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# A NEW EVOLUTIONARY METHOD TO PARAMETERS AND ORDERS IDENTIFICATION AND SYNCHRONIZATION OF CHAOTIC FRACTIONAL-ORDER SYSTEMS

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## Abstract

*System identification is an important task in control theory. Classical control theory is usually known for integer-order processes. Nowadays real processes are fractional order usually. According to a large number of fractional-order systems, the identification of these systems is so important. This paper aims to evaluate an improved Biogeography-based Optimization (BBO) approach to estimate the parameters and orders of fractional-order systems. After that, a method based on this algorithm has been introduced to synchronization of chaotic systems. Results show that the proposed scheme has high accuracy.*

**Keywords:** Fractional-order system, System identification, Biogeography-based Optimization

## I. Introduction

The main point of system identification is the estimation of the parameters of a process in good situations [X, XIV]. The system identification is the basis of controller design in the system. Fractional order dynamics identification has been done in some studies. These methods are numerical [XV], or analytical [XI, XXII]. Many classical methods have been used for the online identification of systems. Calculations on the fractional-order system are so complicated then system identification of fractional order systems seems to be a challenging problem [XIX]. The fractional-order systems usually have been identified using the integer version. The usual way for a fractional-order system is approximation methods with integer systems. This point causes errors to form real model and identified model. Order identification on fractional-order systems is a new point of view on system identifications. In this paper, a new method has been introduced to the parameters and orders identification of a fractional-order system. This method is using Biogeography-based Optimization (BBO). BBO is a new type of optimization method. This algorithm is obtained from biogeography [VI]. The basis of this algorithm is n the immigration and emigration of the animal species among islands [VIII, IX].

This algorithm shares the information among solutions using the adaption of the migration operator. This feature makes BBO applicable to a large number of problems.

We use this technique to find the parameters and orders of the fractional-order system. This method enables us to achieve a very good approximation. The basic step of fractional order system identification is understanding the theory of fractional calculus. This section deals with fractional order integration and derivation which describes in the next part. The studied system is chaotic. These systems have complex behavior then identification of these systems is so important.

Evolutionary algorithms for the identification have been done in many studies [I, XII].

These algorithms can be used in some cases for the fractional-order system but new modified algorithms can be improved this task. About chaotic systems, this task becomes more challenging. To the best of our knowledge, a specified method for order identification of fractional order systems has not been introduced yet. Especially in the chaotic case, it is an important task, because the order of a chaotic system in the fractional-order chaotic system plays an important role. A few changes in the fractional order of a chaotic system change this system to a stable, unstable, or hyperchaotic system. Then exact order identification besides the parameter identification gives a better view for fractional-order chaotic systems.

Synchronization between chaotic systems is the one of most applicable topics on the chaotic system. A large variety of applications are introduced for chaos synchronization [XXIV, XXI, II].

Synchronization between fractional-order chaotic systems is a new topic in recent decades [XVII, XVIII]. Different methods have been introduced in this field [XX, V, VII].

The main organization of this paper is as follows. In section 2, basic definitions of fractional calculations have been introduced. The general description of the biography-based optimization has been written in section3. In section 4, a fractional-order chaotic system is defined and parameters and orders identifications using this method have been done.

This section shows the simulation results of the proposed method. The main conclusions of the paper are mentioned in section 5.

## II. Experimental Study

### Materials

Euler's gamma function is one of the usable functions on fractional calculations. This function is defined as follows.

$$T(n) - \int_0^x t^{n-1} e^{-t} dt \qquad (1)$$

Three types of definitions are well-known definitions for the fractional-order operator. The RL definition of the fractional-order operator is defined as follows [1].

$$\frac{d^\alpha}{dt^\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^\alpha}{dt^\alpha} \int_0^t \frac{f^{(n)}(T)}{(t-T)^{n-\alpha+1}} dT \quad (2)$$

The Laplace transform of the fractional derivative is defined as follows.

$$\mathcal{L}\left(\frac{d^\alpha}{dt^\alpha} f(t)\right) = s^\alpha \mathcal{L}(f(t)) - \sum_{k=0}^{n-1} [\frac{d^{\alpha-1-k} f(t)}{dt^{\alpha-1-k}}]_{t=0} \quad (3)$$

where $L$ shows the Laplace transformer. For solving ordinary differential equations Laplace transform is an effective method. Also for the fractional-order systems, this transform has many capabilities.

## III. Results and Discussions

### Biogeography based optimization

The basis of BBO is the study of biological organisms' geographical distribution. In the science of biogeography, a habitat is an ecological area that is inhabited by particular plant or animal species and geographically isolated from other habitats. In comparison with other evolutionary optimization methods, BBO has some unique characteristics. In the BBO algorithm, habits identified the problem possible solutions. Operators of inhabit share information between the problem solutions. These operators are based on the concept of migration.

The main four parameters of BBO are as follows.

1- Suitability index variable (SIV)

shows habitability in an island variable

2- Habitat suitability index (HSI)

shows the goodness of the solution, like fitness score in Genetic algorithm

3- Immigration rate ($\lambda$)

shows how likely a solution is to accept features from other solutions

4- Emigration rate ($\mu$)

shows how likely a solution is to share its features with other solutions.

A bad habit is that low emigration and high emigration is happed on it. It means, when HIS increases, the number of species becomes more. This makes more ability for the species to leave the island. This event leads to high emigration. HSI has a direct relation to the emigration rate and population. Low immigration rate happens in the habitats which are already saturated with species. Also, HSI has a reverse relation with the immigration rate.

The value of HSI for the low HSI habitats is increased by the immigration of species from other habitats. If this condition happens but HSI doesn't increase then species in that habitat go extinct which is the cause of additional immigration.

In the BBO, the population of solutions is vectors of integers. Each integer in a solution vector shows SIV. In the BBO, the emigration rate $\mu_k$ and immigration rate $\lambda_k$of each habitat can be calculated as follows.

$$\begin{cases} \lambda_k = \frac{Ek}{P} \\ \mu_k = I(1-\frac{k}{p}) \end{cases} \qquad (4)$$

Where $k$ is the number of habitats and $P$ is the individual number.
The main steps of improved BBO are defined as follows:
1- Generation of habitats and consider immigration and emigration rate to these habitats.
2- Ordering the generated habitats.
3-Using the habitats rank, define $\lambda$ , $\mu$
4- Destination $j$ is selected using the$\mu$.
5- Immigration from $x_{ij}$o $x_{ik}$is done ($i$ shows the number of habitats and $k$ is an integer number).

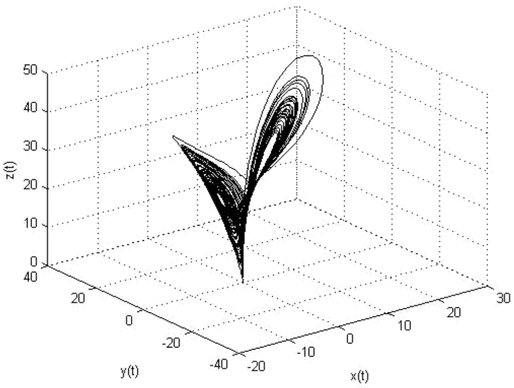



**Figure 1.** The chaotic attractor of the fractional-order Lorenz system

6- Applying the mutation operator.
7- For each habitat, repeat steps 4-6.
8- For each integer $k$ in $i$th habitat, repeat steps 5-6.
9- Modify the habitats with the new generations.
11- Return to step 4 if the final criteria do not satisfied.

## PROBLEM FORMULATION AND APPLICATION

*Parameter identification*

An n-dimensional fractional-order system as follows is considered.

$$D^{\alpha}Y(t) = f(Y(t).t.\theta) \qquad (5)$$

where $Y$ ($t$) = ($Y$1($t$); $Y$2($t$)…, $Y_n$ ($t$))$T$ 2 $R^n$ is the state vector of system.$0$ = $(0_1 0_2. \dots . 0_n)^T$ represents the parameters and $\alpha = \alpha_1.\alpha_2. \dots .\alpha_n)(0 < \alpha_i < 1)$are orders. $F$ ($Y$ ($t$); $t$; $h$) =$(f_1(Y(t).t.\theta) . f2(Y(t).t.\theta). \dots . f_n(Y(t).t.h))^T$.

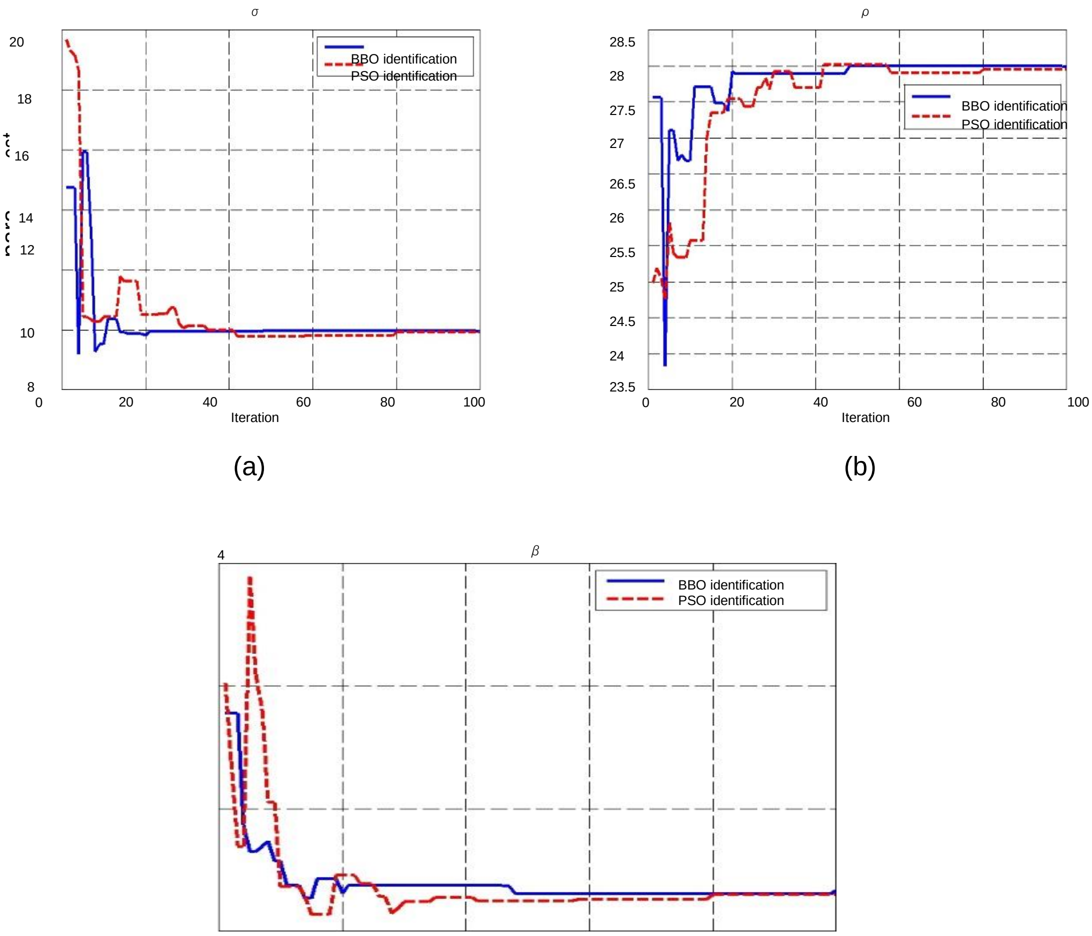
The main advantage of the proposed method in this paper is that we introduced a method that can identify orders and parameters of a fractional-order system simultaneously. Estimation of the system (5) is written as follows.

$$D^{\alpha}\hat{Y}(t) = f\big(\hat{Y}(t).t.\hat{\theta}\big) \tag{6}$$

where $\hat{\theta} = (\hat{\theta}_1 . \hat{\theta}_2 . \ldots . \hat{\theta}_n)^T$ shows the estimated parameters and $\hat{\alpha} = (\hat{\alpha}_1 . \hat{\alpha}_2 . \ldots . \hat{\alpha}_n)$ is the
estimated orders. Also $\hat{Y}(t) = (\hat{Y}_1(t). \hat{Y}_2(t). \ldots . \hat{Y}_n(t))^T$ represents the vector of the estimated states. For identification, the following equation is minimized.

$$F = \sum_{k=1}^{N} ||Y(t) - \hat{Y}(t)||^2 \tag{7}$$

where $k = 1 . 2 . \ldots . N$ is the number of sampling data. The second norm of a vector is shown using the $||\ .\ ||$ . States of the main and the estimated system are shown as $Y_k$ and $\hat{Y}_k$, respectively.




(a) (b)

(c)

**Figure 2**. Parameters estimation of a fractional-order chaotic system, $\sigma$ (a) $\rho$ (b) _ (c)

Now the minimum of the function (7) leads to the estimated parameters and orders of the system. In this paper, we employ the BBO algorithm to identify the parameters and order of the system (5). The main advantage of the BBO algorithm in comparison to the other methods is that this algorithm needs the minimum parameter tuning in compared with the other optimization methods then the implementation of this algorithm is so easy.
In this section, the fractional-order Lorenz system as an example is given to verify the effectiveness of the system identification method for fractional-order systems with the BBO algorithm.
Discretization of fractional-order systems is based on the generalization of the Adams-Bashforth Moulton method [XVI].

***. Fractional-order Lorenz chaotic system***

The fractional version of the Lorenz chaotic system is studied in some cases [III, IV]. The system is formulated as follows.

$$\begin{cases} D^{\alpha 1}x = \sigma(y - x) \\ D^{\alpha 2}y = px - xz - y \\ D^{\alpha 3}z = xy - \beta z \end{cases} \quad (8)$$

where $0 < \alpha_i < 1$ $(i = 1\,, 2\,, 3)$ is the order. The integer version of the system (8) is chaotic with $(\sigma\,.\,\rho\,.\,\beta) = \left(10\,.\,28\,.\,\frac{8}{3}\right)$. When the sum of three fractional orders is bigger than 2.91, the fractional-order Lorenz system is chaotic [11, 12]. For the simulation, we selected the parameters as $(\sigma\,.\,\rho\,.\,\beta) = \left(10\,.\,28\,.\,\frac{8}{3}\right)$ and the orders as $(\alpha_1\,.\,\alpha_2\,.\,\alpha_3) = (0.991\,.\,0.993\,.\,0.996)$ .Fig.1 shows the chaotic behavior of the fractional-order Lorenz system with the mentioned
parameters and orders.

***Synchronization***

Synchronization problem is defined with two drive and response systems as follows.

$$D^{\alpha}x_d = f(x_d) \quad (9)$$

and

$$D^{\alpha}x_r = g(x_r) + U(x_d\,.\,x_r) \quad (10)$$

where $x_r\,, x_d \in R^n$are the vectors which show the states of the drive and response systems
, $f$ is a vector that shows the nonlinear functions of the drive system and $g$ is the same vector
for the response system, $\alpha \in R^n$is the vector of fractional orders. In the definition of synchronization, the synchronization error is defined as:

$$e = x_r - x_d \quad (11)$$

The main object is t find $U$ to achieve synchronization between the drive system (9) and the response system (10). We can write

$$D^{\alpha}e = D^{\alpha}x_r - D^{\alpha}x_d \quad (12)$$

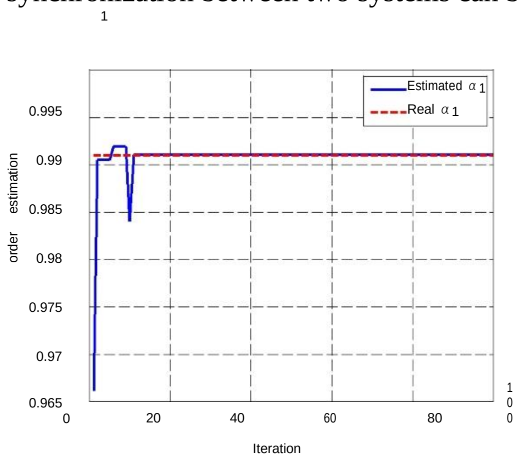
Put Eq.(9) and (10) into Eq.(12) leads to

$$D^{\alpha}e = (B)x_r + G(x_r) + U(x_r.x_d) - \big((A)x_d + F(x_d)\big) \qquad (13)$$

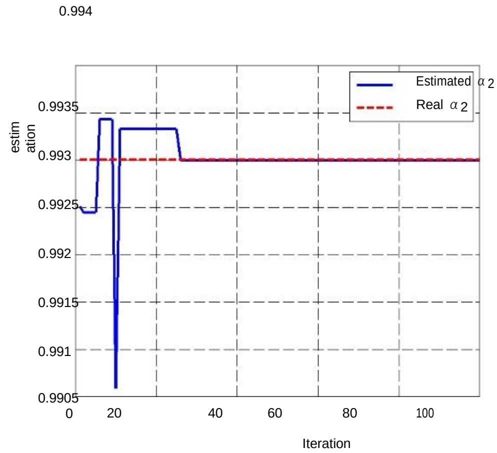
Now active control method is used to design controllers [24], then

$$U(x_d, x_r) = -Bx_r - G(x_r) + \big(Ax_d + F(x_d)\big) + K(x_r - x_d) \qquad (14)$$

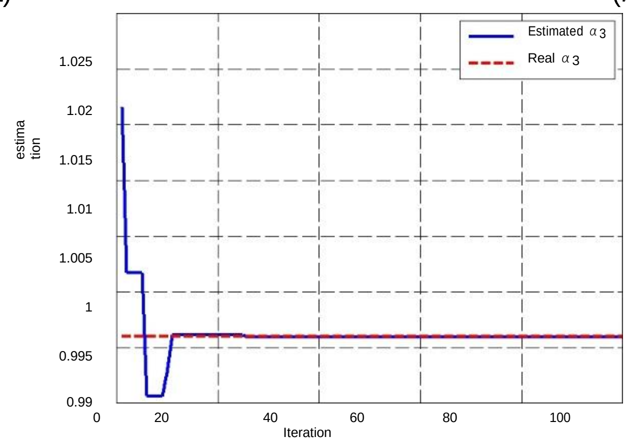
where $K = [k_1\ ;\ k_2\ ; \ldots . k_n]^T \in R^{n\times n}$ is a matrix for the control parameters and $k_i = [k_{i1}\ .\ k_{i2}\ . \ldots . k_{in}]$ is the control vector in each state controller. The selection of these elements must be based on the stability theory of fractional-order systems. Generally, a diagonal matrix for simplifies the calculations is selected but we use the BBO algorithm to minimizing the synchronization time. Eq.(13) proves that by choosing the appropriate K matrix that satisfies the stability condition of Eq.(13) means$|\arg(\ s\,pec(K))| > \alpha\pi/2$, the synchronization errors are converging to zero, and synchronization between two systems can be achieved.




(a)




(b)




(c)

**Figure 3**. Orders estimation of a fractional-order chaotic system, $\alpha_1$ (a) $\alpha_2$ (b) $\alpha_3$ (c)

## V. Simulation example

Initial conditions for the states of drive and response systems are selected as (0:1; 0:2; 0:3) and (0:2; 0:3; 0:4) with mentioned parameters and orders. For comparison, we simulate the proposed method in [XIII], which is using the Particle swarm optimization (PSO) method. Fig.2 shows the parameters identification of two methods also Fig.3 shows orders identification. The result shows that the proposed method identifies parameters and orders of a fractional-order chaotic system with high accuracy. Synchronization of two systems is done with the mentioned method and this method is compared with the active control method [XXIV]. The results of this comparison are shown in Fig.4

## VI. Conclusion

In this paper, a new method for the estimation of parameters and orders of a fractional-order chaotic system was proposed. This method is based on a Biogeography-based Optimization algorithm. The proposed method provides results with high accuracy compared with other methods. The result shows that the proposed method is a good performance in parameters and orders identification of fractional-order chaotic systems.

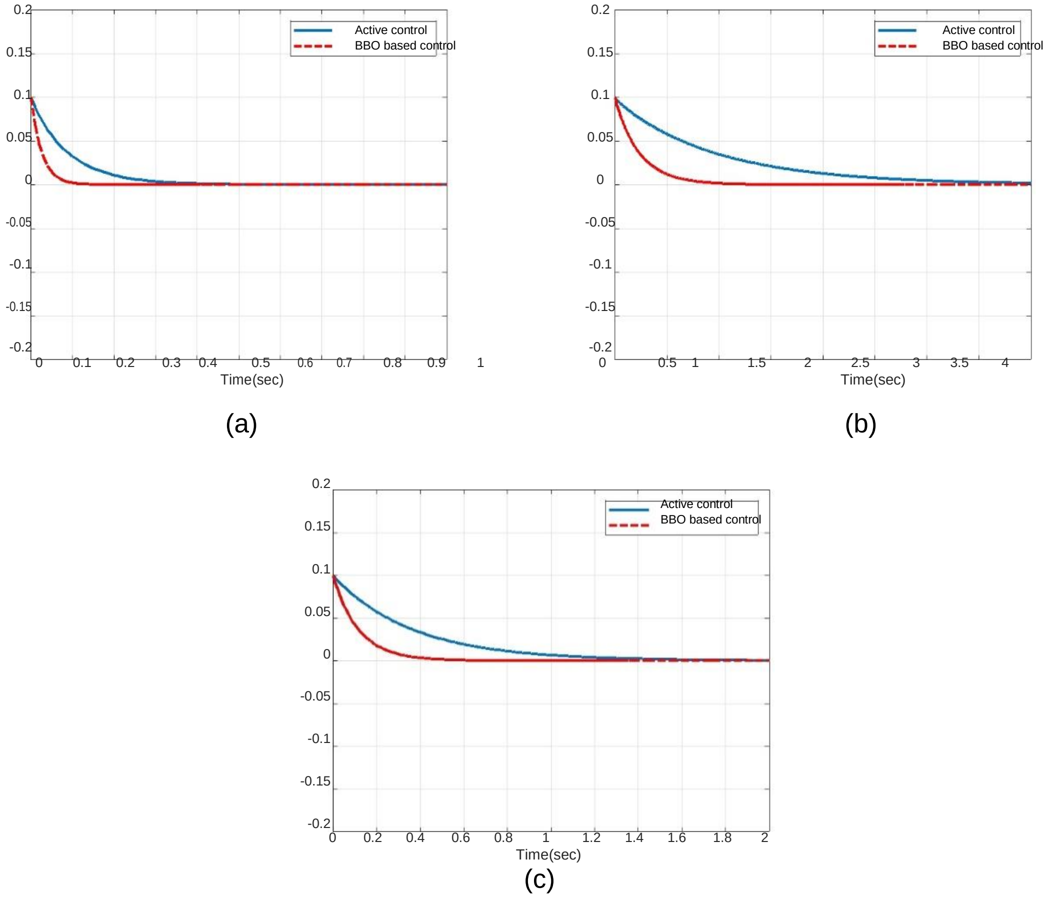



**Figure 4**. Orders estimation of a fractional-order chaotic system, $\alpha_1$ (a) $\alpha_2$ (b) $\alpha_3$ (c)

**Conflicts of Interest:**

The authors declare that they have no conflicts of interest to report regarding the present study.

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