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THE FIVE PARAMETER LOGISTIC (5PL) FUNCTION AND COVID-19 EPIDEMIC IN ICELAND

Pinaki Pal[1], Asish Mitra[2]

[1]Ram Krishna Mahavidyalaya, Tripura, India.

[2]College of Engineering & Management, Kolaghat, WB, India.

[1]pinakipalagt@gmail.com, [2]asish@cemk.ac.in

Corresponding Author: **Asish Mitra**

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Abstract

Right now, investigations are rigorously carried out on modelling the dynamic progress of (Covid-19) pandemic around the globe. Here we introduce a simple mathematical model for analyzing the dynamics of Covid-19, considering only the number of cumulative cases. In the present work, the 5PL function is applied to study the Covid-19 spread in Iceland. The cumulative number of infected persons C(t) has been accurately fitted with the 5PL equation, giving rise to different epidemiological parameters. The result of the current examination reveals the effectiveness and efficacy of the 5PL function for exploring the Covid 19 dynamics in Iceland. The mathematical model is simple enough such that practitioners knowing algebra and non-linear regression analysis can employ it to examine the pandemic situation in different countries.

Keywords: 5PL Function, Covid-19 Pandemic, Daily Growth Rate, Iceland, Simulation, Tipping Point.

I. Introduction

In December of 2019, a novel coronavirus disease (COVID-19) was first identified in Wuhan City, Hubei, China, and the virus is identified as severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). Now it is a worldwide health concern affecting many different countries worldwide. By 13 April 2021, it has affected more than 138 million people around the globe and more than 29,77,000 deaths have been reported due to COVID-19 [XIX].

The virus was first traced in Iceland in February 2020. The total number of cases registered was 6,258, of which 6,126 had recovered and 29 deaths had occurred till 08 April 2021 [XIX].

SarS-CoV-2 (Covid-19) is relatively a new virus, that has not been studied enough by virologists. Researchers from all disciplines such as epidemiologists, mathematicians, physicists, biologists, chemists, and economists are seriously associated in the investigation of this pandemic for an understanding of the spread dynamics of Covid-19. Armed with this knowledge, policymakers can decisions to battle and control this pandemic.

There are three key elements for the development of an epidemic: (1) source: pathogens and their repositories; (2) susceptible persons with a route for the infection to enter the body; (3) transmission: a way or mechanism by which viruses moved to other susceptible persons.

The present study focuses on using phenomenological models for modeling disease spread without detailed microscopic foundations [II, III, VIII]. Previous epidemics [VIII-XI, XXVIII] have been analyzed using the classical logistic growth model, the generalized logistic model (GLM), the generalized Richards model (GRM) and the generalized growth model (GGM). In the present study, we will demonstrate the 5PL growth curve and model this curve to COVID19 disease dynamics in terms of the cumulative number of infected people in Iceland.

The paper is prepared as follows: in Section II, we explain the model and data in detail. In Section III and IV, we perform simulation, i.e., calibrate the model to the cumulative number of infected cases in the COVID-19 epidemics for the whole of Iceland, and discuss the results. We talk about several limitations of our methods in Section V and then conclude in Section VI.

II. Mathematical Model & Materials

Generalized Logistic Function or Curve (or Richards' curve)

Richards' curve, otherwise called the generalized logistic function or curve, initially created for growth modeling, is an extension of the logistic or sigmoid functions, taking into consideration more adaptable S-shaped curves (XXIV, XII):

$$Y(t) = A + \frac{K - A}{[C + Qe^{-Bt}]^{\gamma}} \tag{1}$$

where Y(t) = weight, height, size etc., and t = time.

The five parameters are:

A: the lower asymptote;

K: the upper asymptote when C=1. If A=0 and C=1 then K is called the carrying capacity;

B: the growth rate;
$\nu>0$: affects near which asymptote maximum growth occurs. It rules the degree of asymmetry of the curve (XXII, XXIII).

Q: is related to the value Y(0)

C: typically takes a value of 1. Otherwise, the upper asymptote is

$$A + \frac{K - A}{C^{\gamma}}$$

The equation can also be written:

$$Y(t) = A + \frac{K - A}{[C + e^{-B(t-M)}]^{\gamma}} \tag{2}$$

where M can be considered as a starting time, t_0 at which

$$Y(t_0) = A + \frac{K - A}{[C + 1]^{\gamma}} \tag{3}$$

Counting both Q and M can be advantageous:

$$Y(t) = A + \frac{K - A}{[C + Qe^{-B(t-M)}]^{\gamma}} \tag{4}$$

This model is sometimes known as "Richards' curve" after F. J. Richards. When Q= ν =1, eq (4) attains the maximum growth rate at time M.

The model [Eq. (4)] is illustrated as a function of time in Fig. 1.

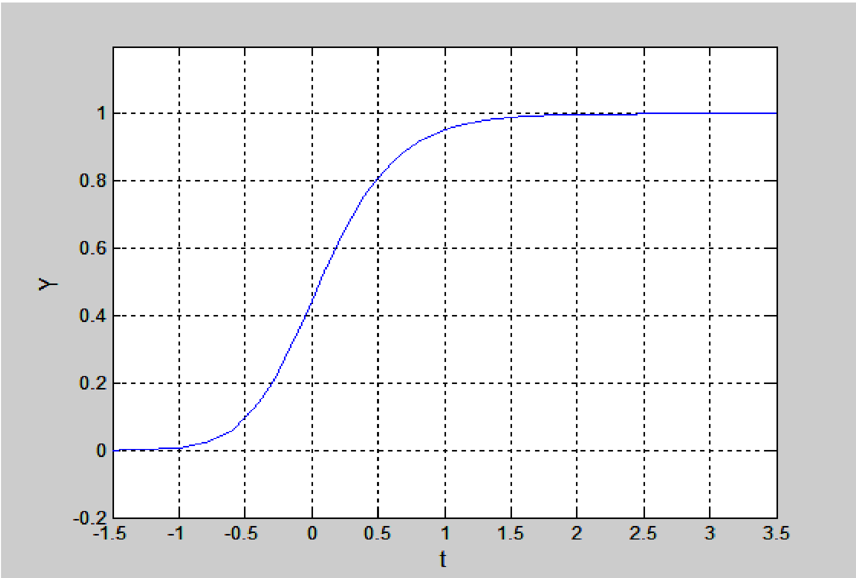

Fig. 1. A typical plot of model [Eq. (4)] for A=M=0, B=3, C=K=1, Q= 0.5, ν=2

Modeling Covid19 Infection Trajectory

Richards growth curve (or so-called the generalized logistic curve) is useful especially for population studies with asymmetrical growth about the point of inflection [XXV, I]. The curve also describes various biological processes [XXVI]. It is now extensively used in epidemiology for real-time prediction of an outbreak of diseases like SARS [XIV, XV], dengue fever [XVI, XVII], pandemic influenza H1N1 [XVIII], and COVID-19 outbreak [XXVII].

In the literature [IV-VI, XX], there is a variety of parametrized forms of the Richards curve, and we use the following 5PL function for modeling COVID-19 infection trajectories [XXI]:

$$C(t) = C_{min} + \frac{C_{max} - C_{min}}{[1 + e^{-r(t-\tau)}]^g} \qquad (5)$$

where C_{min}, C_{max} and rare real numbers, and g is a positive real number. The parameter g controls the flexibility of the curve: (i) the curve becomes the logistic function for g=1, and (ii) if g goes to zero, then the curve reduces to the Gompertz function.

In the context of epidemiological modeling, each of the parameters can be interpreted as follows:

C(t): number of cumulative cases of COVID-19 at the time, t.
C_{min}: initial minimum number of cases. (February 28, 2020, when the first case was reported in Iceland.)
C_{max}: maximum number of individuals infected during the epidemic.
r: daily exponential growth rate. A larger number of r implies a faster spread of the virus around the country.
τ: a tipping point when the number of new daily cases begins to level off and afterward to diminish.
g : asymmetric parameter describing the skewness of the distribution of daily new cases. g =1 indicates asymmetric distribution centered at τ; g > 1 indicates faster increases in new cases before τ and slower after τ; and the reverse if g < 1.

Data

For the present study data are the daily cumulative cases of COVID-19 in Iceland from February 28, 2020, to April 08, 2021. Centers for Disease Control and Prevention (CDC) piles up this real-time data and is available on their website [VII].

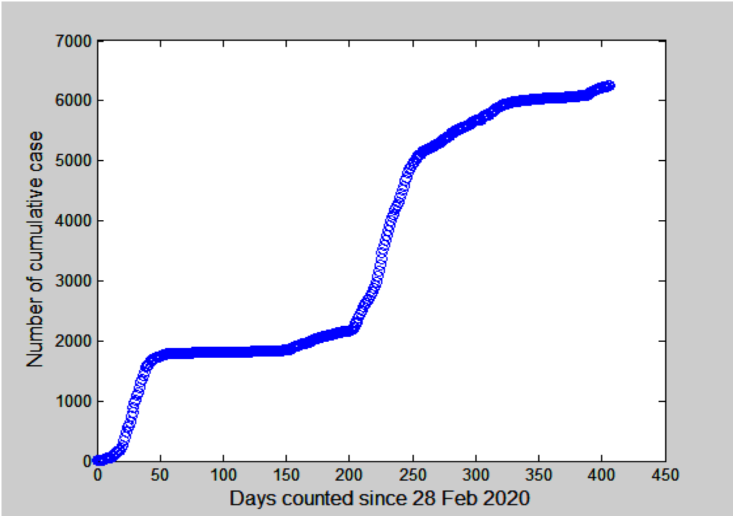

Fig. 2. Cumulative Covid 19 case

Fig. 2 shows the daily cumulative data on COVID-19 confirmed cases nationwide. This data includes 406 observations covering a period from February 28, 2020, to April 8, 2021. The figure dictates two pandemic waves. The first wave occurs from the 1st week of March 2020 to the 1st week of April 2020. The figure confirms a second pandemic wave where a sharp rise occurs in the number of infections in mid to late September 2020.

III. Simulations

Nonlinear least-squares parametric and interval estimations [XIII]

Assume that a set of data x_i, y_i, i = 1, 2, · · ·, N is available satisfying a given model y(x) = f(**a**, x), where **a** is the vector describing undetermined coefficients. As the measured data is corrupted with noises, we write the original function as y(x) = f(**a**, x) + ε, , where ε represents the residual error. The undetermined parameters **a** can be estimated by minimizing the following function I:

$$I = \underbrace{min}_{a} \sum_{1}^{N} [y_i - y(x_i)]^2 = \underbrace{min}_{a} \sum_{1}^{N} [y_i - f(\boldsymbol{a}, x_i)]^2 \tag{6}$$

Substituting back the estimated values of the parameters **a** in the objective function, the residue error $\varepsilon_i = y_i - f(\mathbf{a}, x_i)$ can be obtained. The Matlab command nlinfit(), based on non-linear least-squares fitting with Gauss-Newton's algorithm, is used to find the least square fitting. We can estimate the confidence interval (95%) with the help of function nlparci(). The syntax of the function is:

[a, r, J] = nlinfit(x, y, fun, $\mathbf{a_0}$) % least squares estimation

ci = nlparci(a, r, J) % confidence interval with 95% confidence where x and y contain the measured data. “fun” indicates the objective function. $\mathbf{a_0}$ is initial guess value of the undetermined parameters. a gives the estimated parameters, r returns the residue error vector for the estimation and matrix J is the Jacobian. ci provides 95% confidence intervals of the parameters, a.

The statistical measure for goodness of fit between the data and model is the regression coefficient, R^2, and is estimated as

$$R^2 = \frac{explained\ variance}{total\ variance} = 1 - \frac{sum\ of\ the\ squares\ of\ the\ residue\ errors}{\sum_1^N (y_i - \hat{y}_i)^2} \quad (7)$$

Cumulative data C(t) fitting to 5PL model [Eq (5)]

We fit the 5PL growth model to the cumulative case for Covid 19 pandemic 1st wave and 2nd wave in Iceland separately and using the above-mentioned Matlab command nlinfit(), get the best fit values of the parameters **a** of the model. The command nlparci() provides a 95% confidence interval. Table 1a and Table 1b show the best-fit values of the parameters and their confidence intervals (CIs).

Table 1a. Parameter estimation for 1st wave (1st week of March to 1st week of April 2020).

Parameter	Best-fit value	Lower 95% CI	Upper 95% CI
C_{min}	1	-3	2
C_{max}	1803	1794	1811
r	0.176	0.165	0.187
τ	29.5	28.0	31.0
g	0.96	0.77	1.10
R^2	0.9995		

Table 1b. Parameter estimation for 2nd wave (mid to late September 2020).

Parameter	Best-fit value	Lower 95% CI	Upper 95% CI
C_{min}	1940	1915	1965
C_{max}	5260	5064	5456
r	0.114	0.087	0.140
τ	86.2	83.4	89.0
g	0.610	0.392	0.828
R^2	0.9976		

With the best fit values of the parameters of the model, we have reconstructed the cumulative case for the two pandemic waves and are shown in figures (Fig. 3a and Fig. 3b).

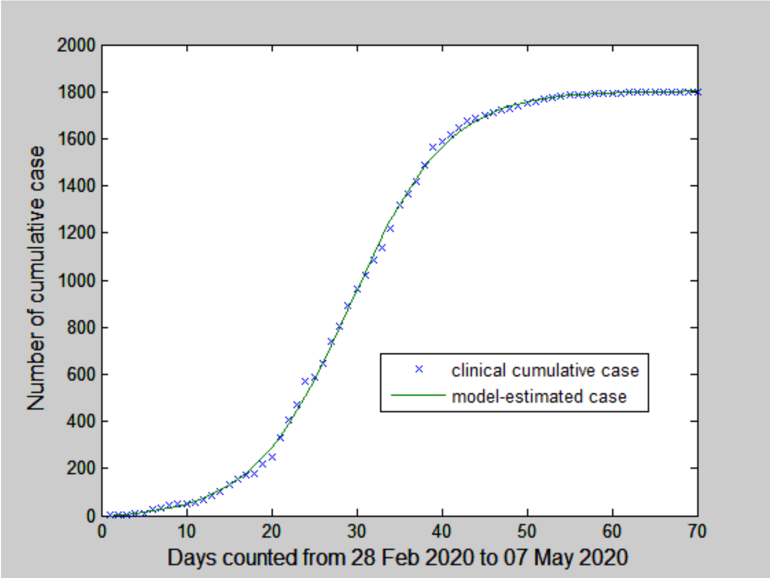

Fig. 3a. Observed and model–estimated cumulative case for 1st wave

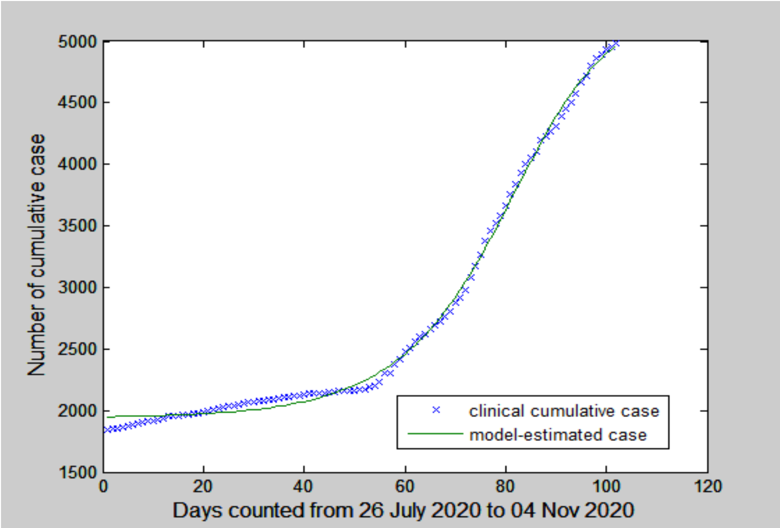

Fig. 3b. Observed and model–the estimated cumulative case for 2nd wave

IV. Discussions

For the 1st pandemic wave, our model predicts the maximum number of people infected at the end of the 1st wave (C_{max}) as 1803 which matches well with 1801 on 07 May 2020. The estimated tipping point (τ) for the new daily cases is on 28 March 2020, 30 days from the beginning of the epidemic on 28 Feb 2020. This indicates that the epidemic flattens on 28 March 2020. The daily exponentially growth rate of the Covid-19 spread is about 17.6%. Accordingly, the number of total

COVID-19 cases in Iceland doubles every four days in the absence of any anti-epidemic measure (see appendix). The estimated asymmetric parameter g is 0.96, which is statistically close to g = 1.0. This result indicates that the pattern is symmetrically distributed on 28 March 2020. The regression coefficient (R^2) between the data and model is 0.9995, which indicates excellent goodness of fit.

The value of C_{max} for the 2nd pandemic wave is 5260, as predicted by our model. It happens around 21 Nov 2020. After this day, we observe a rise in the number of infections. The estimated tipping point (τ) for the new daily cases is on 21 October 2020, 236 days from the beginning of the epidemic on 28 Feb 2020. This indicates that the 2nd wave flattens on 21 October 2020. The daily exponentially growth rate of the Covid-19 spread is about 11.4%, lower than that of the 1st wave, accordingly, total cases double every 6 and half days (see appendix). The estimated asymmetric parameter g is 0.61, which is statistically away from g = 1.0. So the pattern is not symmetrically distributed on 21 October 2020; indicating slower increases in new cases before 21 October 2020 and faster after 21 October 2020. The regression coefficient (R^2) between the data and model is 0.9976, which again indicates excellent goodness of fit for the 2nd wave.

V. Limitations

It is vital to specify that the data on confirmed cases are those that were successfully recognized and announced, accordingly, the genuine total confirmed cases of COVID-19 are probably much higher. The parameters of the Model C_{min}, the minimum number of cases at the beginning of an epidemic, are highly defective due to a lack of testing protocols and perhaps a lack of knowledge at the initial stage of the epidemic. During the study, we have not considered the differences in population density, the existing capacity of the health systems, the current level of interventions and socio-demographic and economic situation across and within the states and districts of the entire country. Accordingly, the predicted results from the model could also fluctuate in a significant way from the observed ones.

VI. Conclusions

In this report, a numerical investigation is carried out on Covid-19 in Iceland. We use a mathematical model, the 5PL function, robustly used in different fields, namely, biology, chemistry, physics, and economics, to model the Covid-19 spread in Iceland. A solid match between the estimated and the observed data on total confirmed cases is observed. Our study presents a simple mathematical model, 5PL curve, for describing the dynamics of the Covid-19 pandemic in Iceland. It provides specific information regarding the exponential growth rate, the doubling time for the epidemic, and the tipping point when daily new cases will level off. Despite some limitations as mentioned before, researchers can conduct their study on the Covid-19 pandemic using this model for different countries/regions.

Appendix A. Calculation of doubling time for the total cases.

Formula: $$A = P(1 + \frac{r}{100})^n$$

A=2P, in doubling time, n days

$$2P = P(1 + \frac{r}{100})^n$$

$$(1 + \frac{r}{100})^n = 2$$

For 1st wave: r=17.6%

$$(1.176)^n = 2$$

$$nlog(1.176) = log2$$

$$n = \frac{log2}{log(1.176)} = 4.27\ days$$

For 2nd wave: r=17.6%

$$(1.114)^n = 2$$

$$nlog(1.114) = log2$$

$$n = \frac{log2}{log(1.114)} = 6.42\ days$$

Conflicts of Interest:

The authors declare that they have no conflicts of interest to report regarding the present study.

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