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INTUITIONISTIC FUZZY ENTROPY AND ITS APPLICATIONS TO MULTICRITERIA DECISION MAKING WITH IF-TODIM

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Abstract

The intuitionistic fuzzy entropy (IFE) is being used to measure the degree of uncertainty of a fuzzy set (FS) with alarming accuracy and precision more accurately than the fuzzy set theory. Entropy plays a very important role in managing the complex issues efficiently that we often face in our daily life. In this paper, we first review several existing entropy measures of intuitionistic fuzzy sets (IFSs) and then suggest two new entropies of IFSs better than the existing ones. To show the efficiency of proposed entropy measures over existing ones, we conduct a numerical comparison analysis. Our entropy methods are not only showing better performance but also handle those IFSs amicably which the existing method fails to manage. To show the practical applicability and reliability, we utilize our methods to build intuitionistic fuzzy Portuguese of interactive and multicriteria decision making (IF-TODIM) method. The numerical results show that the suggested entropies are convenient and reasonable in handling vague and ambiguous information close to daily life matters.

Keywords: Intuitionistic Fuzzy Sets, Entropy Measure, Multicriteria Decision Making, IF-TODIM.

I. Introduction

The exactness of mathematics has indebted its accomplishment in huge measures to the exertions of Aristotle and the philosophers who lead him. In their exertions to formulate a brief theory of logic, the so-called "Laws of Thought" [XI] were introduced. After that, the "Law of the Excluded Middle", and the conditions that “each proposition necessity can also be True or False”. Thus, when Parmenides started to the future, the initial description of this law (around 400 B.C.) there were robust and abrupt hostilities, for example, Heraclitus suggested that things could be concurrently true and not true. The man who was Plato placed the basis, for what prayed become fuzzy logic, represents that there is a third region outside False “no” and True “yes” wherever these contraries "tumbled out." Thus at the start of the 1900s, Lukasiewicz [XII] labeled a three-valued logic, besides with the mathematics to convoy it and he reconnoitered four-valued logics, five-valued logics, and then infinite-valued logic Siddique [XIX]. Therefore, the complications and restrictions of human empathy and furthermore, the controlling complications encompass a lot of fuzzy notions. Consequently, the fuzzy set was primarily suggested by Zadeh [XXV], fuzzy sets have been useful in numerous areas, particularly the entropy created on decision-making problems based on fuzzy sets have been extensively premeditated and functional to numerous areas such as selection criteria for job, best candidate selection in elections, military weapon system evaluation, selection of manufacturers etc.

The fuzzy set theory is the adapted method of the crisp set. The crisp set is also called the traditional set. To find vagueness and uncertainty of our daily life problems, fuzzy sets are very useful. Probability theory also finds uncertainty but there is a huge difference between fuzzy set and probability theory. The difference between fuzzy and crisp sets is that the crisp function defines only “yes” or “no”. In crisp set “1” is for membership and “0” is for non-membership.

The fuzzy set theory is used for decision making and the decision-makers repeatedly show some hesitation because of the complication and the ambiguity of the data of authentic control problem. To label the hesitancy degree, Atanassov [I] anticipated the notion of intuitionistic fuzzy sets. Intuitionistic fuzzy numbers (IF-numbers) cannot merely define the overall variety of these fuzzy numbers, but also can define the vagueness of decision makings on the findings of the decision-makers. IFSs also studies the three features of evidence i.e. membership degree, non-membership degree and hesitancy degree. The extension of fuzzy sets (FSs) in intuitionistic fuzzy sets (IFSs). The fuzzy set describes only the membership and non-membership but there are many causes by which the hesitancy may occur, thus the fuzzy set cannot define the degree of hesitancy. Due to such causes, we may use the intuitionistic fuzzy sets (IFSs). To deal with decision-making problems many intuitionistic fuzzy multi-criteria decisions making (MCDM) methods are established such as Li [XIII], Liu [XIV], Wei et al [XXIV], Wang and Wang [XXIII].

Entropy is being used to measure the degree of fuzziness of a fuzzy set. Fuzzy entropy has also been extensively functional in grouping examination, image processing, MCDM problem and design gratitude. Zadeh [XXVI], initially familiarized the entropy of a fuzzy occurrence in 1968. Later, in 1972, Deluca and Termini [V] contributed to the characterization of fuzzy entropy, and they also anticipated fuzzy entropy centered on Shannon [XXI] function. Many researchers have comprehended the importance of entropy and have fabricated fuzzy entropy procedures from different perspectives [19-21]. Yang [IX] suggested an interval-based entropy (2008) based on Yager [XXIV] entropy which is more effective than several other entropies. Gau and Buehrer [VIII] familiarized the notion of a vague set.

But Bustince and Burillo [III] proved that vague sets are intuitionistic fuzzy sets. They also defined the intuitionistic entropy of IFS. Like a Fuzzy set, entropy is a degree that measures the degree of intuitionism of a set, that is, the amount of parting of intuitionistic fuzzy sets not from ordinary sets but from fuzzy sets. Thus, we shall call this degree intuitionistic fuzzy entropy (IFE). In the recent past, Li [XIII] explain and introduced a new entropy for intuitionistic fuzzy sets and proved validity through an example. In this work, we have used our new suggested intuitionistic entropy and justified it with examples. For this entropy, we have developed and modified the algorithm intuitionistic fuzzy TODIM (IF-TODIM) method for multicriteria decision making (MCDM) based on a new entropy measure.

The rest of the paper is arranged as follows. In section 2, we analyze some existing entropy measures of IFSs. In section 3, we introduced two new entropy measures based on IFSs and gave the proof of all axioms. Section 4, is dedicated to numerical comparisons and examples to show the validity of our proposed ones. In section 5, we utilize our proposed IF-TODIM to manage a daily life problem involving complex multicriteria decision-making. We stated the conclusion in section 6.

II. Preliminaries

These sections include basic definitions of IFSs and a review of some existing entropies of IFS.

Definition 1 [I]. The intuitionistic fuzzy set (IFS) $\dddot{E}$ of the universal set $\dddot{G}$ is defined as follows;

$$\dddot{E}=\left\{\left\langle\dddot{\varepsilon}_i,\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right),\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right\rangle:\dddot{\varepsilon}_i\in\dddot{G}\right\}$$

where $\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right):\dddot{G}\rightarrow[0,1]$ and $\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right):\dddot{G}\rightarrow[0,1]$. Thus, $\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)$ define the degree of membership and $\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)$ define the degree of non-membership for all $\dddot{\varepsilon}_i\in\dddot{G}$ and

$$0\le\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)+\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\le 1,\ \forall\dddot{\varepsilon}_i\in\dddot{G}.$$

The degree of non-determinacy of intuitionistic fuzzy sets (IFSs) is defined as,

$$\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)+\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)+\dddot{\pi}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)=1,\ \forall\dddot{\varepsilon}_i\in\dddot{G}.$$

Obviously, $\dddot{\pi}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)=1-\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)-\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)$.

Definition 2 [I]. Let $\dddot{S}$ be the universal set and $\dddot{F}\ and\ \dddot{H}$ are subsets of $\dddot{S}$ then union $\dddot{F}\cup\dddot{H}$ is defined as,

$$\dddot{\kappa}_{\dddot{F}\cup\dddot{H}}\left(\dddot{\varepsilon}_i\right)=\left\{\left\langle\dddot{\varepsilon}_i,max\left(\dddot{\kappa}_{\dddot{F}}\left(\dddot{\varepsilon}_i\right),\dddot{\kappa}_{\dddot{H}}\left(\dddot{\varepsilon}_i\right)\right),min\left(\dddot{\lambda}_{\dddot{F}}\left(\dddot{\varepsilon}_i\right),\dddot{\lambda}_{\dddot{H}}\left(\dddot{\varepsilon}_i\right)\right)\right\rangle\right\},\ \forall\dddot{\varepsilon}_i\in\dddot{G}.$$

Definition 3 [I]. Let $\dddot{S}$ be the universal set and $\dddot{F}\ and\ \dddot{H}$ are subsets of $\dddot{S}$ then intersection $\dddot{F}\cap\dddot{H}$ is defined as,

$$\dddot{\kappa}_{\dddot{F}\cap\dddot{H}}\left(\dddot{\varepsilon}_i\right)=\left\{\left\langle\dddot{\varepsilon}_i,min\left(\dddot{\kappa}_{\dddot{F}}\left(\dddot{\varepsilon}_i\right),\dddot{\kappa}_{\dddot{H}}\left(\dddot{\varepsilon}_i\right)\right),max\left(\dddot{\lambda}_{\dddot{F}}\left(\dddot{\varepsilon}_i\right),\dddot{\lambda}_{\dddot{H}}\left(\dddot{\varepsilon}_i\right)\right)\right\rangle\right\},\ \forall\dddot{\varepsilon}_i\in\dddot{G}.$$

Definition 4 [I]. The complement of fuzzy set $\ddot{F}$ can be denoted as $\ddot{F}^{c}$. Membership degree and non-membership degree can be calculated as,

$$\ddot{\kappa}_{\ddot{F}^c}(\ddot{\varepsilon}_i)=\left\{\left\langle \ddot{\varepsilon},\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i),\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\right\rangle\right\},\ \forall\ddot{\varepsilon}_i\in\dddot{G}$$

Definition 5 [I]. The weighted Hamming distance between fuzzy sets $\ddot{F}$ *and* $\ddot{H}$ is defined as,

$$d^{\infty}(\ddot{F},\ddot{H})=\frac{1}{2}\sum\ddot{w}_j\left\{\left|\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)-\ddot{\kappa}_{\ddot{H}}(\ddot{\varepsilon}_i)\right|+\left|\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i)-\ddot{\lambda}_{\ddot{H}}(\ddot{\varepsilon}_i)\right|+\left|\ddot{\pi}_{\ddot{F}}(\ddot{\varepsilon}_i)-\ddot{\pi}_{\ddot{H}}(\ddot{\varepsilon}_i)\right|\right\},\ \forall\ddot{\varepsilon}_i\in\dddot{G}.$$

Definition 6. A map $\dddot{Q}:IFS(\dddot{G})\rightarrow[0,1]$ is said to be intuitionistic fuzzy entropy (IFE), if it holds the following axioms:

$(I)\quad 0\le\dddot{Q}(\ddot{E})\le 1;$

$(II)\quad \dddot{Q}(\ddot{E})=0$ If and only if $\ddot{E}$ is crisp set;

$(III)\quad \dddot{Q}(\ddot{E})=1$ if and only if $\ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)=\ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)=0.5;$

$(IV)\quad \dddot{Q}(\ddot{E})=\dddot{Q}(\ddot{E}^{c});$

$(V)\quad \text{If } \ddot{E}\le\ddot{F}, \text{ then } \dddot{Q}(\ddot{E})\le\dddot{Q}(\ddot{F}),$

$\ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)\le\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ and } \ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\ge\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ when } \ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i),$

$\text{if } \ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)\ge\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ and } \ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ when } \ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\ge\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i).$

Definition 7 [VI]. Let us consider $\ddot{E}^{n}=\left\{\left\langle\ddot{\varepsilon}_i,\ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i),\ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\right\rangle:\ddot{\varepsilon}_i\in\dddot{G}\right\}$, be an IFS in $\dddot{G}=\{\ddot{\varepsilon}_1,\ddot{\varepsilon}_2,\ddot{\varepsilon}_3,...,\ddot{\varepsilon}_i\}, i=1,2,3,...,n$. For any positive real number n the IFS $\ddot{E}^{n}$ define as fallow;

$$\ddot{E}^{n}=\left\{\left\langle\ddot{\varepsilon}_i,\left[\ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)\right]^{n},1-\left[1-\ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\right]^{n}\right\rangle:\ddot{\varepsilon}_i\in\dddot{G}\right\}$$

Definition 8. Let $\ddot{E}$ be the intuitionistic fuzzy set on the universal set $\dddot{G}$, the entropy $\dddot{Q}(\ddot{E})$

is called $\sigma-$entropy if

$$\dddot{Q}(\ddot{E})+\dddot{Q}(\ddot{F})=\dddot{Q}(\ddot{E}\cup\ddot{F})+\dddot{Q}(\ddot{E}\cap\ddot{F}),\quad\forall\ddot{\varepsilon}_i\in\dddot{G}, if$$

$(I)\ \ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)\le\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ and } \ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\ge\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ when } \ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i)\Rightarrow\ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)\le\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)$

$(II)\ \text{if } \ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i)\ge\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ and } \ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i) \text{ when } \ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\ge\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i)\Rightarrow\ddot{\lambda}_{\ddot{E}}(\ddot{\varepsilon}_i)\le\ddot{\lambda}_{\ddot{F}}(\ddot{\varepsilon}_i)\le\ddot{\kappa}_{\ddot{F}}(\ddot{\varepsilon}_i)\le\ddot{\kappa}_{\ddot{E}}(\ddot{\varepsilon}_i).$

Hooda [XVII] proposed entropy

$$\ddot{Q}_0(\dddot{E}) = -\frac{1}{4e}\sum_{i=1}^{r}\left(\frac{2\left|\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right|}{\frac{1}{2}\left(2 - \dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right)} e^{\left|\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right|}\right) + 1, \ \forall \dddot{\varepsilon}_i \in \ddot{G} \quad (1)$$

Lanping Li [XIII] define intuitionistic fuzzy entropy as

$$\ddot{Q}_1(\dddot{E}) = \frac{1}{r}\sum_{i=1}^{r}\frac{1 - \left(\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right)^2}{1 + 3\left(\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right)^2}, \ \forall \dddot{\varepsilon}_i \in \ddot{G}, \quad (2)$$

Szmidt and Kacprzyk [XIX] suggest the following entropy of IFS,

$$\ddot{Q}_2\left(\ddot{E}\right) = \frac{1}{r}\sum_{i=1}^{r}\left(\frac{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) \wedge \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i) + \dddot{\pi}_{\ddot{E}}(\dddot{\varepsilon}_i)}{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) \vee \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i) + \dddot{\pi}_{\ddot{E}}(\dddot{\varepsilon}_i)}\right), \ \forall \dddot{\varepsilon}_i \in \dddot{G} \quad (3)$$

Zeng and Li [4] proposed intuitionistic fuzzy entropy as follow

$$\ddot{Q}_3(\ddot{E}) = 1 - \frac{1}{r}\sum_{i=1}^{r}\left(\left|\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right|\right), \ \forall \dddot{\varepsilon}_i \in \dddot{G} \quad (4)$$

Zhang and Jiang [XXX] suggest intuitionistic fuzzy entropy by maximum of membership and non-membership divide by a minimum of membership and non-membership and define as follow

$$\ddot{Q}_4\left(\ddot{E}\right) = \frac{1}{r}\sum_{i=1}^{r}\left(\frac{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) \wedge \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)}{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) \vee \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)}\right), \ \forall \dddot{\varepsilon}_i \in \dddot{G} \quad (5)$$

Verma and Sharma [XXII] proposed entropy of IFSs as follows,

$$\ddot{Q}_5(E) = \frac{1}{r\left(\sqrt{e} - 1\right)}\sum_{i=1}^{r}\left\{\left(\frac{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) + 1 - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)}{2}\right) e^{\frac{\left(\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) + 1 - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right)}{2}} + \left(\frac{\left(-\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) + 1 + \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right)}{2}\right) e^{\frac{\left(-\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) + 1 + \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)\right)}{2}}\right\} \quad (6)$$

Mishra [XV] proposed the IF entropy of sine function and define it as follows

$$\ddot{Q}_6\left(\ddot{E}\right) = \frac{1}{r}\sum_{i=1}^{r}\left(1 - \sin\left\langle\frac{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)}{2(1 + \dddot{\pi}_{\ddot{E}}(\dddot{\varepsilon}_i)}\right\rangle \pi\right), \ \forall \dddot{\varepsilon}_i \in \dddot{G} \quad (7)$$

Wei et al. [XXIV] suggest the IF entropy of cosine function and define it as

$$\ddot{Q}_7\left(\ddot{E}\right) = \frac{1}{r}\sum_{i=1}^{r}\left(\cos\left\langle\frac{\dddot{\kappa}_{\ddot{E}}(\dddot{\varepsilon}_i) - \dddot{\lambda}_{\ddot{E}}(\dddot{\varepsilon}_i)}{4(1 + \dddot{\pi}_{\ddot{E}}(\dddot{\varepsilon}_i)}\right\rangle \pi\right), \ \forall \dddot{\varepsilon}_i \in \dddot{G} \quad (8)$$

Wang and Wang [XXIII] proposed the IF entropy measure of cotangent function and define as

$$\dddot{Q}_8\left(\dddot{E}\right)=\frac{1}{r}\sum_{i=1}^{r}\left(\cot\left\langle\frac{\pi}{4}+\frac{\left|\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)-\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right|\pi}{4(1+\dddot{\pi}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)}\right\rangle\right),\ \forall\,\dddot{\varepsilon}_i\in\dddot{G} \tag{9}$$

Wang and Wang [XXIII] proposed the IF entropy measure of cotangent function but they can not define the degree of hesitancy in IF entropy but Liu and Ren [XIV] proposed IF entropy of cotangent function and define the degree of hesitancy as follows

$$\dddot{Q}_9\left(\dddot{E}\right)=\frac{1}{r}\sum_{i=1}^{r}\left(\cot\left\langle\frac{\pi}{4}+\frac{\left|\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)-\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right|\times\left(1-\dddot{\pi}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)}{4}\pi\right\rangle\right),\ \forall\,\dddot{\varepsilon}_i\in\dddot{G} \tag{10}$$

III. Construction of new intuitionistic fuzzy entropy

In this section, we are supposed to introduce two new entropy measures and then compared them with the several prevailing ones to check the validity of our suggested entropy measures. Moreover, $\dddot{G}$ is universal set, $\dddot{E}$ is a subset of the universal set $\dddot{G}$. $\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)$ is membership function and $\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)$ is non-membership function. We denote our new entropy measures by $\dddot{Q}_{SS}^{1}\left(\dddot{E}\right)$ and $\dddot{Q}_{SS}^{2}\left(\dddot{E}\right)$. The suggested entropy measures of IFSs $\dddot{Q}_{SS}^{1}\left(\dddot{E}\right)$ and $\dddot{Q}_{SS}^{2}\left(\dddot{E}\right)$ are respectively, defined as follows

$$\dddot{Q}_{SS}^{1}\left(\dddot{E}\right)=1-d_s^{\alpha}(\dddot{E},\dddot{E}^{c}) \tag{11}$$

where

$$d_s^{\alpha}(\dddot{E},\dddot{E}^{c})=\frac{1}{r}\left(\sum_{i=1}^{r}\left(\begin{array}{l}\left(\left|e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)-e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)\right|\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\geq\frac{1}{2}\right]}\\+\left(\left|e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)-e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)\right|+\left(\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)\left(\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)<\frac{1}{2}\right]}\end{array}\right)\right)$$

$$\dddot{Q}_{SS}^{2}\left(\dddot{E}\right)=\dddot{\psi}+\dddot{\tau} \tag{12}$$

Where

$$\dddot{\psi}=\frac{1}{2r}\frac{\sum_{l=1}^{r}\left[\begin{array}{l}e^{\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)+\dddot{\pi}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)}\left(\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}\right)\right)+\\e^{\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)+\dddot{\pi}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)}\left(\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)-1+\left(\dddot{\lambda}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)\left(\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\right)\end{array}\right]}{(e^{0.5}-1)},\forall\,\dddot{\kappa}_{\dddot{E}}\left(\dddot{\varepsilon}_i\right)\neq\frac{1}{2},\dddot{\varepsilon}_i\in\dddot{G},$$

$$\dddot{\tau} = \frac{1}{r}\sum_{l=1}^{r}\frac{\left[e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)+e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-1\right]}{(e^{0.5}-1)}, \forall \dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i) = \dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i) = \frac{1}{2}.$$

Theorem 1: Assume that $\dddot{G} = \{\dddot{\varepsilon}_1, \dddot{\varepsilon}_2, ..., \dddot{\varepsilon}_r\}$ be finite universal set. The proposed intuitionistic fuzzy entropy $\dddot{Q}^1_{SS}(\dddot{E})$ satisfies the conditions (I) to (V) in Definition 6.

Proof: We first give the proof of the condition (I) of Definition 6, since $0 \le d^\alpha_s(\dddot{E}, \dddot{E}^c) \le 1$ implies $0 \le 1 - d^\alpha_s(\dddot{E}, \dddot{E}^c) \le 1$, thus, the condition (I) of Definition 6 is satisfied. We prove the condition (II) of Definition 6. Let $\dddot{E}$ be a crisp set, i.e. $\forall \dddot{\varepsilon}_i \in \dddot{G}$, either $\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i) = 1, \dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i) = 0$ or $\forall \dddot{\varepsilon}_i \in \dddot{G}$ $\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i) = 0, \dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i) = 1$, in both cases $d^\alpha_s(\dddot{E}, \dddot{E}^c) = 1$, consequently $\dddot{Q}^1_{SS}(\dddot{E}) = 1 - d^\alpha_s(\dddot{E}, \dddot{E}^c) = 0.$

Hence, the condition (II) of Definition 6 is proved. For condition (III) of Definition 6, if $\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i) = \dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i) = 0.5$

$$d^\alpha_s(\dddot{E}, \dddot{E}^c) = \frac{1}{r}\left(\sum_{i=1}^{r}\left(\begin{array}{l}\left(\left|e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right|\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\ge\frac{1}{2}\right]}+\\ \left(\left|e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right|+\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)<\frac{1}{2}\right]}\end{array}\right)\right)$$

Implies that, $1 - d^\alpha_s(\dddot{E}, \dddot{E}^c) = 1 - 0 = 1$ we conclude that if $\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i) = \dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i) = 0.5$ then $\dddot{Q}^1_{SS}(\dddot{E}) = 1$. To prove axiom (IV) of Definition 6, we have from Eq.(11),

$$\dddot{E}^c = \left\{\left\langle \dddot{\varepsilon}_i, \dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i), \dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right\rangle : \dddot{\varepsilon}_i \in \dddot{G}\right\}$$

$$\dddot{Q}^1_{SS}(\dddot{E}^c) = 1 - \frac{1}{r}\left(\sum_{i=1}^{r}\left(\begin{array}{l}\left(\left|e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right|\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\ge\frac{1}{2}\right]}+\\ \left(\left|e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right|+\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)<\frac{1}{2}\right]}\end{array}\right)\right)$$

$$= 1 - \frac{1}{r}\left(\sum_{i=1}^{r}\left(\begin{array}{l}\left(\left|e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right|\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\ge\frac{1}{2}\right]}+\\ \left(\left|e^{\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)-e^{\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)}\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right|+\left(\dddot{\lambda}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\left(\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)\right)\right)I_{\left[\dddot{\kappa}_{\dddot{E}}(\dddot{\varepsilon}_i)<\frac{1}{2}\right]}\end{array}\right)\right) =$$

$\dddot{Q}_{SS}^{1}\left(\ddot{E}\right)$. Thus, the axiom (IV) is proved. Finally, we prove the axiom (V). We use Eq.(11) and let $\ddot{\alpha}=\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right),\ \dddot{\beta}=\dddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)$ *and* $\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)=\dddot{Q}_{SS}^{1}\left(\ddot{E}\right)$, $\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)=1-\left|\ddot{\alpha}e^{\dddot{\beta}}-\dddot{\beta}e^{\ddot{\alpha}}\right|+\ddot{\alpha}\dddot{\beta},$ where $\ddot{\alpha},\dddot{\beta}\in[0,1]$.

If $\ddot{\alpha}\le\dddot{\beta}$ *we have* $\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)=1-\left(\dddot{\beta}e^{\ddot{\alpha}}-\ddot{\alpha}e^{\dddot{\beta}}\right)+\ddot{\alpha}\dddot{\beta}$. We need to prove that the function $\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)$ is increasing on $\ddot{\alpha}$ and decreasing on $\dddot{\beta}$, for increasing and decreasing we check the first derivative test.

$$\frac{\partial\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)}{\partial\ddot{\alpha}}=-\left(\dddot{\beta}e^{\ddot{\alpha}}-e^{\dddot{\beta}}\right)+\dddot{\beta}=-\dddot{\beta}e^{\ddot{\alpha}}+e^{\dddot{\beta}}+\dddot{\beta},$$
$$\frac{\partial\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)}{\partial\dddot{\beta}}=-\left(e^{\ddot{\alpha}}-\ddot{\alpha}e^{\dddot{\beta}}\right)+\ddot{\alpha}=-e^{\ddot{\alpha}}+\ddot{\alpha}e^{\dddot{\beta}}+\ddot{\alpha}.$$

When $\ddot{\alpha}\le\dddot{\beta}$, we have

$$\frac{\partial\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)}{\partial\ddot{\alpha}}\ge 0 \text{ and } \frac{\partial\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)}{\partial\dddot{\beta}}\le 0$$

then

$$\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)$$

is increasing with $\ddot{\alpha}$ and decreasing with $\dddot{\beta}$, thus we can says that $\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)\le\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)$

and

$$\dddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)\ge\dddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\text{when}\,\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\le\dddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\ \ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)\le\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\le\dddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\le\dddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right).$$

Thus,

$$\dddot{\xi}\left(\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right),\dddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)\right)\le\dddot{\xi}\left(\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right),\dddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\right).$$

This implies that $\dddot{Q}_{SS}^{1}(\ddot{E})\le\dddot{Q}_{SS}^{1}(\ddot{F}),\ \forall\ddot{\varepsilon}_i\in\ddot{G}.$

Similarly, if, $\ddot{\alpha}\ge\dddot{\beta}$ then $\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)=1-\left(\ddot{\alpha}e^{\dddot{\beta}}-\dddot{\beta}e^{\ddot{\alpha}}\right)+\ddot{\alpha}\dddot{\beta}.$

We need to prove that the function $\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)$ is decreasing on $\ddot{\alpha}$ and increasing on $\dddot{\beta}$, for increasing and decreasing we check the first derivative test.

$$\frac{\partial\dddot{\xi}\left(\ddot{\alpha},\dddot{\beta}\right)}{\partial\ddot{\alpha}}=-\left(e^{\dddot{\beta}}-\dddot{\beta}e^{\ddot{\alpha}}\right)+\dddot{\beta}=-e^{\dddot{\beta}}+\dddot{\beta}e^{\ddot{\alpha}}+\dddot{\beta}$$

and

$$\frac{\partial \ddot{\xi}\left(\ddot{\alpha},\ddot{\beta}\right)}{\partial \ddot{\alpha}} = -\left(\ddot{\alpha}e^{\ddot{\beta}} - e^{\ddot{\alpha}}\right) + \ddot{\alpha} = -\ddot{\alpha}e^{\ddot{\beta}} + e^{\ddot{\alpha}} + \ddot{\alpha}.$$ When $\ddot{\alpha} \geq \ddot{\beta}$,

we have

$$\frac{\partial \ddot{\xi}\left(\ddot{\alpha},\ddot{\beta}\right)}{\partial \ddot{\alpha}} \leq 0$$

and

$$\frac{\partial \ddot{\xi}\left(\ddot{\alpha},\ddot{\beta}\right)}{\partial \ddot{\beta}} \geq 0$$

then

$$\partial \ddot{\xi}\left(\ddot{\alpha},\ddot{\beta}\right)$$

is decreasing with $\ddot{\alpha}$ and increasing with $\ddot{\beta}$, thus we can say that

$$\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \geq \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ and } \ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ when } \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \geq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right).$$

Thus we have,

$$\ddot{\xi}\left(\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)\right) \leq \ddot{\xi}\left(\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\right).$$

Thus, it is clear that

$$\ddot{Q}^1_{SS}(\ddot{E}) \leq \ddot{Q}^1_{SS}(\ddot{F}),\ \forall \ddot{\varepsilon}_i \in \ddot{G}.$$

Now, it is clear that if $\ddot{E} < \ddot{F}$ then we have $\ddot{Q}^1_{SS}(\ddot{E}) \leq \ddot{Q}^1_{SS}(\ddot{F})$. Thus, condition (V) of definition 6 is satisfied. This completes the proof.

Property. The proposed entropy $\ddot{Q}^1_{SS}$ is σ-entropy if it satisfies

$$\ddot{Q}^1_{SS}\left(\ddot{E}\right) + \ddot{Q}^1_{SS}\left(\ddot{F}\right) = \ddot{Q}^1_{SS}\left(\ddot{E} \cup \ddot{F}\right) + \ddot{Q}^1_{SS}\left(\ddot{E} \cap \ddot{F}\right) \text{ if } \forall \ddot{\varepsilon}_i \in \ddot{G}$$

Proof. We prove Eq.(11) is σ-entropy,

$$\ddot{Q}^1_{SS}\left(\ddot{E}\right) + \ddot{Q}^1_{SS}\left(\ddot{F}\right) = \ddot{Q}^1_{SS}\left(\ddot{E} \cup \ddot{F}\right) + \ddot{Q}^1_{SS}\left(\ddot{E} \cap \ddot{F}\right) \text{ if } \forall \ddot{\varepsilon}_i \in \ddot{G}$$

$$(I)\ \ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ and } \ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \geq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ when } \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)$$

$$\Rightarrow \ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right)$$

$$(II) \text{ if } \ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \geq \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ and } \ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ when } \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \geq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right), \ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right).$$

From axiom

$$(I)\quad \ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \leq \ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right) \text{ then } max\left(\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right),\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\right) = \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)$$

$$max\left(\ddot{\kappa}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right),\ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\right) = \ddot{\kappa}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right) \text{ and } min\left(\ddot{\lambda}_{\ddot{E}}\left(\ddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right)\right) = \ddot{\lambda}_{\ddot{F}}\left(\ddot{\varepsilon}_i\right),$$

which implies that,

$$\ddot{E}\cup\ddot{F}=\left\{\left\langle \dddot{\varepsilon}_i, max\left(\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right),\ min\left(\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)\right\rangle\right\},\ \forall\,\dddot{\varepsilon}_i\in\dddot{G},$$

that is

$$\ddot{E}\cup\ddot{F}=\left\{\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right\}=\ddot{F}.$$

Clearly,

$$\dddot{Q}^1_{SS}\left(\ddot{E}\cup\ddot{F}\right)=\dddot{Q}^1_{SS}\left(\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)=\dddot{Q}^1_{SS}\left(\ddot{F}\right).$$ Also

$$\ddot{E}\cap\ddot{F}=\left\{\left\langle \dddot{\varepsilon}_i, min\left(\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right), max\left(\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)\right\rangle\right\},$$

$min\left(\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)=\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)$ and $max\left(\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)=\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)$,

then

$$\ddot{E}\cap\ddot{F}=\left\{\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)\right\}=\ddot{E},$$

which implies that

$$\dddot{Q}^1_{SS}\left(\ddot{E}\cap\ddot{F}\right)=\dddot{Q}^1_{SS}\left(\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)\right)=\dddot{Q}^1_{SS}\left(\ddot{E}\right).$$ Hence,

$$\dddot{Q}^1_{SS}\left(\ddot{E}\right)+\dddot{Q}^1_{SS}\left(\ddot{F}\right)=\dddot{Q}^1_{SS}\left(\ddot{E}\cup\ddot{F}\right)+\dddot{Q}^1_{SS}\left(\ddot{E}\cap\ddot{F}\right),\ \ \forall\,\dddot{\varepsilon}_i\in\dddot{G}.$$

Thus, (I) is proved. Now, we give the proof of (II). We have

$$\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)\le\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\le\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\le\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ \forall\,\dddot{\varepsilon}_i\in\dddot{G}.\ max\left(\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)=\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)$$

and $min\left(\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)=\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)$

that is $\ddot{E}\cup\ddot{F}=\left\{\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)\right\}=\ddot{E}.\ \dddot{Q}^1_{SS}\left(\ddot{E}\cup\ddot{F}\right)=\dddot{Q}^1_{SS}\left(\ddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)\right)=\dddot{Q}^1_{SS}\left(\ddot{E}\right).$

Then

$$\ddot{E}\cap\ddot{F}=\left\{\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right\}=\ddot{F}$$

implies that

$$\dddot{Q}^1_{SS}\left(\ddot{E}\cap\ddot{F}\right)=\dddot{Q}^1_{SS}\left(\ddot{\kappa}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right),\ddot{\lambda}_{\ddot{F}}\left(\dddot{\varepsilon}_i\right)\right)=\dddot{Q}^1_{SS}\left(\ddot{F}\right).$$

Hence, $\dddot{Q}^1_{SS}\left(\ddot{E}\right)+\dddot{Q}^1_{SS}\left(\ddot{F}\right)=\dddot{Q}^1_{SS}\left(\ddot{E}\cup\ddot{F}\right)+\dddot{Q}^1_{SS}\left(\ddot{E}\cap\ddot{F}\right),\ \forall\,\dddot{\varepsilon}_i\in\dddot{G}.$

Thus, this completes the proof.

Theorem 2. Assume that $\dddot{G}=\left\{\dddot{\varepsilon}_1,\dddot{\varepsilon}_2,...,\dddot{\varepsilon}_r\right\}$ be a finite universe of discourse. The proposed intuitionistic fuzzy entropy $\dddot{Q}^2_{SS}\left(\ddot{E}\right)$ satisfies the conditions (I) to (V) in Definition 6 and property of σ-entropy.

Proof. Similar to Theorem 1.

IV. Numerical results, discussions and comparisons

This section includes numerical examples to check the validity of our proposed entropy measure. We also conduct the comparative analysis of our proposed entropy measures with several existing entropy measures to show the reasonability and comparatively better performance of our newly suggested entropy measures.

Example1. Suppose that $\dddot{G}$ is finite universal set with $\dddot{M}$ in $\dddot{G}$. Let an IFS

$\dddot{M} = \{\langle \dddot{\varepsilon}_1, 0.49, 0.48\rangle, \langle \dddot{\varepsilon}_2, 0.45, 0.49\rangle\}$, $\forall \dddot{\varepsilon}_i \in \dddot{G}$ be the IFS. The existing entropies $\dddot{Q}_5$, $\dddot{Q}_7$, $\dddot{Q}_8$ are not consistent, which is proved by Manfeng Liu[8]. We only conduct comparison analysis of our proposed entropy measures $\dddot{Q}^1_{ss}$ and $\dddot{Q}^1_{ss}$ with the existing entropies $\dddot{Q}_0$, $\dddot{Q}_1$, $\dddot{Q}_2$, $\dddot{Q}_3$, $\dddot{Q}_4$, $\dddot{Q}_6$ and $\dddot{Q}_9$ respectively with the help of linguistic hedges like concentration, dilation etc., using definition 4, as follows

$$Concentration = CON(\dddot{M}) = \dddot{M}^2; \qquad Dilation = DIL(\dddot{M}) = \dddot{M}^{\frac{1}{2}}$$

$$Very\ Large\ (\dddot{M}) = \dddot{M}^3; \qquad Very\ very\ Large(\dddot{M}) = \dddot{M}^4.$$

To check the validity and reasonability of the entropy measures Eq.(1)-Eq.(12), we apply the following mathematical requirement Liu and Ren [XIV];

$$\dddot{Q}\left(\dddot{M}^{\frac{1}{2}}\right) > \dddot{Q}\left(\dddot{M}\right) > \dddot{Q}\left(\dddot{M}^2\right) > \dddot{Q}\left(\dddot{M}^3\right) > \dddot{Q}\left(\dddot{M}^4\right).$$

Because the membership of IFS is $\dddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right) : \dddot{G} \to [0,1]$ when we vary the power the behavior of membership is changed and follow this pattern.

$$\dddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)^{\frac{1}{2}} > \dddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right) > \dddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)^2 > \dddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)^3 > \dddot{\kappa}_{\ddot{E}}\left(\dddot{\varepsilon}_i\right)^4$$

Thus, IFS entropy must follow the above pattern.

If any entropy is failed to fulfill the above mathematical requirement, then it would be considered mathematically not well suited and logically not acceptable. Based on the above mathematical requirement we construct the following linguistic hedges for an IF $\dddot{M}$.

$$\dddot{M}^{\frac{1}{2}} = \{\langle \dddot{\varepsilon}_1, 0.700, 0.2930\rangle, \langle \dddot{\varepsilon}_2, 0.6710, 0.2858\rangle\}; \quad \dddot{M}^2 = \{\langle \dddot{\varepsilon}_1, 0.2400, 0.7296\rangle, \langle \dddot{\varepsilon}_2, 0.2025, 0.7399\rangle\};$$

$$\dddot{M}^3 = \{\langle \dddot{\varepsilon}_1, 0.1176, 0.8590\rangle, \langle \dddot{\varepsilon}_2, 0.0900, 0.8670\rangle\}; \dddot{M}^4 = \{\langle \dddot{\varepsilon}_1, 0.0576, 0.9270\rangle, \langle \dddot{\varepsilon}_2, 0.0410, 0.9320\rangle\}.$$

The valuations of all nine entropy of intuitionistic fuzzy sets are given in below table 1.

Table 1: Comparison of existing entropy measures with proposed entropy measures

	$\dddot{Q}_0$	$\dddot{Q}_1$	$\dddot{Q}_2$	$\dddot{Q}_3$	$\dddot{Q}_4$	$\dddot{Q}_6$	$\dddot{Q}_9$	$\dddot{Q}^1_{ss}$	$\dddot{Q}^2_{ss}$
$\dddot{M}^{\frac{1}{2}}$	0.5769	0.5933	0.442	0.604	0.422	0.439	0.5279	0.6589	0.5596
$\dddot{M}$	0.9875	0.9966	0.954	0.965	0.949	0.963	0.9637	0.5230	0.4699
$\dddot{M}^2$	0.3948	0.4116	0.341	0.486	0.301	0.256	0.5236	0.2194	0.3385
$\dddot{M}^3$	0.0368	0.1555	0.153	0.241	0.117	0.057	0.2120	0.1094	0.1915
$\dddot{M}^4$	-0.395	0.0679	0.113	0.119	0.053	0.013	0.1090	0.0568	0.0960

Based on the result of Table 1, we have the following

$$\ddot{Q}_0\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_0\left(\dddot{M}\right)>\ddot{Q}_0\left(\dddot{M}^2\right)>\ddot{Q}_0\left(\dddot{M}^3\right)>\ddot{Q}_0\left(\dddot{M}^4\right)$$

$$\ddot{Q}_1\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_1\left(\dddot{M}\right)>\ddot{Q}_1\left(\dddot{M}^2\right)>\ddot{Q}_1\left(\dddot{M}^3\right)>\ddot{Q}_1\left(\dddot{M}^4\right)$$

$$\ddot{Q}_2\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_2\left(\dddot{M}\right)>\ddot{Q}_2\left(\dddot{M}^2\right)>\ddot{Q}_2\left(\dddot{M}^3\right)>\ddot{Q}_2\left(\dddot{M}^4\right)$$

$$\ddot{Q}_3\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_3\left(\dddot{M}\right)>\ddot{Q}_3\left(\dddot{M}^2\right)>\ddot{Q}_3\left(\dddot{M}^3\right)>\ddot{Q}_3\left(\dddot{M}^4\right)$$

$$\ddot{Q}_4\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_4\left(\dddot{M}\right)>\ddot{Q}_4\left(\dddot{M}^2\right)>\ddot{Q}_4\left(\dddot{M}^3\right)>\ddot{Q}_4\left(\dddot{M}^4\right)$$

$$\ddot{Q}_6\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_6\left(\dddot{M}\right)>\ddot{Q}_6\left(\dddot{M}^2\right)>\ddot{Q}_6\left(\dddot{M}^3\right)>\ddot{Q}_6\left(\dddot{M}^4\right)$$

$$\ddot{Q}_9\left(\dddot{M}^{\frac{1}{2}}\right)<\ddot{Q}_9\left(\dddot{M}\right)>\ddot{Q}_9\left(\dddot{M}^2\right)>\ddot{Q}_9\left(\dddot{M}^3\right)>\ddot{Q}_9\left(\dddot{M}^4\right)$$

$$\ddot{Q}^1_{ss}\left(\dddot{M}^{\frac{1}{2}}\right)>\ddot{Q}^1_{ss}\left(\dddot{M}\right)>\ddot{Q}^1_{ss}\left(\dddot{M}^2\right)>\ddot{Q}^1_{ss}\left(\dddot{M}^3\right)>\ddot{Q}^1_{ss}\left(\dddot{M}^4\right)$$

$$\ddot{Q}^2_{ss}\left(\dddot{M}^{\frac{1}{2}}\right)>\ddot{Q}^2_{ss}\left(\dddot{M}\right)>\ddot{Q}^2_{ss}\left(\dddot{M}^2\right)>\ddot{Q}^2_{ss}\left(\dddot{M}^3\right)>\ddot{Q}^2_{ss}\left(\dddot{M}^4\right).$$

The numerical comparison results from Table 1, show that the entropy measures Eq.(1)- Eq.(10) do not fulfill the above mathematical requirement. On the other hand, our proposed entropy measures satisfy the above mathematical requirement $\dddot{Q}^1_{ss}$ and

$\dddot{Q}_{ss}^{2}$. We can articulates that our proposed entropies $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ of intuitionistic fuzzy sets are effective and reasonable. Thus, our proposed entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ are intuitionally acceptable and well suited in measuring the fuzziness of intuitionistic fuzzy sets.

V. Construction of TODIM based on intuitionistic fuzzy entropy

Numerous procedures are designed to explain multi-criteria decision-making to solve our everyday complications, these are TOPSIS, VOKER, TODIM etc. TODIM is one of the preeminent techniques for MCDM. In this chapter, we use IF-TODIM based on our proposed entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ to handle daily life issues. TODIM method is developed based on prospect theory[26]. Consider a set of *m* alternatives to be well-ordered in the occurrence of *n* criteria, and take one of them as a situation criterion. Afterward the explanations of these elements, specialists are inquired to assessment, for each of the criteria, the involvement of individually alternative $\dddot{D}_i$ to the independent concomitant with the criterion.

The steps to solve IF-TODIM are as follows;

Step 1: The decision matrix of each alternative $\dddot{D}_i$ with respect to each criteria $\dddot{G}_j$ is constructed in this step, label the intuitionistic fuzzy decision matrix $\dddot{L} = \left(\ddot{r}_{ij}\right)_{m\times n} = \left(\ddot{\kappa}_{ij}\left(\ddot{\varepsilon}_i\right), \dddot{\lambda}_{ij}\left(\ddot{\varepsilon}_i\right)\right)_{m\times n}$ identified by the decision-makers in the MCDM hitches, where $\ddot{r}_{ij} = \left(\ddot{\kappa}_{ij}\left(\ddot{\varepsilon}_i\right), \dddot{\lambda}_{ij}\left(\ddot{\varepsilon}_i\right)\right)_{m\times n}$ is a IFSs.

$$\dddot{L} = \left(\ddot{r}_{ij}\right)_{m\times n} = \begin{array}{c|cccccc} & \dddot{G}_1 & \dddot{G}_2 & . & . & . & \dddot{G}_j \\ \hline \dddot{D}_1 & r_{11} & r_{12} & . & . & . & r_{1j} \\ \dddot{D}_2 & r_{21} & r_{22} & . & . & . & r_{2j} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ \dddot{D}_i & r_{i1} & r_{i2} & . & . & . & r_{ij} \end{array}$$

Step 2: We transmute the decision matrix $\dddot{M} = (\ddot{r}_{ij})_{m\times n}$ into a normalized IF decision matrix

$$\dddot{L} = \left(\dddot{l}_{ij}\right)_{m\times n} = \dddot{l}_{ij} = \begin{cases} \ddot{r}_{ij} = \left(\ddot{\kappa}_{ij}\left(\ddot{\varepsilon}_i\right), \dddot{\lambda}_{ij}\left(\ddot{\varepsilon}_i\right)\right)_{m\times n} & for\ beneficial\ criteria \\ \left(\ddot{r}_{ij}\right)^c = \left(\dddot{\lambda}_{ij}\left(\ddot{\varepsilon}_i\right), \ddot{\kappa}_{ij}\left(\ddot{\varepsilon}_i\right)\right)_{m\times n} & for\ \cos t\ criteria. \end{cases}$$

When the criteria is a benefit there is no change in the construction of matrix i.e. $\dddot{r}_{ij} = \left(\dddot{\kappa}_{ij}\left(\dddot{\varepsilon}_i\right), \dddot{\lambda}_{ij}\left(\dddot{\varepsilon}_i\right)\right)$ but for cost criteria, we take the complement $\left(\dddot{r}_{ij}\right)^c = \left(\dddot{\lambda}_{ij}\left(\dddot{\varepsilon}_i\right), \dddot{\kappa}_{ij}\left(\dddot{\varepsilon}_i\right)\right)$ of the matrix in normalize form.

Step 3: We govern the weight of separate criteria $\dddot{G}_j$ in this step by using the following instruction

$$\dddot{w}_j = \frac{\dddot{Q}_j}{\sum_{j=1}^{n} \dddot{Q}_j}$$

where $\dddot{Q}_j$ is our proposed entropy measures are given in Eq.(11) and Eq.(12). There are many methods to find weight criteria but in this paper, we define weight by using our suggested entropy of IFSs $\dddot{Q}_j$. Where j is criteria, we find entropy $\dddot{Q}_j$ for each criteria and $j = 1, 2, \ldots, n.$

Step 4: The relative weight is to be concluded over each criterion $\dddot{G}_j$, in this step, using

$$\dddot{w}_{jr} = \frac{\dddot{w}_j}{\dddot{w}_r} \text{ where } \dddot{w}_j \text{ is the weight of criterion } \dddot{G}_j$$

$$\dddot{w}_r = max\left[\dddot{w}_j : j = 1, 2, 3, \ldots, n\right] \; and \;\; 0 \le \dddot{w}_j \le 1$$

Step 5:The determination of the dominance degree of the alternative $\dddot{D}_i$ over separately alternative $\dddot{D}_t$ with reverence to the criterion by $\dddot{G}_j$ is in this step, using

$$\dddot{\phi}_j\left(\dddot{D}_i, \dddot{D}_t\right) = \begin{cases} \sqrt{\dfrac{\dddot{w}_{jr}\, \dddot{d}_{\tilde{h}}\left(\dddot{I}_{ij}, \dddot{I}_{tj}\right)}{\sum_{j=1}^{n} \dddot{w}_{jr}}}, & if \;\; \dddot{I}_{ij} > \dddot{I}_{tj} \\ 0, & if \;\; \dddot{I}_{ij} = \dddot{I}_{tj} \\ -\dfrac{1}{\dddot{\theta}} \sqrt{\dfrac{\sum_{j=1}^{n} \dddot{w}_{jr}\, \dddot{d}_{\tilde{h}}\left(\dddot{I}_{ij}, \dddot{I}_{tj}\right)}{\dddot{w}_{jr}}}, & if \;\; \dddot{I}_{ij} < \dddot{I}_{tj}. \end{cases}$$

where $\dddot{\phi}_j\left(\dddot{D}_i, \dddot{D}_t\right)$ signifies dominance degree of the alternative $\dddot{D}_i$ over individually alternative $\dddot{D}_t$ with respect to the criterion by $\dddot{G}_j$ and equating alternatives i with alternatives t. The classification of alternatives i and t be $\dddot{I}_{ij}\ and\ \dddot{I}_{tj}$, $\left(\dddot{I}_{ij}=\dddot{\kappa}_{ij}\left(\dddot{\varepsilon}_{i}\right)-\dddot{\lambda}_{ij}\left(\dddot{\varepsilon}_{i}\right), and\ \dddot{I}_{tj}=\dddot{\kappa}_{tj}\left(\dddot{\varepsilon}_{i}\right)-\dddot{\lambda}_{tj}\left(\dddot{\varepsilon}_{i}\right)\right)$, for each criteria $\dddot{G}_{j}$. Thus, $\dddot{d}_{\tilde{h}}\left(\dddot{I}_{ij},\dddot{I}_{tj}\right)$ signifies hamming distance between each alternative $\dddot{I}_{ij}\ and\ \dddot{I}_{tj}$ for each criteria $\dddot{G}_{j}$. The gain interval is represented if $\dddot{I}_{ij}>\dddot{I}_{tj}$ and use it in $\sqrt{\frac{\dddot{w}_{jr}\,\dddot{d}_{\tilde{h}}\left(\dddot{I}_{ij},\dddot{I}_{tj}\right)}{\sum_{j=1}^{n}\dddot{w}_{jr}}}$

but, when $\dddot{I}_{ij}<\dddot{I}_{tj}$ signifies loss and it is used in $-\frac{1}{\dddot{\theta}}\sqrt{\frac{\sum_{j=1}^{n}\dddot{w}_{jr}\,\dddot{d}_{\tilde{h}}\left(\dddot{I}_{ij},\dddot{I}_{tj}\right)}{\dddot{w}_{jr}}}$.

When $\dddot{I}_{ij}=\dddot{I}_{tj}$, it is nil. Thus $\sum_{j=1}^{n}\dddot{w}_{jr}=\dddot{\theta}$ is called the attenuation factor of the losses.

Step 6: The overall dominance degree of $\dddot{D}_{i}$ over separately alternative $\dddot{D}_{t}$ is determined in this stage using

$$\dddot{\delta}\left(\dddot{D}_{i},\dddot{D}_{t}\right)=\sum_{j=1}^{n}\dddot{\phi}_{j}\left(\dddot{D}_{i},\dddot{D}_{t}\right)\quad (i,t=1,2,\ldots,n).$$

When we sum all matrix of the dominance degree then the outcomes are overall dominance degree.

Step 7: The values of each alternative $\dddot{D}_{i}$ will be derived by using

$$\dddot{\varepsilon}_{i}=\frac{\sum_{t=1}^{n}\dddot{\delta}\left(\dddot{D}_{i},\dddot{D}_{t}\right)-min\left(\sum_{t=1}^{n}\dddot{\delta}\left(\dddot{D}_{i},\dddot{D}_{t}\right)\right)}{max\left\{\sum_{t=1}^{n}\dddot{\delta}\left(\dddot{D}_{i},\dddot{D}_{t}\right)\right\}-min\left(\sum_{t=1}^{n}\dddot{\delta}\left(\dddot{D}_{i},\dddot{D}_{t}\right)\right)}.$$

Thus, $\dddot{\varepsilon}_{i}$ stretches the rank of each alternative $\dddot{D}_{i}$. The greater value of $\dddot{\varepsilon}_{i}$ is best alternatives.

Example 3. Let us consider that, a person wants to buy land for his residence. Let three places (alternatives $\dddot{D}_{i}\left(i=1,2,3\right)$) are available. The customer wants to select three criteria $\dddot{G}_{j}$ to decide which place is best for his residence, the criteria are given below,

$\dddot{G}_{1}$: Price of land; $\dddot{G}_{2}$: Health facilities; $\dddot{G}_{3}$: Education Facilities.

We note that $\dddot{G}_{1}$ is cost criteria but $\dddot{G}_{2}$ and $\dddot{G}_{3}$ are benefit criteria.

Step 1. Construction of decision matrix

Step 2. We first transform the decision matrix into the normalized matrix. $\dddot{G}_1$ is cost criteria therefore, we take its complement and we keep the remaining criteria.

Table-2: Decision Making Matrix

	$\dddot{G}_1$	$\dddot{G}_2$	$\dddot{G}_3$
$\dddot{D}_1$	$(0.6, 0.3)$	$(0.7, 0.2)$	$(0.6, 0.2)$
$\dddot{D}_2$	$(0.5, 0.3)$	$(0.6, 0.3)$	$(0.6, 0.3)$
$\dddot{D}_3$	$(0.5, 0.2)$	$(0.6, 0.2)$	$(0.4, 0.4)$

Table-3: Normalized Decision Matrix

	$\dddot{G}_1$	$\dddot{G}_2$	$\dddot{G}_3$
$\dddot{D}_1$	$(0.3, 0.6)$	$(0.7, 0.2)$	$(0.6, 0.2)$
$\dddot{D}_2$	$(0.3, 0.5)$	$(0.6, 0.3)$	$(0.6, 0.3)$
$\dddot{D}_3$	$(0.2, 0.5)$	$(0.6, 0.2)$	$(0.4, 0.4)$

Table-4: Construction of Weight

	$\dddot{w}_1$	$\dddot{w}_2$	$\dddot{w}_3$
Q_{ss}^1	0.309	0.321	0.370
Q_{ss}^2	0.322	0.330	0.348

Table-5: Relative Weight

	$\dddot{w}_{1r}$	$\dddot{w}_{2r}$	$\dddot{w}_{3r}$
Q_{ss}^1	0.835	0.868	1.000
Q_{ss}^2	0.925	0.948	1.000

Table-6: Dominance Degree of Criteria $\dddot{G}_1$ for Q_{ss}^1

	$\dddot{D}_1$	$\dddot{D}_2$	$\dddot{D}_3$
$\dddot{D}_1$	0.0000	-0.2105	0.0000
$\dddot{D}_2$	0.1758	0.0000	0.1758
$\dddot{D}_3$	0.0000	-0.2105	0.0000

Table-7: Dominance Degree of Criteria $\dddot{G}_2$ for Q_{ss}^1

	$\dddot{D}_1$	$\dddot{D}_2$	$\dddot{D}_3$
$\dddot{D}_1$	0.0000	0.1792	0.1792
$\dddot{D}_2$	-0.2064	0.0000	-0.2064
$\dddot{D}_3$	-0.2064	0.1792	0.0000

Table-8: Dominance Degree of Criteria $\dddot{G}_3$ for Q_{ss}^1

	$\dddot{D}_1$	$\dddot{D}_2$	$\dddot{D}_3$
$\dddot{D}_1$	0.0000	0.1923	0.2720
$\dddot{D}_2$	-0.1923	0.0000	0.2720
$\dddot{D}_3$	-0.2720	-0.2720	0.0000

Table-9: Dominance Degree of Criteria $\dddot{G}_1$ for Q_{ss}^2

	$\dddot{D}_1$	$\dddot{D}_2$	$\dddot{D}_3$
$\dddot{D}_1$	0.0000	-0.1940	0.0000
$\dddot{D}_2$	0.1794	0.0000	0.1794
$\dddot{D}_3$	0.0000	-0.1940	0.0000

Table-10: Dominance Degree of Criteria $\dddot{G}_2$ for Q_{ss}^2

	$\dddot{D}_1$	$\dddot{D}_2$	$\dddot{D}_3$
$\dddot{D}_1$	0.0000	0.1816	0.1816
$\dddot{D}_2$	-0.1916	0.0000	-0.1916
$\dddot{D}_3$	-0.1916	0.1816	0.0000

Table-11: Dominance Degree of Criteria $\dddot{G}_3$ for Q_{ss}^2

	$\dddot{D}_1$	$\dddot{D}_2$	$\dddot{D}_3$
$\dddot{D}_1$	0.0000	0.1866	0.2638
$\dddot{D}_2$	-0.1866	0.0000	0.2638
$\dddot{D}_3$	-0.2638	-0.2638	0.0000

We calculate overall dominance degree in the following Table 12, 13.

Table-12: Overall Dominance Degree for entropy Q_{ss}^{1}

	$\ddot{D}_1$	$\ddot{D}_2$	$\ddot{D}_3$	$\sum_{t=1}^{n}\ddot{\delta}_j\left(\ddot{D}_i,\ddot{D}_t\right)$
$\ddot{D}_1$	0.0000	0.1610	0.4512	0.6122
$\ddot{D}_2$	-0.2229	0.0000	0.2414	0.0185
$\ddot{D}_3$	-0.4784	-0.3033	0.0000	-0.7781

Table-13: Overall Dominance Degree for entropy Q_{ss}^{2}

	$\ddot{D}_1$	$\ddot{D}_2$	$\ddot{D}_3$	$\sum_{t=1}^{n}\ddot{\delta}_j\left(\ddot{D}_i,\ddot{D}_t\right)$
$\ddot{D}_1$	0.0000	0.1742	0.4454	0.6196
$\ddot{D}_2$	-0.1988	0.0000	0.2516	0.0528
$\ddot{D}_3$	-0.4554	-0.2762	0.0000	-0.7316

Table-14: Overall Value of Alternatives

Q_{ss}^{1}	$\ddot{\varepsilon}_i$	Q_{ss}^{2}	$\ddot{\varepsilon}_i$
$\ddot{D}_1$	1.000	$\ddot{D}_1$	1.000
$\ddot{D}_2$	0.573	$\ddot{D}_2$	0.580
$\ddot{D}_3$	0.000	$\ddot{D}_3$	0.000

Finally, we rank the alternatives based on the results in above Table 14.

Table-15: Ranking of Alternatives

Results	Ranking
Q_{ss}^{1}	$\ddot{D}_1 \succ \ddot{D}_2 \succ \ddot{D}_3$
Q_{ss}^{2}	$\ddot{D}_1 \succ \ddot{D}_2 \succ \ddot{D}_3$

In Table 15, we can observe that our proposed entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ rank the alternatives in preference order and select the best alternatives $\ddot{D}_1$ unanimously. We see that there is no conflict in the ranking of alternatives using our proposed entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$.

VI. Conclusion

This research paper is dedicated to frame new entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ for IFSs. We have compared our proposed entropy measures with the most existing entropy Eq.(1)-Eq. (10). The numerical comparison results show that our proposed entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ are more capable and efficient to handle different kinds of IFSs as compared to the existing entropy Eq.(1)-Eq. (10). Furthermore, to convince the readers, we have utilized our proposed entropy measures for IFSs to construct IF-TODIM to solve daily life complex issues involving complex multicriteria decision-making processes. The above discussions reveal that in handling various problems related to linguistic variables and multi-criteria decision-making in intuitionistic fuzzy environments, our proposed entropy measures $\dddot{Q}_{ss}^{1}$ and $\dddot{Q}_{ss}^{2}$ for IFSs are reasonable and well-designed.

Conflicts of Interest:

The authors declare that they have no conflicts of interest to report regarding the present study.

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