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A NOVEL FUZZY ENTROPY MEASURE AND ITS APPLICATION IN COVID-19 WITH FUZZY TOPSIS

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Abstract

Fuzzy sets (FSs) are an important tool to model uncertainty and vagueness. Entropy is being used to measure the fuzziness within a fuzzy set (FS). These entropies are used to find multicriteria decision-making. For measuring uncertainty with TOPSIS techniques an axiomatic definition of entropy measure for fuzzy sets is also given in this paper. The proposed entropy is provided to satisfy all the axioms. Several numerical examples are presented to compare the proposed entropy measure with existing entropies. The corresponding results show that the newly proposed entropy can be computed easily and give reliable results. Finally, the decision-making algorithm TOPSIS (Techniques of ordered preference similarity to ideal solution) is utilized to solve multicriteria decision-making problems (MCDM) related to daily life. In the current situation, COVID-19 has no proper medical treatment. We use the TOPSIS technique to suggest an effective medicine for this pandemic. Numerical results and practical examples show the effectiveness and practical applicability of the proposed entropy.

Key Words: Fuzzy entropy, TOPSIS, Uncertainty, Multicriteria Decision Making

I. Introduction

A fuzzy set is a generalized form of the crisp set which is also called the traditional set Ramot et al [XXI]. A fuzzy set was introduced by Zadeh [XXVIII]. To find the vagueness and uncertainty of our everyday life problems fuzzy sets are found to be very advantageous. Probability theory is also used to handle uncertainty which is based on two-way logic. Since the fuzzy set theory is based on many ways of logic therefore it may treat uncertainty more precisely and accurately than probability theory. Fuzzy sets are widely applicable in many fields such as image processing, pattern recognition, optimization etc. Wang et.al [XXIV] and Horvath [XI]. The membership function of the fuzzy set is based on membership between 0 and 1 and one minus membership represents the non-membership function. After the publication of the fuzzy set theory, the authors generalize the classical notion of a set. The recognition of this theory was raised gradually in the 1960s and 1970s respectively. In about 1970s the first application of a fuzzy rule-based system called fuzzy control improved awareness in this area. The use of fuzzy logic newly extended its peak in 1980 in the use of Japanese products. The extent of degree of fuzziness is an essential phase for practical zone Zimmermann [X] and Grattan [IX].

Entropy is used to find a degree of fuzziness of FSs. Entropy is the measure of disorder Shannon [XXII]. In 1972, De Luca and Termini [VI] introduce non-probabilistic entropy. They conclude that in special cases Shannon probabilistic entropy is used for superior determination. De Luca and Termini [V] initiate the fuzzy literature. They propose the parameterized entropy measure, inspired by the traditional Shannon entropy function. In 2004, Li and Liu [XVIII] introduced the concept of credibility measure and give the necessary and sufficient conditions for credibility measure. For the study of fuzzy behavior, Li [XVI] introduced credibility measure and refined in 2008 Li and Liu [XVIII]. Credibility theory is construed from the regularity, self-duality, growing and maximally axioms. Kosko [XIV] defines the entropy by using the distance of near set and far set and define the near and far sets. Pal and Pal [XIX] define the exponential entropy using membership and its complement. Hwang & Yang [XIII] introduced new interval-based entropy of FSs.

More recently, a contiguous disease named Coronavirus disease 2019 (COVID-19) has become a global issue Chakraborty et al [II]. Even though we are associated with a field unrelated to medical science but the study aims to play our part in the collective human endeavor to overcome the pandemic. Several medicines are tested nowadays for the treatment of COVID-19 Dong et al [VII]. Therefore, the primary objective of the study is to propose a model to select a medicine for COVID-19. To achieve the objective, we have set the following criteria for the selection of such a medicine, relevance to the pattern of prevalent diseases, efficacy and safety, adequate quality and availability, and desirable pharmacokinetic properties. Using fuzzy MCDM (Multicriteria Decision Making) techniques we can choose the best/appropriate medicine for this pandemic. In this paper, we introduce new fuzzy entropy using the concept of Yang's [XXVII] entropy. Yang's proposed entropy does not work effectively in some conditions e.gif the membership values between interval 0.4 and 0.6 here the new entropy is friendly with the Yang's entropy and also for interval 0.4 and 0.6. Based on the proposed fuzzy entropy and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), we here give a new MCDM decision-making algorithm. This technique is one of the important MCDM techniques to rank and select the best alternative among many alternatives. We apply TOPSIS for the selection of appropriate medicine based on the above-mentioned criteria.

The rest of the paper is organized as follows: In section 2, we review already existing entropy measures of a fuzzy set. We put forward an axiomatic definition of entropy of FS and suggest new entropy of a fuzzy set. We give a detailed proof of all axioms. In section 3, we give some numerical examples and compare them with Yang's entropy. In section 4, the fuzzy TOPSIS method is used for the selection of effective medicine for COVID-19. Finally, the conclusion is stated in section 5.

II. Entropy of Fuzzy Set

We use the following notation in the definition of entropy. Let $\tilde{R}^{+}$ is the set of real numbers, $\tilde{S}$ be a universal set, $\tilde{F}\left(\tilde{S}\right)$ denotes a class of fuzzy sets, $\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)$ is the membership function of fuzzy set $\tilde{T}$, $\tilde{P}\left(\tilde{S}\right)$ is the class of crisp set, $\tilde{F}$ is a subclass of $\tilde{F}\left(\tilde{S}\right)$ and $\tilde{N}$ shows the entropy of FSs .

Definition 1. The function $\tilde{N}:\tilde{F}\to\tilde{R}^{+}$ is the entropy of $\tilde{F}$ if $\tilde{N}$ has following properties

(1) $0 \le \tilde{N}(\tilde{T}) \le 1;$

(2) $\tilde{N}(\tilde{D}) = 0 \; if \; \tilde{D} \in \tilde{P}(\tilde{S});$

(3) $\tilde{N}\left(\left[\frac{1}{2}\right]\right) = 1;$

(4) $\tilde{N}(\tilde{T}) \le \tilde{N}(\tilde{R})$

if $\tilde{T}$ is crisper than $\tilde{R}$;

$$\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right) \le \tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)$$

and

$$\left(1-\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)\right) \ge \left(1-\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)\right)$$

when

$$\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right) \le \left(1-\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)\right) \text{or}$$

$$\left(1-\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)\right) \le \left(1-\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)\right) \le \tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right) \le \tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right) \;\; \tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right) \ge \tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)$$

and

$$\left(1-\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)\right) \le \left(1-\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)\right)$$

when

$$\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right) \ge \left(1-\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)\right) \text{or} \left(1-\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)\right) \le \left(1-\tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right)\right) \le \tilde{\mu}_{\tilde{R}}\left(\tilde{s}_i\right) \le \tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right).$$

$$\tilde{N}(\tilde{T}^{c}) = \tilde{N}(\tilde{T}), \forall \tilde{T} \in \tilde{F}.$$

Next, we propose new entropy of fuzzy set and compare it with entropy defined by Hwang & Yang [XI].

Hwang & Yang [XI] defined entropy as;

$$\tilde{N}_{\tilde{o}} = \frac{\sum_{i=1}^{r}\left(1-e^{-\tilde{\mu}_{\tilde{T}^c}\left(\tilde{s}_i\right)}\right) I\left[\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right) \ge \frac{1}{2}\right] + \left(1-e^{-\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)}\right) I\left[\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right) < \frac{1}{2}\right]}{r\left(1-e^{\frac{-1}{2}}\right)}, \forall \tilde{T} \in \tilde{F}\left(\tilde{S}\right)$$

Our proposed entropy is given by

$$\tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}\right) = \tilde{\Omega} + \tilde{\Theta} + \tilde{\vartheta}, \; \forall \tilde{T} \in \tilde{F}\left(\tilde{S}\right) \quad (1)$$

where

$$\tilde{\Omega}=\frac{1}{2r}\frac{\sum_{i=1}^{r}\left[\left\{\left(e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)+e^{1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right)-1\right\}+\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right]}{\left(e^{0.5}-1\right)},\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)>\frac{1}{2}$$

$$\tilde{\Theta}=\frac{1}{r}\frac{\sum_{i=1}^{r}\left[\left(e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)+e^{1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right)-1\right]}{\left(e^{0.5}-1\right)},\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)=\frac{1}{2}$$

$$\tilde{\vartheta}=\frac{1}{2r}\frac{\sum_{i=1}^{r}\left[\left(e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)+e^{1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right)-1\right]}{\left(e^{0.5}-1\right)},\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)<\frac{1}{2}.$$

Where

$\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)$ represent the membership function of FSs and $\tilde{\mu}_{\tilde{T}}(\tilde{s}_i):\tilde{F}(\tilde{S})\to[0,1]$.

Theorem 1. The newly proposed entropy defined in Eq.(1) satisfies all axioms of Definition 1.

Proof. We first give the proof of axiom (I) of Definition 1. Obviously, from Eq.(1) $0\le\tilde{\Omega}\le 1$, $0\le\tilde{\Theta}\le 1$ and $0\le\tilde{\vartheta}\le 1$ which follows $0\le\tilde{N}(\tilde{T})\le 1$. Thus, an axiom (I) is satisfied. For the axiom (II), Since D is crisp and possible crisp values are either 0 or 1 and hence in both cases $\tilde{N}_{\tilde{s}\tilde{s}}(\tilde{T})=0$. Thus the axiom of Definition 1 is proved. Next, we prove the axiom (III) of Definition 1. From Eq.(1) it is clear that $\tilde{\Omega}=\tilde{\vartheta}=0,\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\neq\frac{1}{2}$

and

$$\tilde{\Theta}=\frac{1}{r}\frac{\sum_{i=1}^{r}\left[e^{0.5}(1-0.5)+e^{1-0.5}(0.5)-1\right]}{\left(e^{0.5}-1\right)},\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)=\frac{1}{2},$$

$$\tilde{\Theta}(0.5)=\frac{r\left(e^{0.5}-1\right)}{r\left(e^{0.5}-1\right)}=1,\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)=\frac{1}{2}\cdot$$

Thus

$$\tilde{N}_{\tilde{s}\tilde{s}}(\tilde{T})=\tilde{\Omega}+\tilde{\Theta}+\tilde{\vartheta}=1,\ \forall\,\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)=\frac{1}{2}.,$$

an axiom is proved (III). Now, we give proof of axiom (IV) as $\tilde{N}(\tilde{T}) \leq \tilde{N}(\tilde{R})$ if $\tilde{T}$ is crisper than $\tilde{R}$ $\forall\, \tilde{\mu}_{\tilde{T}}(\tilde{s}_i) \leq \tilde{\mu}_{\tilde{R}}(\tilde{s}_i)$, $e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)} \leq e^{\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)}$ implies that $1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i) \geq 1-\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)$, clearly

$$\left[\left\{\left(e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)+\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)e^{1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\right)-1\right\}+\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right]$$

$$\leq\left[\left\{\left(e^{\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)\right)+\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)e^{1-\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)}\right)-1\right\}+\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)\left(1-\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)\right)\right],\ \forall\, \tilde{\mu}_{\tilde{R}}(s_i), \tilde{\mu}_{\tilde{T}}(\tilde{s}_i) > \frac{1}{2},$$

hence

$$\tilde{N}(\tilde{T}) \leq \tilde{N}(\tilde{R}).$$

Similarly,

$$\left[\left(e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)+\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)e^{1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\right)-1\right] \leq$$

$$\left[\left(e^{\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)\right)+\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)e^{1-\tilde{\mu}_{\tilde{R}}(\tilde{s}_i)}\right)-1\right],\ \forall\, \tilde{\mu}_{\tilde{R}}(s_i), \tilde{\mu}_{\tilde{T}}(\tilde{s}_i) \leq \frac{1}{2},$$

thus

$$\tilde{N}(\tilde{T}) \leq \tilde{N}(\tilde{R}).$$

Finally, we prove the axiom (V) of Definition 1. From Eq.(1), we have

$$\tilde{\Omega} = \frac{1}{2r}\frac{\sum_{i=1}^{r}\left[\left\{\left(e^{\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)+e^{1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)}\left(\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right)-1\right\}+\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\left(1-\tilde{\mu}_{\tilde{T}}(\tilde{s}_i)\right)\right]}{\left(e^{0.5}-1\right)},\ \forall\, \tilde{\mu}_{\tilde{T}}(\tilde{s}_i) > \frac{1}{2},$$

we have

$$\tilde{\mu}_{(\tilde{T}^c)}(\tilde{s}) = 1-\tilde{\mu}_{(\tilde{T})}(\tilde{s}), \text{ and } \tilde{\mu}_{(\tilde{T})}(\tilde{s}) = 1-\tilde{\mu}_{(\tilde{T}^c)}(\tilde{s}),$$

$$=\frac{1}{2r}\frac{\sum_{i=1}^{r}\left[\left\{\left(e^{\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\right)+e^{1-\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)}\left(\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\right)\right)-1\right\}+\left\{1-\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\left(\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\right)\right\}\right]}{\left(e^{0.5}-1\right)},\ \forall\, \tilde{\mu}_{\tilde{T}}(\tilde{s}_i) > \frac{1}{2}$$

$$=\frac{1}{2r}\frac{\sum_{i=1}^{r}\left[\left\{\left(e^{1-\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)}\left(\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\right)+e^{\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)}\left(1-\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\right)\right)-1\right\}+\left\{1-\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\left(\tilde{\mu}_{\tilde{T}^c}(\tilde{s}_i)\right)\right\}\right]}{\left(e^{0.5}-1\right)} = \tilde{\Omega}^c$$

Thus $\tilde{\Omega}^c = \tilde{\Omega}$. Similarly, $\tilde{\Theta} = \tilde{\Theta}^c$ *and* $\tilde{\vartheta} = \tilde{\vartheta}^c$ hence $\tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}\right) = \tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}^c\right)$. This completes the proof. □

III. Examples and Comparisons

We have a fuzzy set $\tilde{T} = \left\{\left(\tilde{s}_i, \tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)\right) : \tilde{s}_i \in \tilde{S}\right\}$, the modifier $\tilde{T}^{\tilde{r}}$ for any fuzzy set $\tilde{T}$ is define Huwang and Yang [XI] as:

$$\tilde{T}^r = \left\{\left(s_i, \left(\tilde{\mu}_{\tilde{T}}\left(\tilde{s}_i\right)\right)^r\right) : \tilde{s}_i \in \tilde{S}\right\}$$

The linguistic hedges are as follows:

$Concentration = CON\left(\tilde{T}\right) = \tilde{T}^2$, $\quad$ $Dilation = DIL\left(\tilde{T}\right) = \tilde{T}^{\frac{1}{2}}$

Slightly very $\left(\tilde{T}\right) = \tilde{T}^3$, Slightly more very $\left(\tilde{T}\right) = \tilde{T}^4$.

Example1: Consider the fuzzy set $\tilde{T} = \left\{\left(\tilde{\omega}_1, 0.4\right), \left(\tilde{\omega}_2, 0.49\right)\right\}$

The linguistic hedges on the fuzzy set are defined as:

$$\tilde{T}^{\frac{1}{2}} = \left\{\left(\tilde{\omega}_1, 0.6324\right), \left(\tilde{\omega}_2, 0.7\right)\right\}; \tilde{T}^2 = \left\{\left(\tilde{\omega}_1, 0.16\right)\right), \left(\tilde{\omega}_2, 0.2401\right)\right\};$$

$$\tilde{T}^3 = \left\{\left(\tilde{\omega}_1, 0.064\right), \left(\tilde{\omega}_2, 0.1176\right)\right\}; \tilde{T}^4 = \left\{\left(\tilde{\omega}_1, 0.0256\right), \left(\tilde{\omega}_2, 0.0576\right)\right\}.$$

Table 1. Comparison of Results of Entropy Measures

Entropy	$\tilde{N}_{\tilde{o}}\left(\tilde{T}_i\right)$	$\tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}_i\right)$
$\tilde{T}^{\frac{1}{2}}$	**0.7202**	**0.615**
$\tilde{T}$	**0.9112**	**0.4903**
$\tilde{T}^2$	**0.4591**	**0.3221**
$\tilde{T}^3$	**0.1981**	**0.1524**
$\tilde{T}^4$	**0.1032**	**0.0823**

The results of mathematical requirement from Table 1 are given as follows

$$\tilde{N}_{\tilde{o}}\left(\tilde{T}_1^{\frac{1}{2}}\right) < \tilde{N}_{\tilde{o}}\left(\tilde{T}_1\right) > \tilde{N}_{\tilde{o}}\left(\tilde{T}_1^2\right) > \tilde{N}_{\tilde{o}}\left(\tilde{T}_1^3\right) > \tilde{N}_{\tilde{o}}\left(\tilde{T}_1^4\right)$$

$$\tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}_1^{\frac{1}{2}}\right) > \tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}_1\right) > \tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}_1^2\right) > \tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}_1^3\right) > \tilde{N}_{\tilde{s}\tilde{s}}\left(\tilde{T}_1^4\right).$$

The comparison analysis results which are obtained from Table 1, show that our proposed entropy measure $\tilde{N}_{\tilde{s}\tilde{s}}$ performs better and satisfies the mathematical requirement as compared to the entropy measure given by Yang.

Next, we use it in the construction of Fuzzy TOPSIS (F-TOPSIS) to handle the problem involving complex multicriteria decision making (MCDM) process, to ensure the practical applicability and usefulness of the proposed entropy measure $\tilde{N}_{\tilde{s}\tilde{s}}$.

IV. Construction of Fuzzy TOPSIS Based on New Entropy Measure

In multicriteria decision making (MCDM) we have a finite set of alternatives with a set of criteria. Among each alternative, we have to choose the best one. We utilize TOPSIS algorithm to rank the alternatives based on given criteria. Let the set of alternatives be denoted by $\tilde{T} = \{\tilde{T}_1, \tilde{T}_2, ..., \tilde{T}_r\}$ and the set of criteria of alternative be represented by $\tilde{R} = \{\tilde{R}_1, \tilde{R}_2, ..., \tilde{R}_t\}$. The steps for the proposed TOPSIS are as follows:

Step 1: Construction of fuzzy decision matrix

Suppose that $\tilde{T} = \{\tilde{T}_1, \tilde{T}_2, ..., \tilde{T}_r\}$ is a set of r alternatives on criteria $\tilde{R} = \{\tilde{R}_1, \tilde{R}_2, ..., \tilde{R}_t\}$. Assume that the decision matrix in fuzzy set (FSs) is $\tilde{O} = (\tilde{o}_{ij})_{r\times t}$, where $\tilde{o}_{ij} = \tilde{\mu}_{ij}$ represents membership degree of the alternative $\tilde{T}_i \in \tilde{T}$ on criteria $\tilde{R}_j \in \tilde{R}$ satisfying the condition $0 \le \tilde{\mu}_{ij} \le 1,\ i = 1, 2, ..., r;\ j = 1, 2, ..., t.$ The fuzzy decision matrix is constructed as follows:

$$\tilde{O} = (\tilde{\mu}_{ij})_{r\times t} = \begin{array}{c|cccccc|} & \tilde{R}_1 & \tilde{R}_2 & . & . & . & \tilde{R}_t \\ \hline \tilde{T}_1 & \tilde{\mu}_{11} & \tilde{\mu}_{12} & . & . & . & \tilde{\mu}_{1t} \\ \tilde{T}_2 & \tilde{\mu}_{21} & \tilde{\mu}_{22} & . & . & . & \tilde{\mu}_{2t} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ \tilde{T}_r & \tilde{\mu}_{r1} & \tilde{\mu}_{r2} & & & & \tilde{\mu}_{rt} \\ \hline \end{array} \tag{2}$$

Step 2: Determination of the weights of criteria

The weights $\tilde{\lambda}_j,\ j = 1, 2, ..., t$ of criteria can be obtained by using the newly proposed entropy $\tilde{N}_{ss}$ given in Eq. 1. Suppose the weights of criteria $\tilde{R}_j$, $j = 1, 2, ..., n$ are $\tilde{\lambda}_j$, $j = 1, 2, ..., n$ with $0 \le \tilde{\lambda}_j \le 1$ and $\sum_{j=1}^{t} \tilde{\lambda}_j = 1$. The weighting method based on the proposed entropy is as follows:

$$\tilde{\lambda}_j = \frac{\tilde{N}_j(\tilde{T}_r)}{\sum_{j=1}^{\tilde{r}} \tilde{N}_j(\tilde{T}_r)} \tag{3}$$

where $\tilde{N}_j(\tilde{T}_r)$ is Our proposed entropy given in Eq.1.

Step 3: Determination of positive ideal solution (PIS) and negative ideal solution (NIS) of the MCDM problem.

The criteria can be categorized into two groups which are cost criteria and benefit criteria. Suppose that $\tilde{Z}_1$ is a set of benefit criteria and $\tilde{Z}_2$ is a set of cost criteria. We define the fuzzy positive ideal solution (FPIS) $\tilde{Q}^+$ and fuzzy negative ideal solution (FNIS) $\tilde{Q}^-$ as follows:

$$\tilde{Q}^+ = \left\{ \left\langle \tilde{R}_j \ , \tilde{\mu}_j{}^+ \right\rangle : \ \tilde{R}_j \in \tilde{R} \right\} \tag{4}$$

$$\tilde{Q}^- = \left\{ \left\langle \tilde{R}_j \ , \tilde{\mu}_j{}^- \right\rangle : \ \tilde{R}_j \in \tilde{R} \right\} \tag{5}$$

where $\tilde{\mu}_j^+ = \{1\},\ \tilde{\mu}_j^- = \{0\},\ j \in \tilde{Z}_1 \ and \ \tilde{\mu}_j^- = \{1\},\ \tilde{\mu}_j^+ = \{0\},\ j \in \tilde{Z}_2.$

Step 4: Calculation of distance measures between alternative $\tilde{T}_i$ with PIS and NIS respectively.

The distance measures between alternative $\tilde{T}_i$ with PIS and NIS are calculated respectively as follows:

$$\tilde{E}^+\left(\tilde{T}_r\right) = \tilde{E}_j\left(\tilde{Q}^+\right) = \tilde{d}_{\tilde{h}}\left(\tilde{T}_i, \tilde{Q}^+\right) = \sum_{i=1} \tilde{\lambda}_j \left| \tilde{\mu}_{\tilde{T}_i}(s_i) - \tilde{\mu}_{j^+} \right| \tag{6}$$

$$\tilde{E}^-\left(\tilde{T}_r\right) = \tilde{E}_j\left(\tilde{Q}^-\right) = \tilde{d}_{\tilde{h}}\left(\tilde{T}_i, \tilde{Q}^-\right) = \sum_{i=1} \tilde{\lambda}_j \left| \tilde{\mu}_{\tilde{T}_i}(s_i) - \tilde{\mu}_{j^-} \right| \tag{7}$$

Step 5: Determine the relative closeness coefficient of each alternative.

The closeness coefficient $\tilde{B}\left(\tilde{T}_r\right)$ is calculated as:

$$\tilde{B}\left(\tilde{T}_r\right) = \frac{\tilde{E}^-\left(\tilde{T}_r\right)}{\tilde{E}^+\left(\tilde{T}_r\right) + \tilde{E}^-\left(\tilde{T}_r\right)} \tag{8}$$

Step 6: Rank the alternatives according to the relative closeness coefficient.

Finally, the alternatives are ordered according to the relative closeness coefficient. The ranking of all alternatives is obtained according to ascending order of the relative closeness degrees. The most preferred alternative is the one with the highest relative closeness degree.

V. Numerical Example

To illustrate the application of the proposed entropy measure in handling MCDM problem, we consider the following example.

Example 2. Assume that there are four medicines that we are testing for COVID-19. We consider these medicines as our alternatives $\tilde{T}_1,\ \tilde{T}_2,\ \tilde{T}_3,\ \tilde{T}_4$. We categorize these medicines based on the following criteria.

$\tilde{R}_1$: Relevance to the pattern of prevalent diseases

$\tilde{R}_2$: Efficacy and safety

$\tilde{R}_3$: Adequate quality and availability

$\tilde{R}_4$: Desirable pharmacokinetic properties.

The evaluation values of four possible alternatives $\tilde{T}_1, \tilde{T}_2, \tilde{T}_3, \tilde{T}_4$. under the above four criteria can be denoted by the following FSs.

$$\tilde{T}_1 = \{(\tilde{\omega}_1, 0.7), (\tilde{\omega}_2, 0.9), (\tilde{\omega}_3, 0.8), (\tilde{\omega}_4, 1.00)\},$$

$$\tilde{T}_2 = \{(\tilde{\omega}_1, 0.1), (\tilde{\omega}_2, 0.3), (\tilde{\omega}_3, 0.2), (\tilde{\omega}_4, 0.4)\},$$

$$\tilde{T}_3 = \{(\tilde{\omega}_1, 0.2), (\tilde{\omega}_2, 0.9), (\tilde{\omega}_3, 0.7), (\tilde{\omega}_4, 0.6)\},$$

$$\tilde{T}_4 = \{(\tilde{\omega}_1, 0.8), (\tilde{\omega}_2, 0.4), (\tilde{\omega}_3, 0.6), (\tilde{\omega}_4, 0.2)\}.$$

The following fuzzy decision matrix is constructed by using Eq.(2)

Table 2. Fuzzy Decisions Matrix

Entropy	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$
$\tilde{T}_1$	**0.7**	**0.9**	**0.8**	**1.0**
$\tilde{T}_2$	**0.1**	**0.3**	**0.2**	**0.4**
$\tilde{T}_3$	**0.2**	**0.9**	**0.7**	**0.6**
$\tilde{T}_4$	**0.8**	**0.4**	**0.6**	**0.2**

The weights for each criteria is calculated by utilizing Eq.(3)as follows

Table 3. Criteria Weight of Proposed Entropy

Entropy	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
$\tilde{\lambda}_i$	0.2410	0.2206	0.3087	0.2297

Distance of each alternative $\tilde{T}_r$ from FPIS Eq.(6) and FNIS Eq.(7) and relative closeness degree using Eq.(8) is given in the following Table 5.

Table 4. Calculation of FPIS and FNIS and degree of relative closeness

$\tilde{N}_{\tilde{s}\tilde{s}}$	$\tilde{E}^{+}(\tilde{T}_r)$	$\tilde{E}^{-}(\tilde{T}_r)$	$\tilde{B}(\tilde{T}_r)$
$\tilde{T}_1$	0.1561	0.8439	0.8439
$\tilde{T}_2$	0.7561	0.2439	0.2439
$\tilde{T}_3$	0.3993	0.6006	0.6007
$\tilde{T}_4$	0.4878	0.5122	0.5122

Based on the results of relative closeness degree in Table 4, we rank the alternatives in preferred order as exhibited in following Table 5

Table 5. Ranking of Alternatives

Entropy	Ranking
$\tilde{N}_{\tilde{s}\tilde{s}}$	$\tilde{T}_1 \succ \tilde{T}_3 \succ \tilde{T}_4 \succ \tilde{T}_2$

The final ranking of alternatives based on fuzzy TOPSIS with our proposed entropy measure is made to show the most effective medicine. The ranking is made in preferred order based on the degree of closeness. The medicine with the highest degree of closeness is considered the most effective medicine. Therefore, $\tilde{T}_1$ is considered the most desirable medicine.

VI. Conclusion

Fuzzy sets play an important role in managing complex MCDM problems related to daily life settings. In this communication, we have proposed new fuzzy entropy and perform a comparative analysis with the most existing entropy measure for FS. Finally, the proposed entropy measure of an FS is applied in an application to multicriteria decision-making for the ranking of alternatives. We have used the proposed entropy measure to construct a fuzzy TOPSIS technique for the selection of an appropriate medicine for COVID-19. For this purpose, certain criteria are set for the selection of the best medicine. The proposed technique can efficiently help medical experts to choose the best medicine for COVID-19. Based on obtained results, we conclude that the proposed entropy measures for FS are well suited in handling multicriteria decision-making problems in a fuzzy environment.

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Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

References

I. Asish Mitra, : 'MODIFIED SIRD MODEL OF EPIDEMIC DISEASE DYNAMICS: A CASE STUDY OF THE COVID-19 CORONAVIRUS'. *J. Mech. Cont. & Math. Sci*., Vol.-16, No.-2, February (2021) pp 1-8.

II. Chakraborty, Indranil, and Prasenjit Maity. "COVID-19 outbreak: Migration, effects on society, global environment and prevention."Science of the Total Environment 728, (2020): 138882.

III. Cheng, H. D., Y.-H. Chen, and Y. Sun. "A novel fuzzy entropy approach to image enhancement and thresholding." Signal Processing 75, no. 3 (1999): 277-301.

IV. Ch. Vinay Kumar Reddy, Dr. I. Rajasri, Dr. Sammaiah3 , A. Rajesh, : 'FLATTEN THE CURVE AND RAISE THE LINE TO REDUCE THE RISK OF COVID-19'. *J. Mech. Cont.& Math. Sci*., Vol.-15, No.-8, August (2020) pp 1-7. DOI : 10.26782/jmcms.2020.08.00001.

V. De Luca, A., and S. Termini. "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory." In Readings in Fuzzy Sets for Intelligent Systems, (1993): pp. 197-202.

VI. De Luca, Aldo, and Settimo Termini. "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory." Information and control" 20, no. 4 (1972): 301-312.

VII. Dong, Liying, Shasha Hu, and Jianjun Gao. "Discovering drugs to treat coronavirus disease 2019 (COVID-19). "Drug discoveries & therapeutics" 14, no. 1 (2020): 58-60.

VIII. Dzitac, Ioan, Florin Gheorghe Filip, and Misu-Jan Manolescu. "Fuzzy logic is not fuzzy: World-renowned computer scientist Lotfi A. Zadeh. "International Journal of Computers Communications & Control"12, no. 6 (2017): 748-789.

IX. Grattan-Guinness, I. "Didier Dubois and Henri Prade. Fuzzy sets and systems. Theory and applications. Mathematics in science and engineering, vol. 144. Academic Press, New York etc. 1980, xvii 393 pp."The Journal of Symbolic Logic" 47, no. 3 (1982): 702-703.

X. H.J. Zimmermann. "Fuzzy sets Theory and its Applications." Springer Sciences & Business Media, (2011): 248-255.

XI. Horvath, Juraj. "Image segmentation using fuzzy c-means." In Symposium on Applied Machine Intelligence, (2006).

XII. Hussain, Zahid, and Miin-Shen Yang. "Entropy for hesitant fuzzy sets based on Hausdorff metric with construction of hesitant fuzzy TOPSIS." International Journal of Fuzzy Systems" 20, no. 8 (2018): 2517-2533.

XIII. H. wang, Chao-Ming, and Miin-Shen Yang. "On entropy of fuzzy sets." International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 16, no. 04 (2008): 519-527.

XIV. Kosko, Bart. "Fuzzy entropy and conditioning."Information sciences 40, no. 2 (1986): 165-174.

XV. Lee, K. H.“First course on fuzzy theory and applications.” Springer Science & Business Media,Vol. 27 (2004).

XVI. Lee, Kwang Hyung. “First course on fuzzy theory and applications.” Springer Science & Business Media, 27 (2004).

XVII. Li, Pingke, and Baoding Liu. "Entropy of credibility distributions for fuzzy variables." IEEE Transactions on Fuzzy Systems 16, no. 1 (2008): 123-129.

XVIII. Malone, David, and Wayne Sullivan. "Guesswork is not a substitute for entropy." (2005): 1-5.

XIX. Pal, Nikhil R., and Sankar K. Pal. "Some properties of the exponential entropy." Information sciences 66, no. 1-2 (1992): 119-137.

XX. Ramot, Daniel, Ron Milo, Menahem Friedman, and Abraham Kandel. "Complex fuzzy sets." *IEEE Transactions on Fuzzy Systems* 10, no. 2 (2002): 171-186.

XXI. Shannon, Claude E. "A mathematical theory of communication." The Bell system technical journal 27, no. 3 (1948): 379-423.

XXII. Tsallis, Constantino. "Beyond Boltzmann–Gibbs–Shannon in physics and elsewhere." Entropy 21, no. 7 (2019): 696.

XXIII. Wang, Cong, Witold Pedrycz, ZhiWu Li, MengChu Zhou, and Jun Zhao. "Residual-sparse fuzzy C-means clustering incorporating morphological reconstruction and wavelet frame." IEEE Transactions on Fuzzy Systems, (2020).

XXIV. Xuecheng, Liu. "Entropy, distance measure and similarity measure of fuzzy sets and their relations." Fuzzy sets and systems 52, no. 3 (1992): 305-318.

XXV. Xuecheng, Liu. "Entropy, distance measure and similarity measure of fuzzy sets and their relations." Fuzzy sets and systems 52, no. 3 (1992): 305-318.

XXVI. Yager, Ronald R. "On the measure of fuzziness and negation part I: membership in the unit interval." (1979): 221-229.

XXVII. Zadeh, Lotfi A. "Information and control." Fuzzy sets 8, no. 3 (1965): 338-353.