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NUMERICAL ITERATIVE METHOD OF OPEN METHODS WITH CONVERGE CUBICALLY FOR ESTIMATING NONLINEAR APPLICATION EQUATIONS

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Abstract

Finding the single root of nonlinear equations is a classical problem that arises in a practical application in Engineering, Physics, Chemistry, Biosciences, etc. For this purpose, this study traces the development of a novel numerical iterative method of an open method for solving nonlinear algebraic and transcendental application equations. The proposed numerical technique has been founded from Secant Method and Newton Raphson Method, and the proposed method is compared with the Modified Newton Method and Variant Newton Method. From the results, it is pragmatic that the developed numerical iterative method is improving iteration number and accuracy with the assessment of the existing cubic method for estimating a single root nonlinear application equation.

Keywords: applications equations, cubic methods, open methods, convergence, results.

I. Introduction

During many years, various scientists and researchers have taken interest and given countless techniques to estimate nonlinear problems, which rises in a huge number of application problems in Applied Science and engineering [IV], [V], [VI] for example Distance, rate, time problems, population change, Trajectory of a ball, such as nonlinear equation form as

$$f(x) = 0$$

In fact, solutions of this nonlinear equation mostly can`t be found in Analytic analysis. For that reason, Numerical analysis is most commonly used. Numerical analysis is a significant branch of Mathematics and involves the study of procedures to calculate numerical data. Numerical analysis is also practical to estimating optimization examples. As most commonly used numerical technique includes the secant method [III]. This method is a useful open technique that requires two initial guesses. The secant method is superlinear order of convergence, while in some cases, the secant method struggles due to sluggish convergence. Besides, one of the useful open technique is Newton Raphson Method but it is also not reliable because in some cases it is keeping pitfall, but it is the most useful and second-order convergence method [II], [XIV],

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}$$

Where n=0,1,2, …

However, this is the most valuable and modest numerical technique. In recent years, several improvements had been done in literature by Adomian's Decomposition Technique, difference operator and Taylor series in Newton Technique to resolve nonlinear problems [VIII], [XIII], [XV]. Furthermore, increasing computational competence and convergence analysis by using Newton Raphson Method [IX], [XI], [XIII], [XVI]. Correspondingly, in this paper, a numerical iterative method has been recommended. The proposed method is an assortment of secant methods and the Newton Raphson Method. The numerical method has been compared with Variant of Newton Raphson Method [XVI],

$$y_n = x_n - \frac{f(x_n)}{f`(x_n)}$$

$$x_{n+1} = x_n - \frac{2f(x_n)}{f`(x_n) + f`(y_n)}$$

and Modified Newton Raphson Method [VI],

$$x_{n+1} = x_n - \frac{f(x_n)f`(x_n)}{f`^2(x_n) - f(x_n)f``(x_n)}$$

From the comparison of existing methods with the proposed method of application problems in result Table-1, it can be experiential that the proposed method is executed well, more effectual and easier to employ with the judgement of variant of newton raphson method and modified newton raphson method.

II. Numerical Iterative Method

This segment has developed a third-order iterated method for estimating nonlinear application equations, such as

$$f(x) = 0$$

A developed numerical iterative method is established with the help of Secant Method and Newton Raphson Method, such as

$$x_{n+2} = x_{n+1} - \frac{x_{n+1}-x_n}{f(x_{n+1})-f(x_n)} f(x_{n+1}) \quad (1)$$

and

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)} \quad (2)$$

By using Eq: (2) in Eq: (1), generally we get

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)} - \frac{x_n - \frac{f(x_n)}{f`(x_n)} - x_n}{f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right) - f(x_n)} f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)} + \frac{f(x_n) f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)}{f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right) f`(x_n) - f(x_n) f`(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)} + \frac{f(x_n) f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)}{f`(x_n)[f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right) - f(x_n)]}$$

Finally, we get

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}\left[1 + \frac{f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)}{f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right) - f(x_n)}\right] \quad (3)$$

Hence Eq: (3) is a numerical iterative method for estimating a single root of nonlinear application equations.

III. Convergence Analysis

The following segment will be shown that the proposed numerical iterative method is keeping third order of convergence, such as

$$e_{n+1} = c^2 e_n{}^3 + o(e_n{}^4)$$

Proof:

Using the relation $e_n = x_n - a$ in Taylor series, therefore from Taylor series to estimate $f(x_n), f`(x_n)$ $and$ $f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)$ with c = $\frac{f``(a)}{2f`(a)}$ and ignoring higher-order term for easy to solve, such as

$$f(x_n) = f`(a)(e_n + c e_n{}^2) \quad (i)$$

$$f`(x_n) = f`(a)(1 + 2c e_n) \quad (ii)$$

By using (i) and (ii) in (2), we get

$$x_n - \frac{f(x_n)}{f`(x_n)} = e_n - \frac{e_n f`(a) + \frac{e^2{}_n f``(a)}{2}}{f`(a) + 2e_n \frac{f``(a)}{2}}$$

$$x_n - \frac{f(x_n)}{f`(x_n)} = e_{n-} e_n(1 + c e_n)(1 + 2c e_n)^{-1}$$

$$x_n - \frac{f(x_n)}{f`(x_n)} = e_{n-} e_n(1 + c e_n)(1 - 2c e_n)$$

$$x_n - \frac{f(x_n)}{f`(x_n)} = e_{n-} e_n(1 - c e_n)$$

$$x_n - \frac{f(x_n)}{f`(x_n)} = \mathrm{c}e^2{}_n$$

Thus,

$$f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right) = f`(a)(\mathrm{c}e^2{}_n \ + c^3 {e_n}^4)$$

$$f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right) = \mathrm{c}e^2{}_n f`(a)(1 + c^2 {e_n}^2) \qquad (iii)$$

By using (i), $(ii) and\ (iii)\ in$ (3), we get

$$e_{n+1} = e_{n-}e_n(1 - ce_n)[1 + \frac{\mathrm{c}e^2{}_n f`(a)(1+c^2{e_n}^2)}{\mathrm{c}e^2{}_n f`(a)(1+c^2{e_n}^2) - f`(a)(e_n + c{e_n}^2)}]$$

$$e_{n+1} = e_{n-}e_n(1 - ce_n)[1 + \frac{\mathrm{c}e_n(1-c^2{e_n}^2)}{(1-c^3{e_n}^3)}]$$

$$e_{n+1} = e_{n-}e_n(1 - ce_n)[1 + \mathrm{c}e_n(1 - c^2{e_n}^2)(1 - c^3{e_n}^3)^{-1}\ ]$$

$$e_{n+1} = e_{n-}e_n(1 - ce_n)[1 + \mathrm{c}e_n(1 - c^2{e_n}^2 + c^3{e_n}^3\ ]$$

$$e_{n+1} = e_{n-}e_n(1 - ce_n)[1 + ce_n - c^3{e_n}^3\ ]$$

$$e_{n+1} = e_{n-}e_n[1 + \mathrm{c}e_n - c^3{e_n}^3 - \mathrm{c}e_n - c^2{e_n}^2]$$

$$e_{n+1} = e_{n-}e_n + c^3{e_n}^4 + c^2{e_n}^3$$

$$e_{n+1} = c^2{e_n}^3 + o({e_n}^4)$$

Hence this has proven that the (3) is converge cubically.

IV. Results and Discussions

In this segment, all calculations have been completed by MATLAB/C++ with stopping criteria for computer programs are:

$|x_{n+1} - x_n| < \varepsilon$, where $\varepsilon > e^{-10}$

The developed numerical method is comparing by Variant of Newton Raphson Method [XVI],

$$y_n = x_n - \frac{f(x_n)}{f`(x_n)}$$

$$x_{n+1} = x_n - \frac{2f(x_n)}{f`(x_n) + f`(y_n)}$$

and Modified Newton Raphson Method [VII],

$$x_{n+1} = x_n - \frac{f(x_n)f`(x_n)}{f`^2(x_n) - f(x_n)f``(x_n)}$$

From application functions in Table-1 to exemplify the competence of developed numerical iterative technique by Variant of Newton Raphson Method and Modified

Newton Raphson Method. The functions belonging from the mass of the jumper, the volume of the gas depends on the temperatures, anti-symmetric buckling of a beam, projectile motion of any system and (pollutant bacteria concentration) such as

i. $f(x) = Sinx - x + 1 \; with \; x = 2$
ii. $f(x) = 2x - lnx - 7 \; with \; x = 4$
iii. $f(x) = x\boldsymbol{e^x} - 2 \; with \; x = 3$
iv. $f(x) = \boldsymbol{x^3} \; - 9x + 1 \; with \; x = 0$
v. $f(x) = \boldsymbol{e^x} - 5x \; with \; x = 2$
vi. $f(x) = cosx - \boldsymbol{x^3} \; with \; x = 1$
vii. $f(x) = \boldsymbol{x^2} - \boldsymbol{e^x} - \boldsymbol{3x} + \boldsymbol{2} \; with \; x = 1.5$

From Table-1, it has perceived that the developed numerical iterative method is taking good accuracy as well as iteration perception with the judgment of existing cubic methods, such as

Numerical Table No. 1

FUNCTION	METHODS	ITERATIONS	X	A E%
Sinx-x+1 x=2	Modified Newton Method Variant Newton Method Numerical Iterative Method	4 3 2	1.93456	$1.67012e^{-4}$ $1.19209e^{-7}$ $3.01600e^{-5}$
$x^2 - e^x - 3x + 2$ x=1.5	Modified Newton Method Variant Newton Method Numerical Iterative Method	6 4 3	0.25753	$5.96046e^{-8}$ $1.49012e^{-7}$ $2.86102e^{-6}$
$xe^x - 2$ x=3	Modified Newton Method Variant Newton Method Numerical Iterative Method	19 10 5	0.852605	$5.96046e^{-8}$ $5.96046e^{-8}$ $2.98023e^{-7}$
$x^3 - 9x + 1$ $x = 0$	Modified Newton Method Variant Newton Method Numerical Iterative Method	4 3 2	2.94284	$1.78814e^{-7}$ $1.17209e^{-5}$ $1.21027e^{-5}$
$e^x - 5x$ $x = 2$	Modified Newton Method Variant Newton Method Numerical Iterative Method	5 4 3	2.54264	$2.21729e^{-5}$ $1.45435e^{-5}$ $5.10421e^{-7}$
$Cosx-x^3$ x=1	Modified Newton Method Variant Newton Method Numerical Iterative Method	4 6 2	0.865474	$5.96046e^{-8}$ $1.78814e^{-7}$ $1.76102e^{-3}$
2x-lnx-7 x=4	Modified Newton Method Variant Newton Method Numerical Iterative Method	3 2 2	4.21991	$4.76837e^{-6}$ $1.19209e^{-5}$ $2.86102e^{-6}$

V. Conclusions

This research describes a numerical iterative method for estimating nonlinear application equations that have been proposed. The proposed method is converging cubically and is derived from Secant Method and Newton Raphson Method. We can conclude from numerical outcomes that the developed Numerical Iterative Method finds a good accuracy as well as iteration perception in the judgement of variant of the newton raphson method and modified newton raphson method. Hence, the novel numerical method is moderately modest, stable and more efficient in the estimating of nonlinear application equations.

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Conflicts of Interest:

The authors declare that they have no conflicts of interest to report regarding the present study.

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