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AN INTRODUCTION TO RECTANGULAR BHATTACHARYYA'S CO-ORDINATES: A NEW CONCEPT

Prabir Chandra Bhattacharyya

P-9/1, LIC Township, Madhyamgram, Kolkata, West Bengal – 700129

Email : prabirbhattacharyya@yahoo.com

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Abstract

Co-ordinate geometry is a particular branch of mathematics where geometry is studied with the help of algebra. According to the concept of Bhattacharyya's Co-ordinate System, plane co-ordinate geometry consists of Four Positive dimensions or axes. In four-dimensional co-ordinate geometry, the position of a point can be determineda uniquely by two real positive numbers on a plane. Here, we shall discuss only the plane co-ordinate geometry consisting of four dimensions or axes. The author introduced four positive dimensions or axes to solve the problems with the help of 'Rectangular Bhattacharyya's Co-ordinate System' instead of the Rectangular Cartesian Co-ordinate System. This is the new concept which has been developed by the author. The author determined not only the distance between two points but also the direction of the line segment between two points on the plane.

Keywords : Bhattacharyya's Co-ordinate System, Cartesian Co-ordinate System, Four Positive Dimensions, Plane Co-ordinate Geometry, Relative motion.

I. Introduction

According to the new concept of P. C. Bhattacharyya, axes or dimensions or directions started from the origin O (0,0) are positive. The author has introduced this new concept successfully to plane co-ordinate geometry. Therefore, the question of negative sense to any co-ordinate axes which are vertically opposite to any positive axes does not arise. In plane co-ordinate geometry the author introduced OX and OX' axes which are vertically opposite to each other as two abscissas and OY and OY' axes which are vertically opposite to each other as two ordinates (Fig.1). All abscissas and ordinates are denoted as positive axes. So, plane co-ordinate geometry consists of four positive dimensions. This four-dimensional co-ordinate geometry of a plane is called Bhattacharyya's Co-ordinate Geometry.

René Descartes, a French Mathematician and Philosopher first invented co-ordinate geometry of a plane in the 17th century. It was his revolutionary achievement in the field of mathematics. He provided a systematic link between Euclidean Geometry and Algebra (I).

According to the author's concept, there may exist infinite numbers of positive axes or dimensions or directions from the origin O (0,0) but there cannot exist a single negative axis or any axis concerning any other axis having negative sense because origin O (0,0) is the starting point of any axis or dimension or direction. Hence, the sense of negativity of OX' and OY' axes of René Descartes have been invaded by the author.

Galileo Galilei, an Italian astronomer and mathematician is the first person who discovered the theory of relative motion (II). The author determined the distance between two points and the direction of the line segment with the help of Galileo's theory of relative motion.

II. Formulation of the problem and Method of Solution

Rectangular Bhattacharyya's Co-ordinates

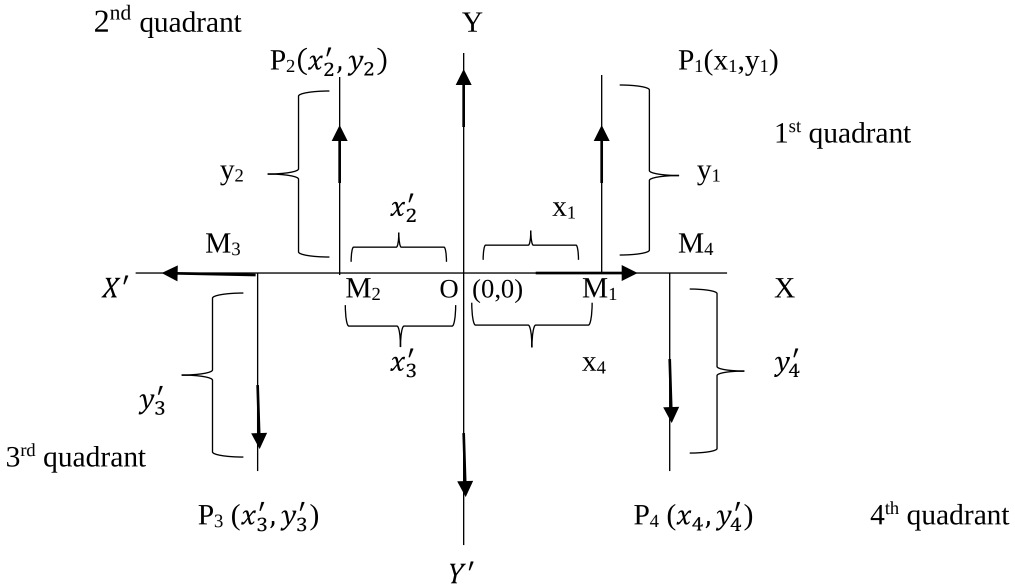

Fig.1

Let us consider O (0,0) to be a fixed point on the plane of this page which is stated as origin (Fig. 1). We draw four straight lines from the origin :

1. Towards the X direction.
2. Towards the Y direction.
3. Towards the X' direction.
4. Towards the Y' direction.

$\overrightarrow{OX}$ and $\overrightarrow{OY}$; $\overrightarrow{OX'}$ and $\overrightarrow{OY}$; $\overrightarrow{OX'}$ and $\overrightarrow{OY'}$; $\overrightarrow{OX}$ and $\overrightarrow{OY'}$, lines are mutually perpendicular to each other and $\overrightarrow{OX}$ and $\overrightarrow{OX'}$ are in the vertically opposite directions and also $\overrightarrow{OY}$ and $\overrightarrow{OY'}$ are in the vertically opposite directions.

The straight line $\overrightarrow{OX}$ is called X-axis, similarly $\overrightarrow{OY}$ is called Y-axis, $\overrightarrow{OX'}$ is called X'-axis and $\overrightarrow{OY'}$ is called Y'-axis. These four lines together divided the plane into four parts of the plane. Each of these parts is said to be a quadrant. The quadrants are named as follows: XOY is the first quadrant, X'OY is the second quadrant, X'OY' is the third quadrant and XOY' is the fourth quadrant. Note that X-axis, Y-axis, X'-axis, Y'-axis are all positive axes or dimensions.

Now we can determine the position of any point on the plane uniquely concerning the co-ordinate axes drawn through O.

Let P_1 (x_1, y_1) be any point on the first quadrant (Fig.1). From P_1 draw $\overrightarrow{M_1 P_1}$ perpendicular on the X-axis (fig. 1). If $\overrightarrow{OM_1}$ and $\overrightarrow{M_1P_1}$ measure x_1 and y_1 units respectively then the position of the point P_1 on the plane is determined uniquely. First of all, we have to move from O through a distance x_1 unit along $\overrightarrow{OX}$ and then proceed through a distance y_1 unit in the direction parallel to $\overrightarrow{OY}$. Similarly, we can find a point P_2 (x_2', y_2). First of all, we have to move from O through a distance x_2 unit along $\overrightarrow{OX'}$ and then proceed through a distance y_2 unit in the direct parallels to $\overrightarrow{OY}$. Similarly, we can find the point P_3 (x_3', y_3') and P_4 (x_4, y_4') in the 3rd and 4th quadrant. So, it is possible to find four different points on the plane. The position of the points can be differentiated by introducing the following notations :

1) The distance measured from O along $\overrightarrow{OX}$ axis (or parallel in the direction to $\overrightarrow{OX}$ axis) putting without any notation over the unit is positive.
2) The distance measured from O along to $\overrightarrow{OY}$ axis (or parallel in the direction to $\overrightarrow{OY}$ axis) putting without any notation over the unit is positive.
3) The distance measured from O along $\overrightarrow{OX'}$ axis (or parallel in the direction to $\overrightarrow{OX'}$ axis) putting a notation das ($'$) over the unit is positive.
4) The distance measured from O along $\overrightarrow{OY'}$ axis (or parallel in the direction to $\overrightarrow{OY'}$ axis) putting a notation dash ($'$) over the unit is positive.
5) Significance of arrows : Motions are always positive and also the units of motions are always positive. A motion which is away from the starting point is called countup motion and a motion which is towards the starting point is called countdown motion. The units of countup or countdown motion is always positive. To every motion there is a countup motion which is always greater than or equal to countdown motion.
   (i) The Symbol ‘ $\rightarrow$’ means count up motion of a point to another point.
   (ii) The symbol ‘$\leftarrow$’ means count down motion of a point to another point.

Following the above discussion, we can determine uniquely the position of a point on a plane referred to mutually perpendicular co-ordinate axes through an origin, we require without dash or with one dash ($'$) over positive real numbers. These two positive real numbers may be both without dash or both with one dash, also it may be one without dash and the other with a dash together are called the Rectangular Bhattacharyya's Co-ordinate. To represent a point we write two positive real numbers in braces by putting a comma between them where the first number is the distance from the origin along $\overrightarrow{OX}$ or $\overrightarrow{OX'}$ and the second number is the distance from the origin along $\overrightarrow{OY}$ or $\overrightarrow{OY'}$ respectively.

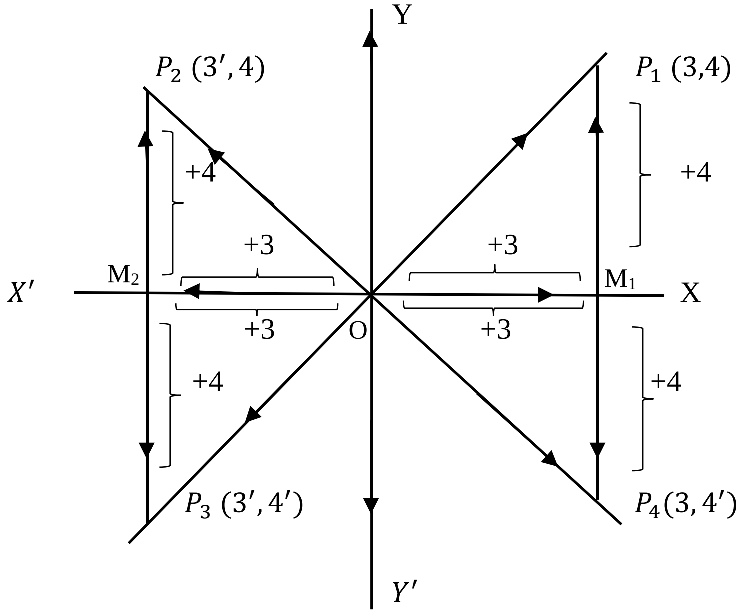

Fig.2

Let us take an example. The position of any point on the plane to co-ordinate axes drawn through O can be determined. Let P_1 be any point in the first quadrant (Fig.2). From P_1 draw $\overrightarrow{M_1P_1}$ perpendicular on X-axis (Fig. 2.). If $\overrightarrow{OM_1}$ and $\overrightarrow{M_1P_1}$ measure 3 and 4 units respectively then the position of P_1 on the plane is determined.

Similarly, we can find the position of P_2, P_3, P_4 in the 2nd, 3rd, and 4th quadrant respectively, and the distance of each of them along the X-axis and X'- axis is 3 unit and Y-axis, and Y'- axis is 4 unit. It is possible to have four different points on the plane at equal distances along the co-ordinate axes. To differentiate regarding the positions of such point we have already introduced notations along the co-ordinate axes.

So,

$$\overrightarrow{OP_1^2} = \overrightarrow{OM_1^2} + \overrightarrow{M_1P_1^2} = 3^2 + 4^2$$

$$\therefore \overrightarrow{OP_1} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \quad [\because \text{all units are positive}]$$

Again

$\overrightarrow{OP_2^2} = \overrightarrow{OM_2^2} + \overrightarrow{M_2P_2^2} = 3^2 + 4^2$ [∵ unit of $3'$ is 3 and positive]

$\therefore \overrightarrow{OP_2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$

Similarly

$\overrightarrow{OP_3^2} = \overrightarrow{OM_2^2} + \overrightarrow{M_2P_3^2} = 3^2 + 4^2$

[unit of $3'$ and $4'$ are 3 and 4 respectively and positive]

$\therefore \overrightarrow{OP_3} = \sqrt{3^2+4^2} = 5$

Similarly

$\overrightarrow{OP_4^2} = \overrightarrow{OM_1^2} + \overrightarrow{M_1P_4^2} = 3^2 + 4^2$ [∵ unit of $4'$ is 4 and positive]

$\therefore \overrightarrow{OP_4} = \sqrt{3^2+4^2} = 5$

So, we can determine the position of four points P_1, P_2, P_3, P_4 in four different quadrants having equal distance from the origin.

In Rectangular Bhattacharyya's Co-ordinate system $\overrightarrow{OX}$, $\overrightarrow{OX'}$ are called abscissas and $\overrightarrow{OY}$, $\overrightarrow{OY'}$ are called ordinates. So, there are two abscissas and two ordinates. All abscissas and ordinates are positive, and also ordered pairs of numbers such as (x,y), (x',y), (x',y'), and (x,y') are all positive real numbers. So, any ordered pair of a point lying in any quadrant are positive real numbers.

So, the point

Having co-ordinate (x_1, y_1) lies in the 1st quadrant.
Having co-ordinate (x'_2, y_2) lies in the 2nd quadrant.
Having co-ordinate (x'_3 y'_3) lies in the 3rd quadrant.
Having co-ordinate (x_4,y'_4) lies in the 4th quadrant.

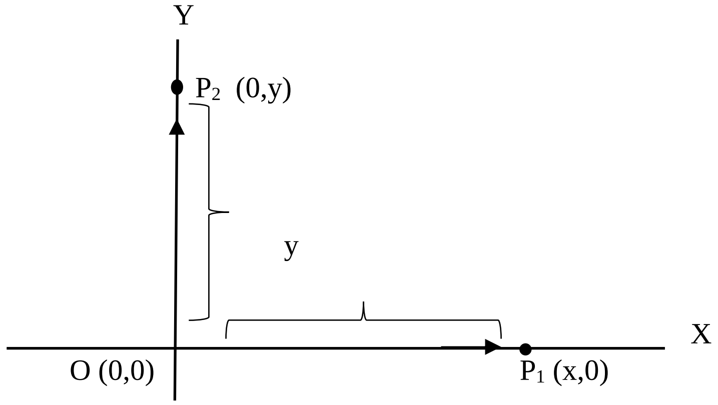

Fig. 3.

It is to be noted that the ordinate of any point on the X-axis is zero and abscissa of any point on the Y-axis is zero. Both abscissa and ordinate of the origin O are zero (Fig.3). So, co-ordinate of a point on X-axis are of the form P_1 (x,0), the co-ordinate of a point on the Y-axis is of the form P_2 (0,y). The co-ordinates of the origin O are always (0,0).

II. To find the distance between two given points.

(a) To find the distance of a given point from the origin

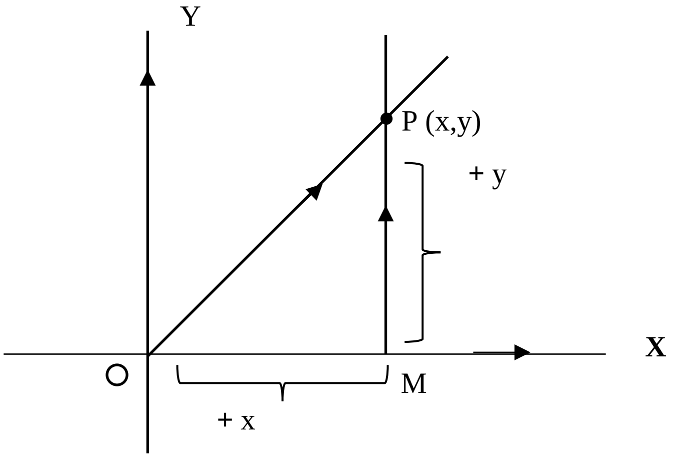

Fig. 4.

let $\overrightarrow{OX}$ and $\overrightarrow{OY}$ be the Rectangular Bhattacharyya's Co-ordinate axes on the plane of reference and the co-ordinates of a point P on the plane be (x,y) (Fig.4). We have to find the distance of the point P from the origin O. From P draw $\overrightarrow{MP}$ perpendicular on $\overrightarrow{OX}$. Now $\overrightarrow{OM}= \vec{x}$ and $\overrightarrow{MP} = \vec{y}$ [x and y are positive units of $\vec{x}$, $\vec{y}$ respectively]

Then, from the right-angled Triangle OMP, we get

$$\overrightarrow{OP^2} = \overrightarrow{OM^2} + \overrightarrow{MP^2}$$

$$\overrightarrow{OP} = \sqrt{\overrightarrow{x^2} + \overrightarrow{y^2}}$$

$$\therefore \text{OP} = \sqrt{x^2 + y^2}$$

$\therefore$ length of $\overrightarrow{OP} = \sqrt{x^2 + y^2}$ [The sign → over any unit indicate direction only].

(b) To find the distance between two points lying in the same quadrant and whose Rectangular Bhattacharyya's Co-ordinates are given :

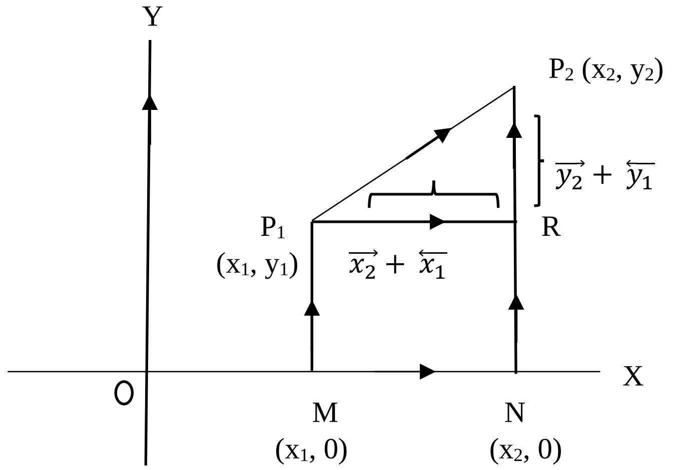

Fig.5

Let (x_1,y_1) and (x_2,y_2) are the Bhattacharyya's Co-ordinates of the points P_1 and P_2 respectively referred to as rectangular co-ordinate axes $\overrightarrow{OX}$ and $\overrightarrow{OY}$ (Fig.5). We have to find the distance between the points P_1 and P_2. Let us draw $\overrightarrow{MP_1}$ and $\overrightarrow{NP_2}$ perpendiculars from P_1 and P_2 respectively on $\overrightarrow{OX}$ and then draw $\overrightarrow{P_1R}$ perpendicular from P_1 on $\overrightarrow{NP_2}$. So, it is evident that

$\overrightarrow{OM} = \overrightarrow{x_1}$, $\overrightarrow{ON} = \overrightarrow{x_2}$, $\overrightarrow{MP_1} = \overrightarrow{y_1}$ and $\overrightarrow{NP_2} = \overrightarrow{y_2}$ and $\overrightarrow{MN} = \overrightarrow{P_1R}$

So, $\overrightarrow{P_1R} = \overrightarrow{x_2} + \overleftarrow{x_1}$ [Using theory of relative motion].

and $\overrightarrow{RP_2} = \overrightarrow{NP_2} + \overleftarrow{NR} = \overrightarrow{NP_2} + \overleftarrow{MP_1}$ [$\because \overrightarrow{MP_1} = \overrightarrow{NR}$ and using the theory of relative motion]

$= \overrightarrow{y_2} + \overleftarrow{y_1}$

From the right-angled tringle P_1 R P_2 we get

$\overrightarrow{P_1P_1^2} = \overrightarrow{P_1R^2} + \overrightarrow{RP_2^2} = (\overrightarrow{x_2} + \overleftarrow{x_1})^2 + (\overrightarrow{y_2} + \overleftarrow{y_1})^2$ [$if\ x_2 \geqslant x_1\ and\ y_2 \geqslant y_1$]

$\therefore \overrightarrow{P_1P_2} = \sqrt{(\overrightarrow{x_2} + \overleftarrow{x_1})^2 + (\overrightarrow{y_2} + \overleftarrow{y_1})^2}$

$\overrightarrow{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note that minus sign (-) represents as subtraction operator.

It is evident from Rectangular Bhattacharyya's Co-ordinate system.

$\overrightarrow{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is positive.

We need not require to say that distance of $\overrightarrow{P_1P_2}$ is positive because distance can not be negative.

(c) To find the distance between two points lying not in the same quadrant and having common ordinate whose Rectangular Bhattacharyya's Co-ordinates are given.

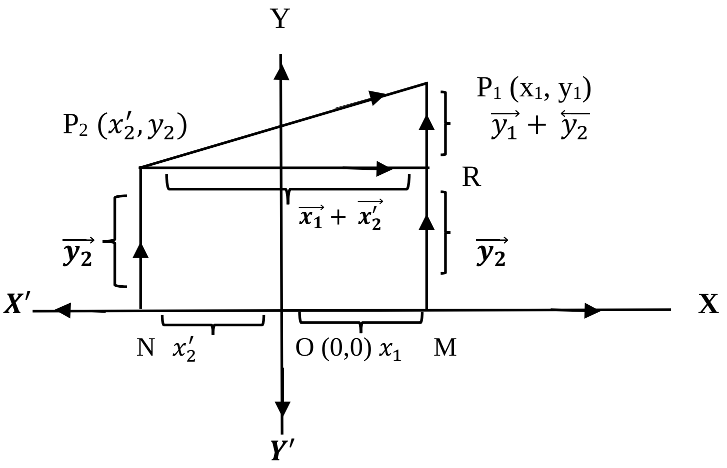

Fig.6

Let (x_1, y_1) be the Bhattacharyya's Co-ordinate of point P_1 referred to rectangular co-ordinate axes $\overrightarrow{OX}$ and $\overrightarrow{OY}$ (Fig.6). So, the point P_1 lies in the first quadrant. Similarly, (x_2', y_2) be the co-ordinate referred to rectangular co-ordinate axes $\overrightarrow{OX'}$ and $\overrightarrow{OY}$. So, the point P_2 lies in the second quadrant. Here, $\overrightarrow{OY}$ is the common ordinate. Now, we have to find the distance between P_2 and P_1. Now, from P_1 draw $\overrightarrow{MP_1}$ perpendicular on $\overrightarrow{OX}$ and from P_2 draw $\overrightarrow{NP_2}$ perpendicular on $\overrightarrow{OX'}$.

So,

$$\overrightarrow{OM} = \overrightarrow{x_1}, \overrightarrow{ON} = \overrightarrow{x_2'}, \overrightarrow{MP_1} = \overrightarrow{y_1}, \ \overrightarrow{NP_2} = \overrightarrow{y_2} \text{ and } \overline{NM} = \overrightarrow{P_2R}, \overrightarrow{MR} = \overrightarrow{NP_2}$$

Now,

$$\overrightarrow{P_2R} = \overrightarrow{OM} + \overrightarrow{ON} = \overrightarrow{x_1} + \overrightarrow{x_2'} \text{ [using theory of relative motion]}$$

and

$$\overrightarrow{RP_1} = \overrightarrow{MP_1} + \overleftarrow{MR} = \overrightarrow{y_1} + \overleftarrow{y_2} \text{ [using theory of relative motion]}$$

We know from right-angled triangle P_2 R P_1

$$\overrightarrow{P_2P_1^2} = \overrightarrow{P_2R^2} + \overrightarrow{RP_1^2} = (\overrightarrow{x_1} + \overrightarrow{x_2'})^2 + (\overrightarrow{y_1} + \overleftarrow{y_2})^2 \ \ [if\ y_1 \geqslant y_2]$$

$$\overrightarrow{P_2P_1} = \sqrt{(\overrightarrow{x_1} + \overrightarrow{x_2'})^2 + (\overrightarrow{y_1} + \overleftarrow{y_2})^2} \text{ is positive.}$$

So, $\overrightarrow{P_2P_1} = \sqrt{(x_1 + x_2)^2 + (y_1 - y_2)^2}$

Bhattacharyya's co-ordinate system measures not only the distance between two points but also indicates the position of the line segment on the plane with its direction.

(d) To find the distance between two points lying not in the same quadrant but with common Abscissa and whose Rectangular Bhattacharyya's Co-ordinates are given.

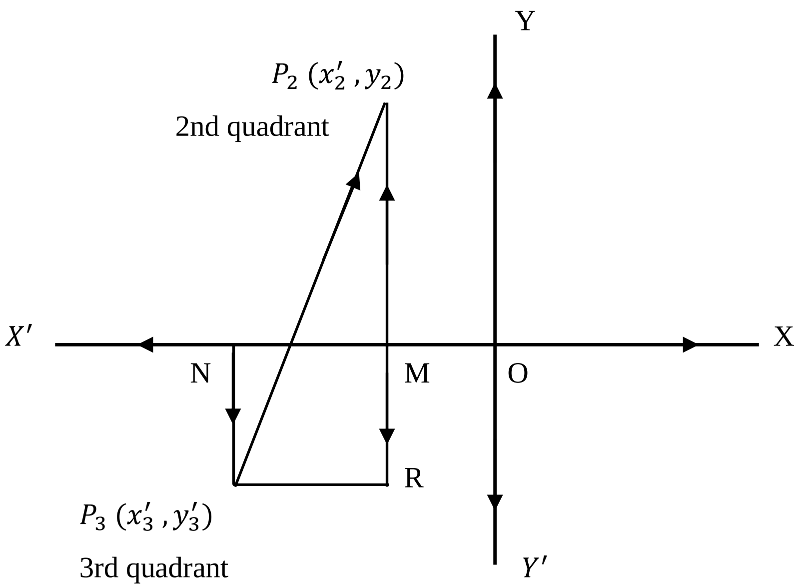

Fig.7.

Let (x_2', y_2) be the Bhattacharyya's co-ordinate of the point P_2 referred to rectangular co-ordinate axes $\overrightarrow{OX'}$ and $\overrightarrow{OY}$ (Fig.7). So, the point P_2 lies in the second quadrant. Similarly, (x_3', y_3') be the co-ordinate referred to rectangular co-ordinate axes $\overrightarrow{OX'}$ and $\overrightarrow{OY'}$. So, point P_3 lies in the third quadrant. Here, $\overrightarrow{OX'}$ is the common abscissa. We have to find the distance between the point P_3 and P_2.

Now, from P_2 draw $\overrightarrow{MP_2}$ perpendicular on $\overrightarrow{OX'}$ axis and from P_3 draw $\overrightarrow{NP_3}$ perpendicular on $\overrightarrow{OX'}$ axis.

So, $\overrightarrow{OM} = \overrightarrow{x_2}$ *and* $\overrightarrow{ON} = \overrightarrow{x_3'}, \overrightarrow{MP_2} = \overrightarrow{y_2}, \overrightarrow{NP_3} = \overrightarrow{y_3'}$ *and* $\overline{NM} = \overline{P_3R}, \overrightarrow{MR} = \overrightarrow{NP_3}$

Now, $\overrightarrow{RP_2} = \overrightarrow{MR} + \overrightarrow{MP_2} = \overrightarrow{y_3'} + \overrightarrow{y_2}$ (using the theory of relative motion)

and $\overrightarrow{P_3R} = \overrightarrow{ON} + \overleftarrow{OM} = \overrightarrow{x_3'} + \overleftarrow{x_2'}$ (using the theory of relative motion) [if $x_3 \geqslant x_2$]

We know from right angle triangle P_3RP_2.

$$\overrightarrow{P_3P_2^2} = \overrightarrow{P_3R^2} + \overrightarrow{RP_2^2} = (\overrightarrow{x_3'} + \overleftarrow{x_2'})^2 + (\overrightarrow{y_3'} + \overrightarrow{y_2})^2$$

$\therefore \overrightarrow{P_3P_2} = \sqrt{(\overrightarrow{x_3'} + \overleftarrow{x_2'})^2 + (\overrightarrow{y_3'} + \overrightarrow{y_2})^2}$ is positive

So, $\overrightarrow{P_3P_2} = \sqrt{(x_3 - x_2)^2 + (y_3 + y_2)^2}$

It is evident from Rectangular Bhattacharyya's Co-ordinate System.

$$\overrightarrow{P_3P_2} = \sqrt{(x_3 - x_2)^2 + (y_3 + y_2)^2} \text{ is positive.}$$

(e) To find the distance between two points lying not in the same quadrant and with different Abscissas and ordinates and whose Rectangular Bhattacharyya's Co-ordinates are given.

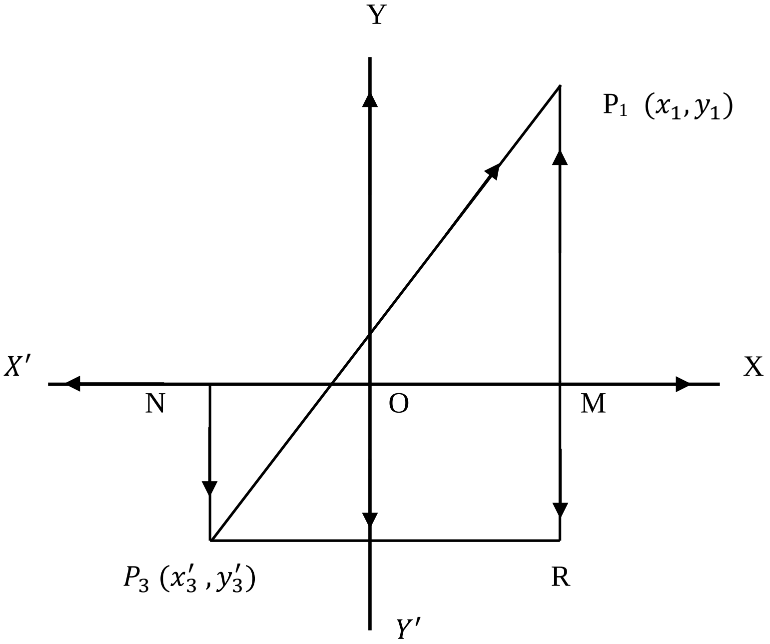

Fig. 8.

Let (x_1, y_1) be the Bhattacharyya's co-ordinate of the point P_1 referred to as rectangular co-ordinate axes $\overrightarrow{OX}$ and $\overrightarrow{OY}$ (Fig. 8). So, the point P_1 lies in the first quadrant. Similarly, (x_3', y_3') be the co-ordinate referred to rectangular co-ordinate $\overrightarrow{OX'}$ and $\overrightarrow{OY'}$. So, point P_3 lies in the third quadrant.

Now, from P_1 draw $\overrightarrow{MP_1}$ perpendicular on $\overrightarrow{OX}$ axis and from P_3 draw $\overrightarrow{NP_3}$ perpendicular on $\overrightarrow{OX'}$ axis.

So, $\overrightarrow{OM} = \overrightarrow{x_1}, \overrightarrow{ON} = \overrightarrow{x_3'}, \overrightarrow{MP_1} = \overrightarrow{y_1}$ *and* $\overrightarrow{NP_3} = \overrightarrow{y_3'}$ *and* $\overrightarrow{P_3R} = \overrightarrow{NM}, \overrightarrow{MR} = \overrightarrow{NP_3}$

Now, $\overrightarrow{P_3R} = \overrightarrow{NM} = \overrightarrow{OM} + \overrightarrow{ON} = \overrightarrow{x_1} + \overrightarrow{x_3'}$ (using theory of relative motion)

and $\overrightarrow{RP_1} = \overrightarrow{MP_1} + \overrightarrow{MR} = (\overrightarrow{y_1} + \overrightarrow{y_3'})$ (using theory of relative motion)

We have from the right-angled triangle P_3RP_1

$$\overrightarrow{P_3P_1^2} = \overrightarrow{P_3R^2} + \overrightarrow{RP_1^2} = (\overrightarrow{x_1} + \overrightarrow{x_3'})^2 + (\overrightarrow{y_1} + \overrightarrow{y_3'})^2$$

$$\therefore \overrightarrow{P_3P_1} = \sqrt{(\overrightarrow{x_1} + \overrightarrow{x_3'})^2 + (\overrightarrow{y_1} + \overrightarrow{y_3'})^2}$$

It is evident from Rectangular Bhattacharyya's Co-ordinate System

$$\overrightarrow{P_3P_1} = \sqrt{(\overrightarrow{x_1} + \overrightarrow{x_3'})^2 + (\overrightarrow{y_1} + \overrightarrow{y_3'})^2} \text{ is positive.}$$

$$\overrightarrow{P_3P_1} = \sqrt{(x_1 + x_3)^2 + (y_1 + y_3)^2}$$

Conclusion

Rectangular Bhattacharyya's Co-ordinate System is the more scientific method to solve the problem of the plane co-ordinate geometry than that of Rectangular Cartesian Co-ordinate System.

Similarly by Bhattacharyya's six-dimensional co-ordinate geometry where all the dimensions or axes are positive the position of a point on a solid body can be determined uniquely by three real positive numbers.

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