in December 2019; a lockdown in Wuhan and different urban communities in the Hubei area could not contain the episode, and it spread to different pieces of the central area of China and throughout the planet. The World Health Organization (WHO) proclaimed a Public Health Emergency of International Concern on 30 January 2020, and a pandemic on 11 March 2020. Since 2021, variations of the infection have arisen or become predominant in numerous nations, with the Delta, Alpha, and Beta variations being the most destructive. As of now, about 211 million cases and 4.42 million deaths have been affirmed, making it perhaps the deadliest pandemic ever [XII].

Suggested preventive measures incorporate social separating, wearing face covers openly, ventilation and air-sifting, hand washing, covering one's mouth when

Pinaki Pal et al

wheezing or hacking, sanitizing surfaces, and checking and self-confinement for individuals uncovered or indicative. A few immunizations have been disseminated in numerous nations since December 2020. Medicines center around tending to indications, yet work is in progress to foster prescriptions that restrain the infection. Specialists worldwide have reacted by carrying out movement limitations, lockdowns and isolates, working environment peril controls, and business terminations. There are additional endeavours to expand the testing limit and follow contacts of the contaminated.

The pandemic has brought about serious worldwide social and financial disturbance, including the biggest worldwide downturn since the Great Depression of the 1930s. It has prompted far and wide stockpile deficiencies exacerbated by alarm purchasing, farming interruption, food deficiencies, and diminished discharges of toxins. Various academic foundations and public regions have been to some extent or completely shut, and numerous occasions have been dropped or deferred. Deception has circled through web-based media and broad communications, and political pressures have been exacerbated. The pandemic has raised issues of racial and geographic segregation, wellbeing value, and the harmony between general wellbeing objectives and individual rights.

Iceland, a Nordic island in the North Atlantic Ocean, is situated among Europe and North America. Numerous different nations have forced severe public lockdowns to control the spread of the Covid, however, Iceland has not depended on such drastic actions. All things considered, it has zeroed in on a thorough arrangement of testing, following, isolate, and segregation — and it has confided in guests and inhabitants to maintain it. Prime Minister Katrín Jakobsdóttir said, "People are respecting the rules." The infection was first detected in Iceland in February 2020. There were 6,258 cases enrolled in total, of which 6,126 had recovered and 29 deaths had occurred as of April 8, 2021 [XII].

The current investigation centers around utilizing phenomenological models for demonstrating the dynamics of infection without biological considerations [I, II, V, XVII]. Past pandemics [V-VIII, XX] have been broken down utilizing the traditional strategic logistic growth model, the generalized logistic model (GLM), the generalized Richards model (GRM), and the generalized growth model (GGM). In the current investigation, we will illustrate the Double Sigmoidal Boltzmann Equation and model this curve to the dynamics of COVID-19 infection in terms of the cumulative number of infected people in Iceland.

The paper is made as follows: in Section II, we clarify the model and COVID-19 data exhaustively. We perform simulation, i.e., align the model to the cumulative number of COVID-19 infections for the entire of Iceland in Section III and IV, and examine the outcomes. We talk around a few constraints of our techniques in Section V and afterward close in Section VI.

II. Mathematical Model & Materials

Double Sigmoidal Boltzmann Equation

The dynamics of sigmoidal and double sigmoidal phenomena are regularly seen in numerous spaces of science (population science, virology [XI], growth of fruits [III, XIII, XVI] growth of animal [XVIII, XIV, XIX] etc.).

Development in numerous organic systems can be considered as happening in two stages. In the first stage, the exponential increase of intensity is observed until saturation is reached and it levels off at some level. In the next stage, a rise of intensity at an asymptotic level is seen (Fig. 1). In between the two stages, “the cold period”, there is a plateau or very slow increasing trend. These two stages can be demonstrated with two sigmoidal bends that depict the connection between time and power [XV].

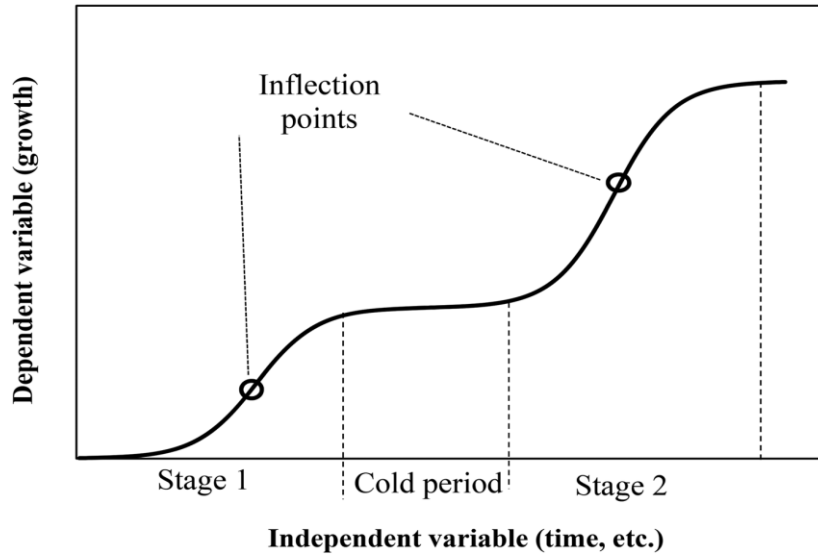


Fig.1. Double Sigmoid Curve

There are at least six parameters, two midpoints and slopes parameters each, a maximum value, and an asymptotic final value (Fig. 1) in a double-sigmoidal curve. These parameters give a better quality of fit, incommensurate with the parsimony principle [X]. It is mathematically expressed by the following double sigmoidal-Boltzmann equation (DSBE) [XI]:

$$y(x) = y_0 + y_{max} \left[\frac{r}{A_1 + e^{\frac{x-c_1}{\alpha_1}}} + \frac{1-r}{A_2 + e^{\frac{x-c_2}{\alpha_2}}} \right] \quad (1)$$

where

y_0 : minimum value of y .

y_{max} : maximum value of y .

r : factor ($0 < r < 1$) for 1st phase.

Pinaki Pal et al

c_1 & c_2 : inflection points for 1st and 2nd phases respectively.

A_1 & A_2 : parameters adjusting the positions of c_1 & c_2 respectively.

α_1 & α_2 : slope factors of the two phases respectively.

Modeling Covid19 Infection Trajectory (cumulative case)

The Coronavirus pandemic has two characterizing worldwide trends. To begin with, total Coronavirus mortality is a lot higher in the Transatlantic area, containing North and South America, and Europe (and the adjacent Central Asian, Middle Eastern, and North African locales). It is fundamentally lower in the remainder of Asia and Africa.

Second, all areas have had two waves, however, there are transient varieties in their spread and peak. The principal wave crested in Europe between 6 March 2020 and 20 April 2020, and the second wave between 6 December 2020 and 21 January 2021. The Americas had a drawn-out first wave that collapsed into a subsequent wave, without seeing any decay between the two, as in Europe.

While there are local varieties, the information uncovers that the second wave is over twice as lethal as the first. During the first-wave period 6 March to 21 October 2020, the average six-weekly global mortality was 29 per million, whereas it is 68 during the second between 21 October 2020 and 6 June 2021. This higher second-wave mortality was also observed in the Spanish Flu a century ago.

With regards to epidemiological context, the dynamics of the COVID-19 pandemic cumulative case can be demonstrated by the following double sigmoidal-Boltzmann equation:

$$C(t) = C_0 + C_{max} \left[\frac{r}{A_1 + e^{\frac{t-c_1}{\alpha_1}}} + \frac{1-r}{A_2 + e^{\frac{t-c_2}{\alpha_2}}} \right] \quad (2)$$

where

$C(t)$: number of cumulative cases of COVID-19 at the time, t .

C_0 : initial minimum number of cases. (February 28, 2020, when the first case was detected in Iceland.)

C_{max} : maximum number of individuals infected during the epidemic.

Since $C_{max} \gg C_0$, we can approximate the eq (2) as

$$C(t) = C_{max} \left[\frac{r}{A_1 + e^{\frac{t-c_1}{\alpha_1}}} + \frac{1-r}{A_2 + e^{\frac{t-c_2}{\alpha_2}}} \right] \quad (3)$$

The daily infection rate V_p , is obtained by differentiating eq (3) with respect to time t

$$V_p(t) = C_{max} \left[\frac{r e^{\frac{t-c_1}{\alpha_1}}}{\alpha_1 \{A_1 + e^{\frac{t-c_1}{\alpha_1}}\}^2} + \frac{(1-r) e^{\frac{t-c_2}{\alpha_2}}}{\alpha_2 \{A_2 + e^{\frac{t-c_2}{\alpha_2}}\}^2} \right] \quad (4)$$

Data

For the current investigation, data are the day-by-day cumulative cases of COVID-19 in Iceland from February 28, 2020, to March 01, 2021. Real-time data were stored by

Pinaki Pal et al

Centres for Disease Control and Prevention (CDC) and are made available on their website [IV].

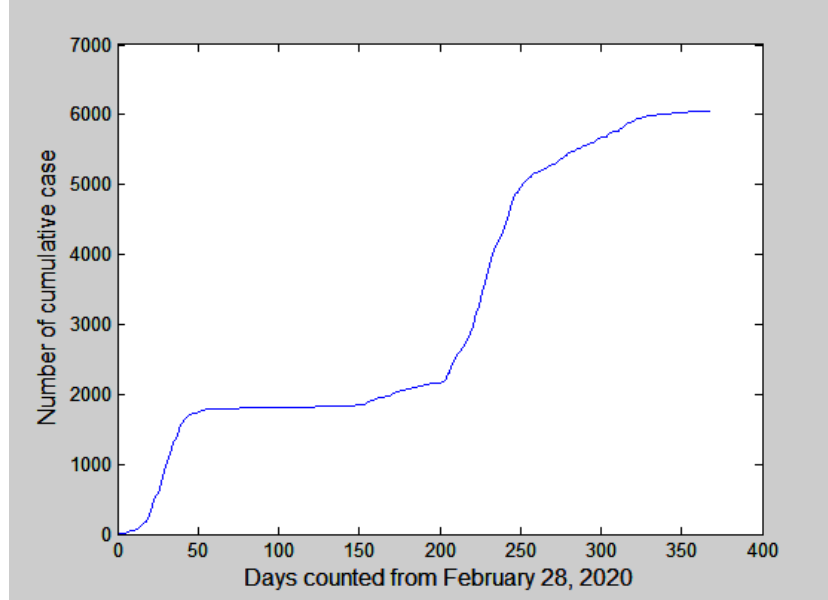


Fig. 2. Cumulative Covid 19 case in Iceland

Fig. 2 gives an idea of the daily cumulative COVID-19 case nationwide. This data consists of 368 days of observations covering a period from February 28, 2020 (the day of the first detected case) to March 01, 2021. The figure delineates two pandemic waves. The first wave spans from 1st week of March 2020 to 1st week of April 2020. The pandemic rebounds in mid-September 2020 indicating a rise of infection again.

III. Simulations

Nonlinear least-squares parametric and interval estimations [IX]

We will develop a model $y(x) = f(\mathbf{a}, x) + \varepsilon$, where \mathbf{a} is the vector describing undetermined coefficients and ε represents the residual error, from the available data: $x_i, y_i, i = 1, 2, \dots, N$. We estimate the undetermined parameters \mathbf{a} by minimizing the following function I:

$$I = \min_{\mathbf{a}} \sum_{i=1}^N [y_i - y(x_i)]^2 = \min_{\mathbf{a}} \sum_{i=1}^N [y_i - f(\mathbf{a}, x_i)]^2 \quad (5)$$

The residue error $\varepsilon_i = y_i - f(\mathbf{a}, x_i)$ can then be obtained by substituting back the estimated values of the parameters \mathbf{a} in the model objective function. In this process, we utilize the Matlab command `nlinfit()` which is based on non-linear least-squares fitting with Gauss-Newton's algorithm. The regression coefficient, R^2 , a statistical measure for goodness of fit between the data and model, is then estimated as

$$R^2 = \frac{\text{explained variance}}{\text{total variance}} = 1 - \frac{\text{sum of the squares of the residue errors}}{\sum_{i=1}^N (y_i - \hat{y}_i)^2} \quad (6)$$

Cumulative data $C(t)$ fitting to DSBE model [eq. (3)]

We fit the modified DSBE growth model (eq. (3)) to the cumulative case for Covid 19 pandemic in Iceland for the entire period of 368 days since February 28, 2020 (day of the first detected case), and get the best fit values of the model's parameters \mathbf{a} by using the above-mentioned Matlab programme nlinfit(). The best-fit values of the parameters are shown in Table 1.

Table 1. Parameter estimation for multi-waves COVID-19 pandemic in Iceland from February 28, 2020 (day of the first detected case) to April 8, 2021.

Table 1.

Parameter	Best-fit value
C_{\max}	5896
r	0.27
A_1	0.75
c_1	27.8
α_1	5.72
A_2	0.89
c_2	231.6
α_2	17.06
R^2	0.9972

IV. Discussions

The maximum number of individuals infected during the epidemic, C_{\max} , as predicted by our model is 5896, whereas the clinical data value is 6049.

We have reproduced the cumulative case for the multi-waves COVID-19 pandemic in Iceland using the model (eq. (3)) with the best-fit values of the parameters shown in Table 1. This model-estimated cumulative case is plotted in Fig. 3 along with the original clinical cumulative data. The figure shows an excellent match between the two with a regression coefficient of 0.9972.

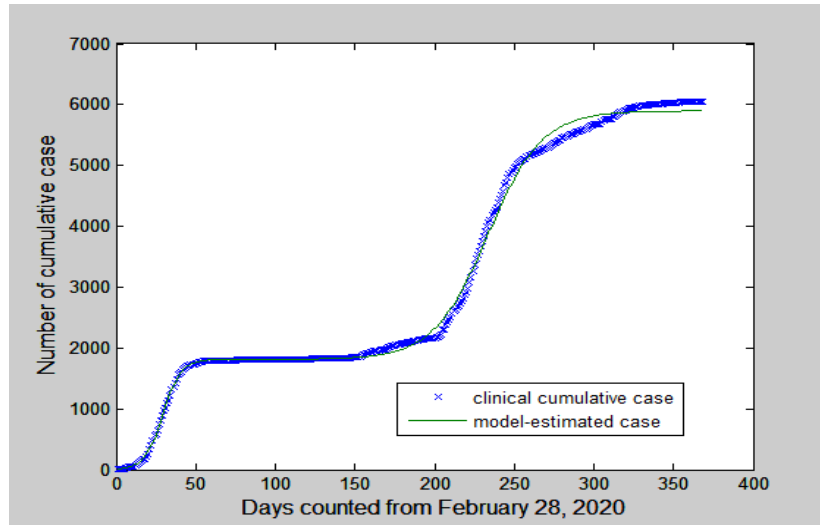


Fig. 3. Observed and model-estimated cumulative case

With the best-fit values of the parameters, we have also estimated the daily infection rate, V_p using the (eq. (4)), and is shown in Fig. 4. We see that the first peak occurs with 79 cases/day on the 28th day since February 28, 2020 (day of the first detected case), which is the value of c_1 of Table 1. On the 232nd day (the value of c_2 of Table 1), the second peak appears with 60 cases/day. The virulence of the second wave is low in comparison to the first one. These may be concluded visually from Fig. 1, showing two sharp rises at those two different time instants. The difference between them (204 days) may be a reflection of the government's action to the pandemic rebound.

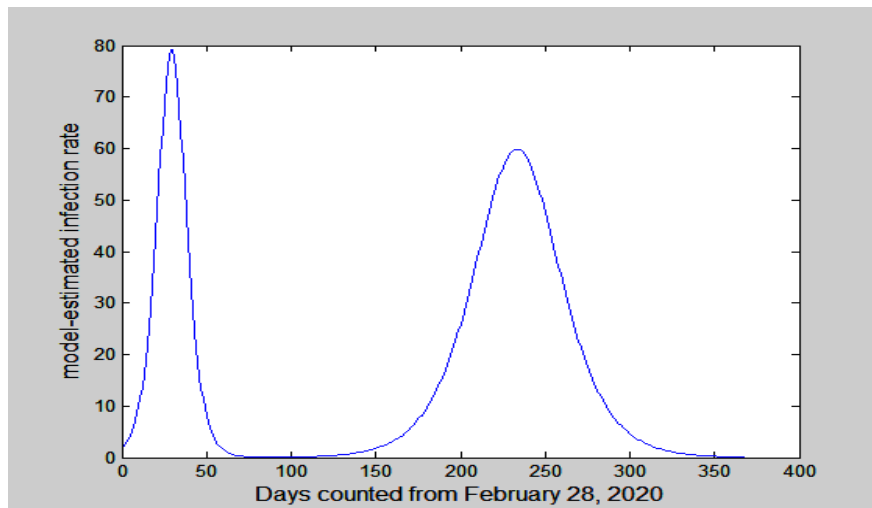


Fig. 4. Model-estimated daily infection rate, V_p

V. Limitations

During the study, we have not considered the variations in population density, present situation of the health systems, government interventions, and socio-demographic and economic situation across and within the states and districts of the entire country. Accordingly, the results from the model could also swing in a significant way from the observed ones.

VI. Conclusions

We propose a phenomenological model, double sigmoidal-Boltzmann equation, to simulate the multi-wave COVID-19 pandemic in Iceland. In the simulation process, the cumulative number of infected persons $C(t)$ has been precisely fitted with the model equation, giving rise to various epidemiological parameters. The results demonstrate that the proposed model can find good estimates for all parameters. Hence, this study could be regarded as a valuable tool for characterizing the transmission dynamics of the multi-wave COVID-19 pandemic and thereby guiding policymakers to adopt precautionary and preventive measures to tone down the damage and fight against this pandemic.

Conflict of Interest:

The authors declare that no conflict of interest to report the present study.

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